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By

John W. Nystrom

Prop^r

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Pocket-Book
OF
MECHANICS
AND
Engineering.

CONTAINING A MEMORANDUM OF FACTS

AND CONNECTION

OF

Practice and Theory.

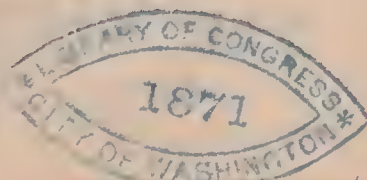
BY

JOHN W. NYSTROM, C. E.

Third Edition Revised, with additional matter.

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MECHANICS

AND

Entered according to Act of Congress in the year 1856, by

JOHN W. NYSTROM,

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CONTAINING A REVISION OF THE

AND CORRECTIONS

Practice and Theory.

STEREOTYPED BY S. DOUGLAS WYETH, No. 7, Pearl Street, Philad'a.

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PREFACE.

LET every Engineer make his own Pocket-Book, as he proceeds in study and practice, it will then suit his particular business; the present work was compiled in this way during a professional career of ten years. It was not originally intended for publication, but had grown so large in manuscript as to be inconvenient; this circumstance, combined with repeated requests to publish it, has lately placed it before the public

The Author hopes the introduction of Algebraical formulas instead of written rules will be favourably received; Written rules adopted by English Authors and Engineers are indeed excellent;—but formulas are better,—because, they not only tell what is to be done, but at a glance, impress the mind with the complete operation. If all the formulas in this book were explained in words, it would be far too large for the pocket.

It is not necessary to understand Algebra for the use of the formulas,—only practise the insertion of numerical values, and perform the arithmetical operations indicated by the particular formula used. The Author has furnished the formulas ready to receive what is given, and refund what is required.

ADVERTISEMENT.

THE undersigned is prepared to furnish Drawings and estimates for Propeller Steamers. He designs Engines and Propellers suitable for any desired description of Vessels. Furnishes Drawings of Vessels, with their whole internal arrangements distinctly shown in sections and details of any desired Scale.

Several years of experience in this profession enable him to furnish complete and *correct* Drawings on very short notice.

It is very important to have full and complete Drawings before the keel of the Vessel is laid, so as to insure unity of action between the Ship and Engine Builders, and to afford a clear understanding in reference to contracts.

The Drawings will contain all new and *useful* improvements.

JOHN W. NYSTROM,

Civil Engineer,

Letters will be promptly attended to. *Philadelphia.*

Mr. Nystrom, the Author of this Pocket-Book, has been in my employ since the year 1849, as Constructor and Draughtsman, in which capacity he has given me the greatest satisfaction. During that time my attention was constantly called to the readiness and accuracy of his calculations, which were made by means of a Pocket-Book then in manuscript. I have frequently requested him to publish it, and am now gratified by receiving its pages in print.

In it there is a Drawing of my Propeller, illustrating the expanding pitch first adopted by me, and now generally used. This is the principle upon which Propellers have been constructed, under my direction by Mr. Nystrom.

R. F. LOPER.

March, 1854.

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10. A series of sections of a leaf, and showing the structure of the same.

INTRODUCTION.

Quantity is that which can be increased or diminished by augments or abatements of homogeneous parts. Quantities are of two essential kinds, *Geometrical* and *Physical*.

1st, *Geometrical* quantities are those which occupy space; as *lines, surfaces, solids, liquids, gases, &c.*

2nd, *Physical* quantities are those which exist in the time but occupy no space, they are known by their character and action upon geometrical quantities; as *attraction, light, heat, electricity and magnetism, colours, force, power, &c. &c.*

To obtain the magnitude of a quantity we compare it with a part of the same, this part is imprinted in our mind as a *unit*, by which the whole is measured and conceived. No quantity can be measured by a quantity of another kind, but any quantity can be compared with any other quantity, and by such comparison arises what we call *calculation* or *Mathematics*.

MATHEMATICS.

Mathematics is a science by which the comparative value of quantities are investigated; it is divided into:

1st, **Arithmetic**,—that branch of Mathematics, which treats of the nature and property of numbers; it is subdivided into *Addition, Subtraction, Multiplication, Division, Involution, Evolution and Logarithms*.

2nd, **Algebra**,—that branch of Mathematics which employs letters to represent quantities, and by that means performs solutions without knowing or noticing the *value* of the quantities. The subdivisions of Algebra are the same as in Arithmetic.

3rd, **Geometry**,—that branch of Mathematics which investigates the relative property of quantities that occupies space; its subdivisions are *Longemetry, Planemetry, Stereometry, Trigonometry, and Conic Sections*.

4th, **Differential-calculus**,—that branch of Mathematics, which ascertains the mean effect, produced by group of continued variable causes.

5th, **Integral-calculus**,—the contrary of Differential, or that branch of Mathematics which investigates the nature of a continued variable cause, that has produced a known effect.

ARITHMETIC.

The art of manœuvring numbers, and to investigate the relationship of quantities.

Figures—1, 2, 3, 4, 5, 6, 7, 8, 9. Arabic dignets, about nine hundred years old.
Ciphers—0, 0, 0. Sometimes called noughts, it is the beginning of figures and things.

Number is the expression of one or more figures and ciphers.

Integer is a whole number or unit.

Fraction is a part of a number or unit.

When figures are joined together in a number, the relative dignity expressed by each figure, depends upon its position to the others. Thus,

Quadrillions.

Trillions.

Billions.

Millions.

Thousands.

Hundreds.

Tens.

Units.

674,385 ; 496,345 ; 695,216 ; 505,310 : 685,367 ;

Notation is the setting down of any number by figures and ciphers.

Numeration is the reading of any number in words, from the expression of figures and ciphers.

Characters which describe the operation by numbers (significations).

= Equality, as $6=6$, reads 6 is equal to 6.

+ Plus, Addition, - - - - - $3+6=9$

— Minus, Subtraction, - - - - - $6-2=4$

× Multiplication, - - - - - $3 \times 4 = 12$

÷ or : Division, - - - - - $15 : 5 = 3$

√ Square root, - - - - - $\sqrt{9} = 3$

∛ Cube root, - - - - - $\sqrt[3]{8} = 2$

> Greater, - - - - - $8 > 4$

< Less, - - - - - $6 < 9$

ROMAN NOTATION.

The Romans expressed their numbers by various repetitions and combinations of seven letters in the alphabet; as,

1=I.

2=II.

3=III.

4=IV.

5=V.

6=VI.

7=VII.

8=VIII.

9=IX

10=X.

20=XX.

30=XXX.

40=XL.

50=L.

60=LX.

70=LXX.

80=LXXX.

90=XC.

100=C.

500=D, or L \bar{D} .

{ As often as a character is repeated,
so many times is its value repeated.

{ A less character before a greater
diminishes its value, as IV=I from
V, or 1 subtracted from 5=4.

{ A less character after a greater in-
creases its value, as XI=X+I, or
10+1=11.

1,000=M, or C \bar{C} .

2,000=MM, or II \bar{C} .

5,000= \bar{V} , or L \bar{C} .

6,000= \bar{VI} , or MM \bar{C} .

10,000= \bar{X} , or C \bar{C} .

50,000= \bar{L} , or L \bar{C} .

60,000= \bar{LX} , or MM \bar{C} .

100,000= \bar{C} , or C \bar{C} .

1,000,000= \bar{M} , or C \bar{C} .

2,000,000=MM \bar{C} , or MM \bar{C} .

{ For every \bar{C} annexed, this becomes
10 times as many.

A bar, thus, — over any number, increases
it 1000 times.

EXAMPLES.—1854, MDCCCLIV.

524,365, DXXTVCCCLXV.

An imperfection in the Roman Notation consists in that, there is no signification for the cipher as in the Arabic notation

ALGEBRA.

In **Algebra** we employ certain *characters* or *letters* to represent quantities. These characters are separated by *signs*, which describe the operations; and by that means, simplify the solution.

1. Whatever the value of *any* quantity may be, it can be represented by a character, as *a*. Another quantity of the same kind, but of different value, being represented by *b*. The sum of these two quantities is of the same kind but of different value.

For **Addition** we have the algebraical sign $+$, (plus) which, when placed between quantities, denotes they shall be added; as $a+b$, reads in the algebraical language, "*a* plus *b*," or *a* is to be added to *b*.

Another algebraical sign $=$, (Equal) denotes that quantities which are placed on each side of this sign, are *equal*. Let the sum of *a* and *b* be denoted by the letter *c*; then we have,

$$a+b=c.$$

This composition is called an *algebraical equation*. The quantity on each side of the equal sign is called a *member*, as $a+b$, is one member, and *c*, the other. When one of the members contains only one quantity, that member is generally placed on the first side of the equal sign, and its value commonly unknown; but the value of the quantities in the other member being given, as $a=4$, and $b=5$, then the practical mode, to insert numerical values in algebraical equations, will appear; as,

$$\text{Equation, } c=a+b, \\ 4+5=9, \text{ the value of } c.$$

2. The sum of three quantities *a*, *b*, and *c*, is equal to *d*, then

$$\text{Equation, } d=a+b+c, \\ 4+5+9=18, \text{ the value of } d.$$

3. For **Subtraction** we have the algebraical sign, $-$, (minus) which, when placed before a quantity, denotes it is to be subtracted as, $a-b$, reads in the algebraical language "*a* minus *b*," or from *a*, subtract *b*. Let the difference be denoted by the letter *c*; and $a=8$. $b=3$

$$\text{Equation, } c=a-b, \\ 8-3=5, \text{ the value of } c.$$

4. From the sum of *a* and *b*, subtract *c*, and the result will be *d*; then,

$$\text{Equation, } d=a+b-c, \\ 8+3-5=6, \text{ the value of } d.$$

5. When two equal quantities are to be added, as $a+a$, it is the same as to take one of them twice, and is marked thus $2a$. The number 2 is called the coefficient of the quantity *a*. If there are more than two equal quantities to be added, the coefficient denotes how many there are of them; as,

$$\begin{aligned} \text{Equation, } & a+a=2a, \\ \text{" } & a+a+a=3a, \\ \text{" } & a+a+a+a=4a, \\ & \text{etc., etc.} \end{aligned}$$

When the quantities are separated by the signs, *plus*, or *minus*, they are called *terms*.

6. **Multiplication**.—When a quantity *a*, is to be multiplied by another quantity *b*, then *a* and *b* are called *factors*; and separated by no sign as ab ; which denotes that *a* is to be multiplied by *b*; but when the values of *a* and *b* are expressed by numbers, they are separated by the sign \times (Multiplication); the result from Multiplication is called the *product*. Let $a=8$, and $b=6$, and the product of *a* and *b*, to be *c*, then,

$$\text{Equation, } c=ab, \\ 8 \times 6=48, \text{ the value of } c.$$

7. The product of *a* and *b*, is to be multiplied by *c*, and the latter product will be equal to *d*; then,

$$\text{Equation, } d=abc, \\ 8 \times 6 \times 48=2304, \text{ the value of } d.$$

8. The sum of a and b , is to be multiplied by c , and the product will be d ; then,

$$\text{Equation, } d = c(a+b), \\ 48(8+6) = 672 \text{ the value of } d.$$

When the sum of two or more quantities is to be multiplied by another quantity, the sum is to be enclosed in parentheses, and denotes itself to be one factor. The other factor is to be placed on the outside of the parentheses, as seen in the preceding example.

9. To the product of a and c , add b , and the result will be d ; then,

$$\text{Equation, } d = ac + b, \\ 8 \times 48 + 6 = 390 \text{ the value of } d.$$

Be particular to distinguish the two Examples 8, and 9.

10. The sum of a and b , to be multiplied by the sum of a and c ; the product will be d ; then,

$$\text{Equation, } d = (a+b)(a+c), \\ (8+6)(8+48) = 784.$$

11. The sum of c and b , to be multiplied by the difference of c and a ; the result will be d ; then,

$$\text{Equation, } d = (c+b)(c-a), \\ (48+6)(48-8) = 2160.$$

12. Division.—When a quantity a , is to be separated into b equal parts, the numbers of parts or b , is called the *divisor*, and the value of each part, is called the *quotient*. The sum of the parts or the whole quantity a , is called the *dividend*; a and b , is separated by the sign : (Division); as $a : b$, reads in the algebraical language, “ a divided by b .” Let the *quotient* be denoted by the letter c ; and $a=18$, $b=6$, then,

$$\text{Equation, } c = a : b, \\ 18 : 6 = 3 \text{ the quotient } c.$$

In Algebra it is found more convenient to set up Division as a fraction, then it will appear as,

13. Divide a , by c , and the quotient will be b . Then,

$$\text{Equation, } b = \frac{a}{c}, \\ \frac{18}{3} = 6 \text{ the quotient } b.$$

14. The product of a and b , to be divided by c ; and the product will be d . Then,

$$\text{Equation, } d = \frac{ab}{c}, \\ \frac{18 \times 6}{3} = 36.$$

15. The sum of d and b , to be multiplied by c , and the product divided by a ; then the result will be e .

$$\text{Equation, } e = \frac{c(d+b)}{a}, \\ \frac{3(36+6)}{18} = 7.$$

16. From the product of a and c , subtract $3b$; divide the remainder by the difference of a , and c ; the result will be h .

$$\text{Equation, } h = \frac{ac-3b}{a-c},$$

$$\frac{18 \times 3 - 3 \times 6}{18 - 3} = 2.4.$$

An old man said to a smart boy, "How old are you?" to which he replied.—
 "To seven times my father's age add yours, divide the sum by double the
 difference of yours and his, and the result will be my age."

Letters will denote,
 a = the old man's age,
 b = the father's age,
 c = the boy's age. Then,

$$\text{Equation, } c = \frac{7b+a}{2(a-b)} \text{ the boy's age.}$$

Now for any number of years of the old man and the father, will be a corresponding age of the boy; suppose,

a = 73 years the age of the old man,

b = 57 years the father's age.

Require the boy's age.

$$c = \frac{7 \times 57 + 73}{2(73 - 57)} = 14\frac{1}{2} \text{ years.}$$

PROPORTION.

THE *relative value* of two quantities, is obtained by dividing one into the other, and the quotient is called the *ratio* of their relationship. If the ratio of two quantities is equal to the ratio of two other quantities, they are said to be in the same proportion; as,

$$a : b = c : d,$$

reads in the algebraical language " a is to b as c is to d ."— a , b , c , and d , are called *terms*, of which a is the first, b the second, c the third, and d the fourth *term*. The first and fourth are called "*the outer terms*," and the second and third, "*the inner terms*." The whole is called an "*analogy*."

A property in the nature of analogies is, that the product of the outer terms ad , is equal to the product of the inner bc . Suppose $a = 4$, $b = 9$, $c = 12$, $d = 27$.

$$4 : 9 = 12 : 27,$$

$$ad = bc, \quad 4 \times 27 = 9 \times 12.$$

If any one of the four quantities are unknown, its value can be calculated by the other three; as,

$$a = \frac{bc}{d} = \frac{9 \times 12}{27} = 4,$$

$$b = \frac{ad}{c} = \frac{4 \times 27}{12} = 9,$$

$$c = \frac{ad}{b} = \frac{4 \times 27}{9} = 12,$$

$$d = \frac{bc}{a} = \frac{9 \times 12}{4} = 27.$$

To Alternate a Proportion.

If $a : b = c : d$,
 then $a : c = b : d$,
 and $ad = bc$.

To Inverse a Proportion.

If $a : b = c : d$,
 then $b : a = d : c$,
 and $bc = ad$.

To Multiply a Proportion.

If $a : b = c : d$,
 then $na : nb = nc : nd$,
 and $\frac{a}{n} : \frac{b}{n} = \frac{c}{n} : \frac{d}{n}$.

To Reduce a Proportion.

If $a : b = c : d$,
 then $na : nb = mc : md$,
 and $\frac{a}{n} : \frac{b}{n} = \frac{c}{m} : \frac{d}{m}$.

Compared Proportions.

If $a : b = c : d$,
 and $c : d = e : f$,
 then $a : b = e : f$.

Continued Proportion.

If $a : b = c : d = e : f$,
 then $af = be = cd$,
 and $ad = bc, cf = de$.

To Compound Proportions.

If $a : d = c : d$,
 and $e : f = g : h$,
 then $ae : df = cg : dh$.

To Compare Proportions.

If $a : b = c : d$,
 and $e : f = g : h$,
 then $\frac{a}{e} : \frac{b}{f} = \frac{c}{g} : \frac{d}{h}$.

To Combine a Proportion.

If $a : b = c : d$,
 then $(a+b) : b = (c+d) : d$,
 and $d(a+b) = b(c+d)$.

To Combine a Proportion Inversely.

If $a : b = c : d$,
 then $a : b = (a+c) : (b+d)$,
 and $a(b+d) = b(a+c)$.

To Dissolve a Proportion.

If $a : b = c : d$,
 then $(a-b) : b = (c-d) : d$,
 and $d(a-b) = b(c-d)$.

To Dissolve a Proportion Inversely.

If $a : b = c : d$,
 then $a : b = (a-c) : (b-d)$,
 and $a(b-d) = b(a-c)$.

If $a : b = d : c$,
 then $a : b = \frac{1}{c} : \frac{1}{d}$,
 and $\frac{1}{a} : \frac{1}{b} = c : d$,
 and $ac = bd$.

To Find the mean Proportion.

$a : x = x : b$
 then $x = \sqrt{ab}$,

x is the mean Proportion.

Proportion of Square and Square Root.

If $a : b = c : d$,
 then $a^2 : b^2 = c^2 : d^2$,
 and $\sqrt{a} : \sqrt{b} = \sqrt{c} : \sqrt{d}$.

Proportion of any Power or Root.

If $a : b = c : d$,
 then $a^n : b^n = c^n : d^n$,
 and $\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{c} : \sqrt[n]{d}$.

SIMPLE INTEREST.

Interest is a profit on money which is lent for a certain time.

Letters will denote.

c = the standing capital, or lent money.

r = interest on the capital c ,

p = per cent. on 100 in the certain time.

Analogy, $c : r = 100 : p.$

If p is the per cent. on 100, in one year, then t = time in years for the standing capital c , and the interest r .

Analogy, $c : r = 100 : pt.$

From this analogy we obtain the equations,

Interest, $r = \frac{cpt}{100}, \quad - \quad - \quad - \quad - \quad 1,$

Per cent., $p = \frac{100 r}{tc}, \quad - \quad - \quad - \quad - \quad 2,$

Capital, $c = \frac{100 r}{pt}, \quad - \quad - \quad - \quad - \quad 3,$

Time in years, $t = \frac{100 r}{cp}, \quad - \quad - \quad - \quad - \quad 4.$

Now for any question in Simple Interest, there is one equation which gives the answer. If the time is given in *months*, *weeks*, or *days*, multiply the 100 correspondingly by 12, 52, 365.

Example 1. What is the interest on \$3789.35, for 3 years and five months, at 6 per cent. per annum?

$t = 3 \times 12 + 5 = 41$ months, from the Equation 1, we have,

Interest, $r = \frac{3789.35 \times 6 \times 41}{12 \times 100} = 776.81 \text{ Dollars.}$

Example 2. A capital $c = \$469.78$, gave an interest $r = 150.72$ dollars, in a time $t = 4$ years and 7 months. Require the per centage per annum?

$t = 4 \times 12 + 7 = 55$ months, from Equation 2, we have,

Per cent., $p = \frac{12 \times 100 \times 150.72}{469.78 \times 55} = 7 \text{ per cent.}$

Example 3. What capital is required to give an interest $r = 345$ Dollars in 6 years, at 5 per cent. per annum? From the Equation 3, we have,

Capital, $c = \frac{100 \times 345}{5 \times 6} = \$1150.$

Example 4. A capital $c = \$2365$ shall stand until the interest will be $r = 550$ Dollars, at $p = 6$ per cent. per annum. How long must the capital stand?

From the Equation 4, we have,

Time, $t = \frac{100 \times 550}{2365 \times 6} = 3.876 \text{ years.}$

$12 \times 0.876 = 10.512$ months, $4 \times 0.512 = 2.048$ weeks, the time $t = 3$ years, 10 months, and 2 weeks.

REBATE OR DISCOUNT.

Rebate or Discount is an allowance on money, which is paid before due. a = amount of money to be paid in the time t . By agreement the amount is paid with a capital c , at the beginning of the time t , but discounted a Rebate r , at p per cent, so that the interest on the capital c , at p per cent., should be equal to the Rebate r , in the time t .

$$a = c + r.$$

$$\text{Rebate,} \quad r = \frac{apt}{100 + pt}, \quad . \quad . \quad . \quad . \quad . \quad 5,$$

$$\text{Capital,} \quad c = \frac{100}{100 + pt}, \quad . \quad . \quad . \quad . \quad . \quad 6,$$

$$\text{Per cent.,} \quad p = \frac{100(a - c)}{ct}, \quad . \quad . \quad . \quad . \quad . \quad 7,$$

$$\text{Time,} \quad t = \frac{100(a - c)}{cp}, \quad . \quad . \quad . \quad . \quad . \quad 8,$$

$$\text{Amount} \quad a = \frac{c}{100}(100 + pt), \quad . \quad . \quad . \quad . \quad . \quad 9,$$

$$\text{Amount,} \quad a = \frac{r}{pt}(100 + pt), \quad . \quad . \quad . \quad . \quad . \quad 10.$$

Now for any question in Rebate or Discount, there is one equation that will give the answer.

Example 5. A sum of money, $a = 78460$ dollars is to be paid after 3 years and 6 months, but by agreement payment is to be made at the present time. What will be the Rebate at 7 per cent.

$$\text{Rebate,} \quad r = \frac{78460 \times 7 \times 3.5}{100 + 7 \times 3.5} = \$15439.91.$$

FELLOWSHIP.

Fellowship, or *partnership*, is a rule by which companies ascertain each fellow's profit or loss, by their stock. Each fellow's part in the stock is called his *share*. The sum of shares is called the *stock*.

Fellowships are of two kinds, *Simple* and *Double*.

Simple Fellowship, when there is no regard to the time, the shares or stock is employed.

Letters will denote,

A = share of either one fellow.

a = profit or loss on the share A .

S = stock or the sum of the shares.

s = gain or loss on the stock S .

$$\text{Then} \quad A : a = S : s.$$

$$\text{Share,} \quad A = \frac{aS}{s}, \quad . \quad . \quad . \quad . \quad . \quad 11,$$

$$\text{Profit or loss,} \quad a = \frac{As}{S}, \quad . \quad . \quad . \quad . \quad . \quad 12,$$

$$\text{Stock,} \quad S = \frac{As}{a}, \quad . \quad . \quad . \quad . \quad . \quad 13,$$

$$\text{Gain or loss,} \quad s = \frac{aS}{A}, \quad . \quad . \quad . \quad . \quad . \quad 14.$$

Example 1. A person had invested $A = \$11645$, in a stock $S = \$64800$, which gave a gain of $s = 13864$. What will be the profit of the person's share?

$$\text{Profit, } a = \frac{11645 \times 13864}{64800} = \$2491.45$$

Double Fellowship. When the different shares are employed at a different length of time, each share is multiplied by its time employed, and the product is the *effect* of the share.

Letters will denote,

t = time for the employed share A .

T = meantime for the employed stock S .

e = effect of the share A .

a = profit of the effect e .

E = effect of the stock.

s = gain of the effect E . Then,

$$e : a = E : s.$$

Effect of A , $e = \frac{aE}{s}$, 15,	Time, $t = \frac{aE}{As}$, 19,
Profit of e , $a = \frac{es}{E}$, 16,	Share, $A = \frac{aE}{ts}$, 20,
Effect of S , $E = \frac{es}{a}$, 17,	Meantime, $T = \frac{es}{aS}$, 21,
Gain of E , $s = \frac{aE}{e}$, 18,	Stock, $S = \frac{es}{aT}$, 22.

Example 2. A canal is to be dug, and requires an effect $E = 76850$ (men and days) to be accomplished; after that it will give a gain $s = 12390$ Dollars. An employer has $A = 168$ laborers. How many days must those laborers be employed at the canal, that the employer will obtain a profit $a = \$5000$?

$$\text{Time, } t = \frac{5000 \times 76850}{168 \times 12390} = 184.6 \text{ days.}$$

PERMUTATION.

Permutation is to arrange a number of things in every possible position. It is commonly used in *games*.

Example 1. How many different values can be written by the three ciphers 1, 2, 3.

$1 \times 2 \times 3 = 6$ different values, namely,

123, 132, 213, 231, 312, 321.

With any three different ciphers can be written six different values. Any three things can be placed in 6 different positions.

Example 2. How many names can be written by the three syllables *mo, ta, la*? The answer is,—Motala, Molata, Tamola, Talamo, Lamota, Latamo.

Example 3. How many words can be written by the five syllables, *mul, tip, li, ca, tion*?

$$1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ words, the answer.}$$

$n - p$

$1 = 1$

$2 = 2$

$3 = 6$

$4 = 24$

$5 = 120$

$6 = 720$

$7 = 5040$

$8 = 40320$

$9 = 362880$

$10 = 3628800$

$11 = 39916800$

$12 = 479001600$

$13 = 6227020800$

$14 = 87178291200.$

The accompanying table shows the permutation of different numbers of things up to 14; which will be convenient in the next coming examples in combination

COMBINATION.

Combination is to arrange a less number of things out of a greater, in every possible position. — It is commonly used in *games*.

Example 1. How many different numbers can be set up by the nine ciphers, 1, 2, 3, 4, 5, 6, 7, 8, 9, and three ciphers in each number?

$$\frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84 \text{ different numbers.}$$

Example 2. How many different variations can a player obtain his cards, when the set contains 52 cards, of which he receives 8 at a time?

$$\frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} = 752538150 \text{ variations.}$$

If they are four players, and *pr.* 4 = 24, they can play $24 \times 752538150 = 18,060,915,600$ different plays.

If it takes half an hour for each play, and they play 8 hours per day, it will take

$$\frac{18060915600}{2 \times 8} = 1128807225 \text{ days} = 3,092,622 \text{ years.}$$

ALLIGATION.

Alligation is to mix together a number of different things of different price or value, and ascertain the mean value of the mixture; or from a given mean value of a mixture ascertain the proportion and value of each ingredient.

Let the different things be *a, b, c, and d, &c.*, their respective price or value per unit, *z, y, x, and w, &c.*

$A = a + b + c + d$ &c., the sum of the things.

$P =$ mean value or price per unit of *A*. Then,

$$AP = az + by + cx + dw + \&c., \quad - \quad - \quad - \quad - \quad - \quad 1,$$

and

$$P = \frac{az + by + cx + dw + \&c.}{A}, \quad - \quad - \quad - \quad - \quad - \quad 2,$$

Example 1. If 3 gallons of wine at \$1.37 per gallon, 2 at \$2.18, and 5 at \$1.75, be mixed together, what is a gallon worth of the mixture?

$$A = 3 + 2 + 5 = 10 \text{ gallons.}$$

$$P = \frac{3 \times 1.37 + 2 \times 2.18 + 5 \times 1.75}{10} = \$1.72 \text{ per gallon.}$$

Alligation of two ingredients a and b, with their respective prices or value per unit, z and y,

$$z > P > y. \quad A = a + b.$$

$$a : b = (P - y) : (z - P) \quad \dots \quad 3,$$

$$a = \frac{b(P - y)}{(z - P)}, \quad \text{and } a = \frac{A(P - y)}{(z - y)}, \quad \dots \quad 4, \text{ \& 5,}$$

Example 2. A Silver-smith will mix two sorts of silver, one at 54 and one at 64 cents per ounce. How much must be taken of each sort to make the mixture worth 60 cents per ounce. (Formula 3.) $P = 60. \quad x = 54. \quad y = 64.$

$$a : b = (60 - 54) : (64 - 60) = 6 : 4, \text{ or,}$$

4 ounces at 54 cents; and 6 ounces at 64 cents.

Alligation of three ingredients, a, b, and c, with their prices or value per unit, z, y and x.

$$a' : c' = (P - x) : (z - P) \quad \dots \quad 6,$$

$$a'' : b = (P - y) : (z - P) \text{ when } z > P > y > x, \quad \dots \quad 7,$$

$$b : c'' = (P - x) : (y - P) \text{ when } z > y > P > x, \quad \dots \quad 8,$$

$$a = a' + a'', \quad c = c' + c''.$$

Example 3. A Farmer will mix wheat at 94 cents per bushel, with barley at 72 cents, rye at 64 cents per bushel. How much of each sort must be taken to make the mixture worth 80 cents per bushel?

(Formula 6.) $z = 94, \quad y = 72, \quad x = 64, \text{ and } P = 80.$

$$a' : c' = (80 - 64) : (94 - 80) = 16 : 14,$$

$$a' : b = (80 - 72) : (94 - 80) = 8 : 14.$$

The wheat $a = 16 + 8 = 24$ bushels at 94 cents per bushel.

“ barley $b = 14$ “ “ 72 “ “

“ rye $c = 14$ “ “ 64 “ “

Alligation of four ingredients a, b, c, and d, respective prices or value per unit; z, y, x, and w.

$$\left. \begin{array}{l} a : d = (P - w) : (z - P) \\ b : c = (P - y) : (x - P) \end{array} \right\} \text{ when } z > y > P > x > w. \quad \left\{ \begin{array}{l} 9, \\ 10, \end{array} \right.$$

$$\left. \begin{array}{l} a' : d = (P - w) : (z - P) \\ a'' : b = (P - y) : (z - P) \\ a''' : c = (P - x) : (z - P) \end{array} \right\} \text{ when } z > P > y > x > w. \quad \left\{ \begin{array}{l} 11, \\ 12, \\ 13, \end{array} \right.$$

$$a = a' + a'' + a''',$$

$$\left. \begin{array}{l} a : d' = (P - w) : (z - P) \\ b : d'' = (P - w) : (y - P) \\ c : d''' = (P - w) : (x - P) \end{array} \right\} \text{ when } z > y > x > P > w. \quad \left\{ \begin{array}{l} 14, \\ 15, \\ 16, \end{array} \right.$$

$$d = d' + d'' + d''',$$

In the same manner, formulæ can be set up for any number of ingredients.

INVOLUTION.

Involution is to multiply a number into itself a number of times; each product is called the *power* of the number, and the *dignity* of the power is marked by a small figure called *exponent*, on the right of the number; thus,

$$a \times a = a^2 \text{ the square of } a.$$

$$a \times a \times a = a^3 \text{ the cube of } a.$$

$$a \times a \times a \times a = a^4 \text{ the bisquare of } a.$$

$$a \times a \times a \times a \times a = a^5 \text{ the fifth power of } a.$$

etc.,

etc.,

etc.

$$3^2 = 3 \times 3 = 9.$$

$$2^3 = 2 \times 2 \times 2 = 8.$$

$$4^4 = 4 \times 4 \times 4 \times 4 = 256.$$

etc.,

etc.,

etc.

Binome is a factor or quantity which contains *two terms*; as $(a + b)$

Binomial-Theorem is the rule which a binome follows, when it is raised to any power.

When a binome is to be multiplied by itself or any other binome, it is set up and performed like the common multiplication by numbers; thus,

$$\text{Example 1.} \quad \begin{array}{r} a+b \\ a+b \end{array} \} (a+b)(a+b),$$

$$\begin{array}{r} +ab+b^2 \\ a^2+ab \end{array} \quad \parallel$$

$$\frac{a^2+2ab+b^2}{a^2+2ab+b^2} = (a+b)^2.$$

Example 2. Suppose $a+b = 358746$, and $a = 358000$, $b = 746$, then,

$$(a+b)^2 = 358746^2,$$

$$\begin{array}{r} \parallel \\ +a^2 = 128164000000 \end{array}$$

$$\begin{array}{r} \parallel \\ +2ab = 534136000 \end{array}$$

$$\begin{array}{r} \parallel \\ +b^2 = 556516 \end{array}$$

$$128698692516 = (a+b)^2.$$

$$\text{Ex. 3. } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

$$\text{Ex. 4. } (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$

$$\text{Ex. 5. } (a-b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - a^2b^5 + 7ab^6 - b^7.$$

Here you will discover the peculiarities of the *Binomial-Theorem*, which is thus expressed in words:

1st. The *exponent* of the first term a in the power, is equal to the exponent of the binome; and in every successive term, the exponent of a is decreased by 1, until the last term of the exponent of a is 0, and, therefore, disappears, because any quantity raised to no power is equal to 1, thus, $a^0 = 1$ and $a^1 = a$.

2d. In the first term of the power, the exponent of b is 0, and therefore b will first appear in the second term with the exponent 1, and in every successive term the exponent of b is increased by 1, until in the last term the exponent will be equal to the exponent of the binome.

3d. The coefficient of the second term in the power, is equal to the exponent of the binome, and the coefficient of any successive term is equal to the product of the coefficient and exponent of a in the foregoing term, divided by the number of terms before the sought coefficient.

4th. When the second term in the binome is negative, the first term in the power will be positive, the second negative, the third positive, the fourth negative, &c., &c. The odd terms are positive, and the even terms are negative.

5th. The number of terms in the power is *one more* than the exponent of the binome.

EVOLUTION.

Evolution is the reverse of Involution, or to find the number that has produced a given power. In this case the given power is called the *number*, and the number which has produced the given power is called the *root* of the number. The symbol $\sqrt{\quad}$ is generally placed over the number of which the root is to be extracted. The dignity of the root is placed thus $\sqrt[3]{\quad}$ of which the figure 3 is called the index of the root; for the square roots the index 2 is always omitted.

$$\text{Example 1.} \quad \sqrt{9} = 3 \quad \text{because } 3^2 = 9.$$

$$\text{" } 2. \quad \sqrt[3]{64} = 4 \quad \text{because } 4^3 = 64.$$

$$\text{" } 3. \quad \sqrt[4]{531441} = 27 \quad \text{" } 27^4 = 531441.$$

dc.,

dc.,

In the accompanying Table are calculated the *Squares*, *Cubes*, *Square Roots* and *Cube Roots* of any number up to 1600. By means of this Table, there will be easy rules to find the *Square Root* and *Cube Root* of numbers exceeding 1600.

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
1	1	1	1.0000000	1.0000000	.100000000
2	4	8	1.4142136	1.2599210	.500000000
3	9	27	1.7320508	1.4422496	.333333333
4	16	64	2.0000000	1.5874011	.250000000
5	25	125	2.2360680	1.7099759	.200000000
6	36	216	2.4494897	1.8171206	.166666667
7	49	343	2.6457513	1.9129312	.142857143
8	64	512	2.8284271	2.0000000	.125000000
9	81	729	3.0000000	2.0800837	.111111111
10	100	1000	3.1622777	2.1544347	.100000000
11	121	1331	3.3166248	2.2239801	.090909091
12	144	1728	3.4641016	2.2894286	.083333333
13	169	2197	3.6055513	2.3513347	.076928077
14	196	2744	3.7416574	2.4101422	.071428571
15	225	3375	3.8729833	2.4662121	.066666667
16	256	4096	4.0000000	2.5198421	.062500000
17	289	4913	4.1231056	2.5712816	.058823529
18	324	5832	4.2426407	2.6207414	.055555556
19	361	6859	4.3588989	2.6684016	.052631579
20	400	8000	4.4721360	2.7144177	.050000000
21	441	9261	4.5825757	2.7589243	.047619048
22	484	10648	4.6904158	2.8020393	.045454545
23	529	12167	4.7958315	2.8438670	.043478261
24	576	13824	4.8989795	2.8844991	.041666667
25	625	15625	5.0000000	2.9240177	.040000000
26	676	17576	5.0990195	2.9624960	.038461538
27	729	19683	5.1961524	3.0000000	.037037037
28	784	21952	5.2915026	3.0365889	.035714286
29	841	24389	5.3851648	3.0723168	.034482759
30	900	27000	5.4772256	3.1072325	.033333333
31	961	29791	5.5677644	3.1413806	.032258065
32	1024	32768	5.6568542	3.1748021	.031250000
33	1089	35937	5.7445626	3.2075343	.030303030
34	1156	39304	5.8309519	3.2396118	.029411765
35	1225	42875	5.9160798	3.2710663	.028571429
36	1296	46656	6.0000000	3.3019272	.027777778
37	1369	50653	6.0827625	3.3322218	.027027027
38	1444	54872	6.1644140	3.3619754	.026315789
39	1521	59319	6.2449980	3.3912114	.025641026
40	1600	64000	6.3245553	3.4199519	.025000000
41	1681	68921	6.4031242	3.4482172	.024390244
42	1764	74088	6.4807407	3.4760266	.023809524
43	1849	79507	6.5574385	3.5033981	.023255814
44	1936	85184	6.6332496	3.5303483	.022727273
45	2025	91125	6.7082039	3.5568933	.022222222
46	2116	97336	6.7823300	3.5830179	.021739130
47	2209	103823	6.8556546	3.6088261	.021276600
48	2304	110592	6.9282032	3.6342411	.020833333
49	2401	117649	7.0000000	3.6593057	.020408163
50	2500	125000	7.0710678	3.6840314	.020000000
51	2601	132651	7.1414284	3.7084298	.019607843
52	2704	140608	7.2111026	3.7325111	.019230769

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
53	2809	148877	7.2801099	3.7562858	.018867925
54	2916	157464	7.3484692	3.7797631	.018518519
55	3025	166375	7.4161985	3.8029525	.018181818
56	3136	175616	7.4833148	3.8258624	.017857143
57	3249	185193	7.5498344	3.8485011	.017543860
58	3364	195112	7.6157731	3.8708766	.017241379
59	3481	205379	7.6811457	3.8929965	.016949153
60	3600	216000	7.7459667	3.9148676	.016666667
61	3721	226981	7.8102497	3.9304972	.016393443
62	3844	238328	7.8740079	3.9578915	.016129032
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0412401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314432	8.2462113	4.0816551	.014705882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8.4261498	4.1408178	.014084517
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013698630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444815	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2736185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4470692	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778688	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857374	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009708738
104	10816	1124864	10.1980390	4.7026694	.009615385

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
105	11025	1157625	10.2469508	4.7176940	.009523810
106	11236	1191016	10.2956301	4.7326235	.009432962
107	11449	1225043	10.3440804	4.7474594	.009345794
108	11664	1259712	10.3923048	4.7622032	.009259259
109	11881	1295029	10.4403065	4.7768562	.009174312
110	12100	1331000	10.4880885	4.7914199	.009090209
111	12321	1367631	10.5356538	4.8058995	.009009009
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008849558
114	12996	1481544	10.6770783	4.8488076	.008771930
115	13225	1520875	10.7238053	4.8629442	.008695652
116	13456	1560896	10.7703296	4.8769990	.008620690
117	13689	1601613	10.8166538	4.8909732	.008547009
118	13924	1643032	10.8627805	4.9048681	.008474576
119	14161	1685159	10.9087121	4.9186847	.008403361
120	14400	1728000	10.9544512	4.9324242	.008333333
121	14641	1771561	11.0000000	4.9460874	.008264463
122	14834	1815848	11.0453610	4.9596757	.008196721
123	15129	1860867	11.0905365	4.9731898	.008130081
124	15376	1906624	11.1355287	4.9866310	.008064516
125	15625	1953125	11.1803399	5.0000000	.008000000
126	15876	2000376	11.2249722	5.0132979	.007936508
127	16129	2048383	11.2694277	5.0265257	.007874016
128	16384	2097152	11.3137085	5.0396842	.007812500
129	16641	2146689	11.3578167	5.0527743	.007751938
130	16900	2197000	11.4017543	5.0657970	.007692308
131	17161	2248091	11.4455231	5.0787531	.007633588
132	17424	2299968	11.4891253	5.0916434	.007575758
133	17689	2352637	11.5325626	5.1044687	.007518797
134	17956	2406104	11.5758369	5.1172299	.007462687
135	18225	2460375	11.6189500	5.1299278	.007407407
136	18496	2515456	11.6619038	5.1425632	.007352941
137	18769	2571353	11.7046999	5.1551367	.007299270
138	19044	2628072	11.7473444	5.1676493	.007246377
139	19321	2685619	11.7898261	5.1801015	.007194245
140	19600	2744000	11.8321596	5.1924941	.007142857
141	19881	2803221	11.8743421	5.2048279	.007092199
142	20164	2863288	11.9163753	5.2171034	.007042254
143	20449	2924207	11.9582607	5.2293215	.006993007
144	20736	2985984	12.0000000	5.2414828	.006944444
145	21025	3048625	12.0415946	5.2535879	.006896552
146	21316	3112136	12.0830460	5.2656374	.006849315
147	21609	3176523	12.1243557	5.2776321	.006802721
148	21904	3241792	12.1655251	5.2895725	.006756757
149	22201	3307949	12.2065556	5.3014592	.006711409
150	22500	3375000	12.2474487	5.3132928	.006666667
151	22801	3442951	12.2882057	5.3250740	.006622517
152	23104	3511908	12.3288280	5.3368033	.006578947
153	23409	3581577	12.3693169	5.3484812	.006535948
154	23716	3652264	12.4096736	5.3601084	.006493506
155	24025	3723875	12.4498996	5.3716854	.006451613
156	24336	3796416	12.4899960	5.3832126	.006410256

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
157	24649	3869893	12.5299641	5.3946907	.006369427
158	24964	3944312	12.5698051	5.4061202	.006329114
159	25281	4019679	12.6095202	5.4175015	.006289208
160	25600	4096000	12.6491106	5.4288352	.006250000
161	25921	4173281	12.6885775	5.4401218	.006211180
162	26244	4251528	12.7279221	5.4513618	.006172840
163	26569	4330747	12.7671453	5.4625556	.006134969
164	26896	4410944	12.8062485	5.4737037	.006097561
165	27225	4492125	12.8452326	5.4848066	.006060606
166	27556	4574296	12.8840987	5.4958647	.006024096
167	27889	4657463	12.9228480	5.5068784	.005988024
168	28224	4741632	12.9614814	5.5178484	.005952381
169	28561	4826809	13.0000000	5.5287748	.005917160
170	28900	4913000	13.0384048	5.5396583	.005882353
171	29241	5000211	13.0766968	5.5504991	.005847953
172	29584	5088448	13.1148770	5.5612978	.005813953
173	29929	5177717	13.1529464	5.5720546	.005780347
174	30276	5268024	13.1909060	5.5827702	.005747126
175	30625	5359375	13.2287566	5.5934447	.005714286
176	30976	5451776	13.2664992	5.6040787	.005681818
177	31329	5545233	13.3041347	5.6146724	.005649718
178	31684	5639752	13.3416641	5.6252263	.005617978
179	32041	5735339	13.3790882	5.6357408	.005586592
180	32400	5832000	13.4164079	5.6462162	.005555556
181	32761	5929741	13.4536240	5.6566528	.005524862
182	33124	6028568	13.4907376	5.6670511	.005494505
183	33489	6128487	13.5277493	5.6774114	.005464481
184	33856	6229504	13.5646600	5.6877340	.005434783
185	34225	6331625	13.6014705	5.6980192	.005405405
186	34596	6434856	13.6381817	5.7082675	.005376344
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7286543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291605
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189517	13.8924400	5.7789966	.005181347
194	37636	7301384	13.9283883	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356688	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284867	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421356	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674673	.004950495
203	41209	8365427	14.2478068	5.87711307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692

Number.	Squares	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5.9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907264	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986486	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941048	14.8996644	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464286
225	50625	11390625	15.0000000	6.0824020	.004444444
226	51076	11543176	15.0332964	6.0991994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6.1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919334	6.2144650	.004166667
241	58081	13997521	15.5241747	6.2230843	.004149378
242	58564	14172488	15.5563492	6.2316797	.004132231
243	59049	14348907	15.5884573	6.2402515	.004115226
244	59536	14526784	15.6204994	6.2487998	.004098361
245	60025	14706125	15.6524758	6.2573248	.004081633
246	60516	14886936	15.6843871	6.2658266	.004065041
247	61009	15069223	15.7162336	6.2743054	.004048583
248	61504	15252992	15.7480157	6.2827613	.004032258
249	62001	15438249	15.7797338	6.2911946	.004016064
250	62500	15625000	15.8113883	6.2996053	.004000000
251	63001	15813251	15.8429795	6.3079935	.003984064
252	63504	16003008	15.8745079	6.3163596	.003968254
253	64009	16194277	15.9059737	6.3247035	.003952569
254	64516	16387064	15.9373775	6.3330256	.003937008
255	65025	16581375	15.9687194	6.3413257	.003921569
256	65536	16777216	16.0000000	6.3496042	.003906250
257	66049	16974593	16.0312195	6.3578611	.003891051
258	66564	17173512	16.0623784	6.3660968	.003875969
259	67081	17373979	16.0934769	6.3743111	.003861004
260	67600	17576000	16.1245155	6.3825043	.003846154

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
261	68121	17779581	16.1554944	6.3906765	.003831418
262	68644	17984728	16.1864141	6.3988279	.003816794
263	69169	18191447	16.2172747	6.4069585	.003802281
264	69696	18399744	16.2480768	6.4150687	.003787879
265	70225	18609625	16.2788206	6.4231583	.003773585
266	70756	18821096	16.3095064	6.4312276	.003759398
267	71289	19034163	16.3401346	6.4392767	.003745318
268	71824	19248832	16.3707055	6.4473057	.003731343
269	72361	19465109	16.4012195	6.4553148	.003717472
270	72900	19683000	16.4316767	6.4633041	.003703704
271	73441	19902511	16.4620776	6.4712736	.003690037
272	73984	20123643	16.4924225	6.4792236	.003676471
273	74529	20346417	16.5227116	6.4871541	.003663004
274	75076	20570824	16.5529454	6.4950653	.003649635
275	75625	20796875	16.5831240	6.5029572	.003636364
276	76176	21024576	16.6132477	6.5108300	.003623188
277	76729	21253933	16.6433170	6.5186839	.003610108
278	77284	21484952	16.6733320	6.5265189	.003597122
279	77841	21717639	16.7032931	6.5343351	.003584229
280	78400	21952000	16.7332005	6.5421326	.003571429
281	78961	22188041	16.7630546	6.5499116	.003558719
282	79524	22425768	16.7928556	6.5576722	.003546099
283	80089	22665187	16.8226038	6.5654144	.003533569
284	80656	22906304	16.8522995	6.5731385	.003522127
285	81225	23149125	16.8819430	6.5808443	.003508772
286	81796	23393656	16.9115345	6.5885323	.003496503
287	82369	23639903	16.9410743	6.5962023	.003484321
288	82944	23887872	16.9705627	6.6038545	.003472222
289	83521	24137569	17.0000000	6.6114890	.003460208
290	84100	24389000	17.0293864	6.6191060	.003448276
291	84681	24642171	17.0587221	6.6267054	.003436426
292	85264	24897088	17.0880075	6.6342874	.003424658
293	85849	25153757	17.1172428	6.6418522	.003412969
294	86436	25412184	17.1464282	6.6493998	.003401361
295	87025	25672375	17.1755640	6.6569302	.003389831
296	87616	25934836	17.2046505	6.6644437	.003378378
297	88209	26198073	17.2336879	6.6719403	.003367003
298	88804	26463592	17.2626765	6.6794200	.003355705
299	89401	26730899	17.2916165	6.6868831	.003344482
300	90000	27000000	17.3205081	6.6943295	.003333333
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543608	17.3781472	6.7091729	.003311258
303	91809	27818127	17.4068952	6.7165700	.003301330
304	92416	28094464	17.4355958	6.7239508	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278689
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
308	94864	29218112	17.5499288	6.7533134	.003246753
309	95481	29503609	17.5783958	6.7606143	.003236246
310	96106	29791000	17.6068169	6.7678995	.003225806
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7824229	.003205128

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968844	.003184713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554496	17.7763888	6.8112847	.003164557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753433	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003048104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313088	.003003093
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6.9520533	.002976190
337	113569	38272753	18.3575598	6.9589434	.002967359
338	114244	38614472	18.3847763	6.9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18.4661853	6.9863681	.002932551
342	116964	40001688	18.4932420	6.9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7.0135791	.002898551
346	119716	41421736	18.6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815417	7.0405860	.002865339
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002808989
357	127449	45499293	18.8944436	7.0939709	.002801120
358	128164	45882712	18.9208879	7.1005885	.002793296
359	128881	46268279	18.9472953	7.1071937	.002785515
360	129600	46656000	18.9736660	7.1137866	.002777778
361	130321	47045831	19.0000000	7.1203674	.002770083
362	131044	47437928	19.0262976	7.1269360	.002762431
363	131769	47832147	19.0525589	7.1334925	.002754821
364	132496	48228544	19.0787840	7.1400370	.002747253

Number.	Squares.	Cubes.	√ Roots.	³ √ Roots.	Reciprocals.
365	133225	48627125	19·1049732	7·1465695	·002739726
366	133956	49027896	19·1311265	7·1530901	·002732240
367	134689	49430863	19·1572441	7·1595988	·002724796
368	135424	49836032	19·1833261	7·1660957	·002717391
369	136161	50243409	19·2093727	7·1725809	·002710027
370	136900	50653000	19·2353841	7·1790544	·002702703
371	137641	51064811	19·2613603	7·1855162	·002695418
372	138384	51478848	19·2873015	7·1919663	·002688172
373	139129	51895117	19·3132079	7·1984050	·002680965
374	139876	52313624	19·3390796	7·2048322	·002673797
375	140625	52734375	19·3649167	7·2112479	·002666667
376	141376	53157376	19·3907194	7·2176522	·002659574
377	142129	53582633	19·4164878	7·2240450	·002652520
378	142884	54010152	19·4422221	7·2304268	·002645503
379	143641	54439939	19·4679223	7·2367972	·002638521
380	144400	54872000	19·4935887	7·2431565	·002631579
381	145161	55306341	19·5192213	7·2495045	·002624672
382	145924	55742968	19·5448203	7·2558415	·002617801
383	146689	56181887	19·5703858	7·2621675	·002610966
384	147456	56623104	19·5959179	7·2684824	·002604167
385	148225	57066625	19·6214169	7·2747864	·002597403
386	148996	57512456	19·6468827	7·2810794	·002590674
387	149769	57960603	19·6723156	7·2873617	·002583979
388	150544	58411072	19·6977156	7·2936330	·002577320
389	151321	58863869	19·7230829	7·2998936	·002570694
390	152100	59319000	19·7484177	7·3061436	·002564103
391	152881	59776471	19·7737199	7·3123828	·002557545
392	153664	60236288	19·7989899	7·3186114	·002551020
393	154449	60698457	19·8242276	7·3248295	·002544529
394	155236	61162984	19·8494332	7·3310369	·002538071
395	156025	61629875	19·8746069	7·3372339	·002531646
396	156816	62099136	19·8997487	7·3434205	·002525253
397	157609	62570773	19·9248588	7·3495966	·002518892
398	158404	63044792	19·9499373	7·3557624	·002512563
399	159201	63521199	19·9749844	7·3619178	·002506266
400	160000	64000000	20·0000000	7·3680630	·002500000
401	160801	64481201	20·0249844	7·3741979	·002493766
402	161604	64964808	20·0499377	7·3803227	·002487562
403	162409	65450827	20·0748599	7·3864373	·002481390
404	163216	65939264	20·0997512	7·3925418	·002475248
405	164025	66430125	20·1246118	7·3986363	·002469136
406	164836	66923416	20·1494417	7·4047206	·002463054
407	165649	67419143	20·1742410	7·4107950	·002457002
408	166464	67917312	20·1990099	7·4168595	·002450980
409	167281	68417929	20·2237484	7·4229142	·002444988
410	168100	68921000	20·2484567	7·4289589	·002439024
411	168921	69426531	20·2731349	7·4349938	·002433090
412	169744	69934528	20·2977831	7·4410189	·002427184
413	170569	70444997	20·3224014	7·4470343	·002421308
414	171396	70957944	20·3469899	7·4530399	·002415459
415	172225	71473375	20·3715488	7·4590359	·002409639
416	173056	71991296	20·3960781	7·4650223	·002406846

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
417	173889	72511713	20.4205779	7.4709991	.002398082
418	174724	73034632	20.4450483	7.4769664	.002392344
419	175561	73560059	20.4694895	7.4829242	.002386635
420	176400	74088000	20.4939015	7.4888724	.002380952
421	177241	74618461	20.5182845	7.4948113	.002375297
422	178084	75151448	20.5426386	7.5007406	.002369668
423	178929	75686967	20.5669638	7.5066607	.002364066
424	179776	76225024	20.5912603	7.5125715	.002358491
425	180625	76765625	20.6155281	7.5184730	.002352941
426	181476	77308776	20.6397674	7.5243652	.002347418
427	182329	77854483	20.6639783	7.5302482	.002341920
428	183184	78402752	20.6881609	7.5361221	.002336449
429	184041	78953589	20.7123152	7.5419867	.002331002
430	184900	79507000	20.7364414	7.5478423	.002325581
431	185761	80062991	20.7605395	7.5536888	.002320186
432	186624	80621568	20.7846097	7.5595263	.002314815
433	187489	81182737	20.8086520	7.5653548	.002309469
434	188356	81746504	20.8326667	7.5711743	.002304147
435	189225	82312875	20.8566536	7.5769849	.002298851
436	190096	82881856	20.8806130	7.5827865	.002293578
437	190969	83453453	20.9045450	7.5885793	.002288330
438	191844	84027672	20.9284495	7.5943633	.002283105
439	192721	84604519	20.9523268	7.6001385	.002277904
440	193600	85184000	20.9761770	7.6059049	.002272727
441	194481	85766121	21.0000000	7.6116626	.002267574
442	195364	86350888	21.0237960	7.6174116	.002262443
443	196249	86938307	21.0475652	7.6231519	.002257336
444	197136	87528384	21.0713075	7.6288837	.002252252
445	198025	88121125	21.0950231	7.6346067	.002247191
446	198916	88716536	21.1187121	7.6403213	.002242152
447	199809	89314623	21.1423745	7.6460272	.002237136
448	200704	89915392	21.1660105	7.6517247	.002232143
449	201601	90518849	21.1896201	7.6574138	.002227171
450	202500	91125000	21.2132034	7.6630943	.002222222
451	203401	91733851	21.2367606	7.6687665	.002217295
452	204304	92345408	21.2602916	7.6744303	.002212389
453	205209	92959677	21.2837967	7.6800857	.002207506
454	206116	93576664	21.3072758	7.6857328	.002202643
455	207025	94196375	21.3307290	7.6913717	.002197802
456	207936	94818816	21.3541565	7.6970023	.002192982
457	208849	95443993	21.3775583	7.7026246	.002188184
458	209764	96071912	21.4009346	7.7082388	.002183406
459	210681	96702579	21.4242853	7.7138448	.002178649
460	211600	97336000	21.4476106	7.7194426	.002173913
461	212521	97972181	21.4709106	7.7250325	.002169197
462	213444	98611128	21.4941853	7.7306141	.002164502
463	214369	99252847	21.5174348	7.7361877	.002159827
464	215296	99897344	21.5406592	7.7417532	.002155172
465	216225	100544625	21.5638587	7.7473109	.002150538
466	217156	101194696	21.5870331	7.7528606	.002145923
467	218089	101847563	21.6101828	7.7584023	.002141328
468	219024	102503232	21.6333077	7.7639361	.002136752

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
469	219961	103161769	21.6564078	7.7694650	.002132196
470	226900	103823066	21.6794834	7.7749861	.002127606
471	221841	104487111	21.7025344	7.7804904	.002123142
472	222784	105154048	21.7255610	7.7859928	.002118644
473	223729	105828817	21.7485632	7.7914875	.002114165
474	224676	106496424	21.7715411	7.7969745	.002109705
475	225625	107171875	21.7944947	7.8024538	.002105263
476	226576	107856176	21.8174242	7.8079254	.002100846
477	227529	108531333	21.8403297	7.8133892	.002096486
478	228484	109213352	21.8632111	7.8188456	.002092056
479	229441	109902239	21.8860686	7.8242942	.002087683
480	230400	110592000	21.9089023	7.8297353	.002083283
481	231361	111284641	21.9317122	7.8351688	.002078902
482	232324	111980168	21.9544984	7.8405949	.002074689
483	233289	112678587	21.9772610	7.8460134	.002070393
484	234256	113379904	22.0000000	7.8514244	.002066116
485	235225	114084125	22.0227155	7.8568281	.002061856
486	236196	114791256	22.0454077	7.8622242	.002057613
487	237169	115501303	22.0680765	7.8676130	.002053388
488	238144	116214272	22.0907220	7.8729944	.002049180
489	239121	116930169	22.1133444	7.8783684	.002044990
490	240100	117649000	22.1359436	7.8837352	.002040816
491	241081	118370771	22.1585198	7.8890946	.002036660
492	242064	119095488	22.1810730	7.8944468	.002032520
493	243049	119823157	22.2036033	7.8997917	.002028398
494	244036	120553784	22.2261108	7.9051294	.002024291
495	245025	121287375	22.2485955	7.9104599	.002020202
496	246016	122023936	22.2710575	7.9157832	.002016129
497	247009	122763473	22.2934968	7.9210994	.002012072
498	248004	123505992	22.3159136	7.9264085	.002008032
499	249001	124251499	22.3383079	7.9317104	.002004008
500	250000	125000000	22.3606798	7.9370053	.002000000
501	251001	125751501	22.3830293	7.9422931	.001996008
502	252004	126506008	22.4053565	7.9475739	.001992032
503	253009	127263527	22.4276615	7.9528477	.001988072
504	254016	128024064	22.4499443	7.9581144	.001984127
505	255025	128787625	22.4722051	7.9633743	.001980198
506	256036	129554216	22.4944438	7.9686271	.001976285
507	257049	130323843	22.5166605	7.9738731	.001972387
508	258064	131096512	22.5388553	7.9791122	.001968504
509	259081	131872229	22.5610283	7.9843444	.001964637
510	260100	132651000	22.5831796	7.9895697	.001960784
511	261121	133432831	22.6053091	7.9947883	.001956947
512	262144	134217728	22.6274170	8.0000000	.001953125
513	263169	135005697	22.6495033	8.0052049	.001949318
514	264196	135796744	22.6715681	8.0104032	.001945525
515	265225	136590875	22.6936114	8.0155946	.001941748
516	266256	137388096	22.7156334	8.0207794	.001937984
517	267289	138188413	22.7376341	8.0259574	.001934236
518	268324	138991832	22.7596134	8.0311287	.001930502
519	269361	139798359	22.7815715	8.0362935	.001926782
520	270400	140608000	22.8035085	8.0414515	.001923077

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
521	271411	141420761	22·8254244	8·0466030	·001919386
522	272484	142236648	22·8473193	8·0517479	·001915709
523	273529	143055667	22·8691933	8·0568862	·001912046
524	274576	143877824	22·8910463	8·0620180	·001908397
525	275625	144703125	22·9128785	8·0671432	·001904762
526	276676	145531576	22·9346899	8·0722620	·001901141
527	277729	146363183	22·9564806	8·0773743	·001897533
528	278784	147197952	22·9782506	8·0824800	·001893939
529	279841	148035889	23·0000000	8·0875794	·001890359
530	280900	148877001	23·0217289	8·0926723	·001886792
531	281961	149721291	23·0434372	8·0977589	·001883239
532	283024	150568768	23·0651252	8·1028390	·001879699
533	284089	151419437	23·0867928	8·1079128	·001876173
534	285156	152273304	23·1084400	8·1129803	·001872659
535	286225	153130375	23·1300670	8·1180414	·001869159
536	287296	153990656	23·1516738	8·1230962	·001865672
537	288369	154854153	23·1732605	8·1281447	·001862197
538	289444	155720872	23·1948270	8·1331870	·001858736
539	290521	156590819	23·2163735	8·1382230	·001855288
540	291600	157464000	23·2379001	8·1432529	·001851852
541	292681	158340421	23·2594067	8·1482765	·001848429
542	293764	159220088	23·2808935	8·1532939	·001845018
543	294849	160103007	23·3023604	8·1583951	·001841621
544	295936	160989184	23·3238076	8·1633102	·001838235
545	297025	161878625	23·3452351	8·1683092	·001834862
546	298116	162771336	23·3666429	8·1733020	·001831502
547	299209	163667323	23·3880311	8·1782888	·001828154
548	300304	164566592	23·4093998	8·1832695	·001824818
549	301401	165469149	23·4307490	8·1882441	·001821494
550	302500	166375000	23·4520788	8·1932127	·001818182
551	303601	167284151	23·4733892	8·1981753	·001814882
552	304704	168196608	23·4946802	8·2031319	·001811594
553	305809	169112377	23·5159520	8·2080825	·001808318
554	306916	170031464	23·5372046	8·2130271	·001805054
555	308025	170953875	23·5584380	8·2179657	·001801802
556	309136	171879616	23·5796522	8·2228985	·001798561
557	310249	172808693	23·6008474	8·2278254	·001795332
558	311364	173741112	23·6220236	8·2327463	·001792115
559	312481	174676879	23·6431808	8·2376614	·001788909
560	313600	175616000	23·6643191	8·2425706	·001785714
561	314721	176558481	23·6854386	8·2474740	·001782531
562	315844	177504328	23·7065392	8·2523715	·001779359
563	316969	178453547	23·7276210	8·2572635	·001776199
564	318096	179406144	23·7486842	8·2621492	·001773050
565	319225	180362125	23·7697286	8·2670294	·001769912
566	320356	181321496	23·7907545	8·2719039	·001766784
567	321489	182284263	23·8117618	8·2767726	·001763668
568	322624	183250432	23·8327506	8·2816255	·001760563
569	323761	184220009	23·8537209	8·2864928	·001757469
570	324900	185193000	23·8746728	8·2913444	·001754386
571	326041	186169411	23·8956063	8·2961903	·001751313
572	327184	187149448	23·9165215	8·3010304	·001748252

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
573	328329	188132517	23·9374184	8·3058651	·001745201
574	329476	189119224	23·9582971	8·3106941	·001742160
575	330625	190109375	23·9791576	8·3155175	·001739130
576	331776	191102976	24·0000000	8·3203353	·001736111
577	332927	192100033	24·0208243	8·3251475	·001733102
578	334084	193100552	24·0416306	8·3299542	·001730104
579	335241	194104539	24·0624188	8·3347553	·001727116
580	336400	195112000	24·0831891	8·3395509	·001724138
581	337561	196122941	24·1039416	8·3443410	·001721170
582	338724	197137368	24·1246762	8·3491256	·001718213
583	339889	198155287	24·1453929	8·3539047	·001715266
584	341056	199176704	24·1660919	8·3586784	·001712329
585	342225	200201625	24·1867732	8·3634466	·001709402
586	343396	201230056	24·2074369	8·3682095	·001706485
587	344569	202262003	24·2280829	8·3729668	·001703578
588	345744	203297472	24·2487113	8·3777188	·001700680
589	346921	204336469	24·2693222	8·3824653	·001697793
590	348100	205379000	24·2899156	8·3872065	·001694915
591	349281	206425071	24·3104996	8·3919428	·001692047
592	350464	207474688	24·3310501	8·3966729	·001689189
593	351649	208527857	24·3515913	8·4013981	·001686341
594	352836	209584584	24·3721152	8·4061180	·001683502
595	354025	210644875	24·3926218	8·4108326	·001680672
596	355216	211708736	24·4131112	8·4155419	·001677852
597	356409	212776173	24·4335834	8·4202460	·001675042
598	357604	213847192	24·4540385	8·4249448	·001672241
599	358801	214921799	24·4744765	8·4296383	·001669449
600	360000	216000000	24·4948974	8·4343267	·001666667
601	361201	217081801	24·5152013	8·4390098	·001663894
602	362404	218167208	24·5356883	8·4436877	·001661120
603	363609	219256227	24·5560583	8·4483605	·001658375
604	364816	220348864	24·5764115	8·4530281	·001655629
605	366025	221445125	24·5967478	8·4576906	·001652893
606	367236	222545016	24·6170673	8·4623479	·001650165
607	368449	223648543	24·6373700	8·4670001	·001647446
608	369664	224755712	24·6576560	8·4716471	·001644737
609	370881	225866529	24·6779254	8·4762892	·001642036
610	372100	226981000	24·6981781	8·4809261	·001639344
611	373321	228099131	24·7184142	8·4855579	·001636661
612	374544	229220928	24·7386338	8·4901848	·001633987
613	375769	230346397	24·7588368	8·4948065	·001631321
614	376996	231475544	24·7790234	8·4994233	·001628664
615	378225	232608375	24·7991935	8·5040350	·001626016
616	379456	233744896	24·8193473	8·5086417	·001623377
617	380689	234885113	24·8394847	8·5132435	·001620746
618	381924	236029032	24·8596058	8·5178403	·001618123
619	383161	237176659	24·8797106	8·5224331	·001615509
620	384400	238328000	24·8997992	8·5270189	·001612903
621	385641	239483061	24·9198716	8·5316009	·001610306
622	386884	240641848	24·9399278	8·5361780	·001607717
623	388129	241804367	24·9599679	8·5407501	·001605136
624	389376	242970624	24·9799920	8·5453173	·001602564

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
625	390625	244140625	25.0000000	8.5498797	.001600000
626	391876	245134376	25.0199920	8.5544372	.001597444
627	393129	246491883	25.0399681	8.5589899	.001594896
628	394384	247673152	25.0599282	8.5635377	.001592357
629	395641	248858189	25.0798724	8.5680807	.001589825
630	396900	250047000	25.0998008	8.5726189	.001587302
631	398161	251239591	25.1197134	8.5771523	.001584786
632	399424	252435968	25.1396102	8.5816809	.001582278
633	400689	253636137	25.1594913	8.5862247	.001579779
634	401956	254840104	25.1793566	8.5907238	.001577287
635	403225	256047875	25.1992063	8.5952380	.001574803
636	404496	257259456	25.2190404	8.5997476	.001572327
637	405769	258474853	25.2388589	8.6042525	.001569859
638	407044	259694072	25.2586619	8.6087526	.001567398
639	408321	260917119	25.2784493	8.6132480	.001564945
640	409600	262144000	25.2982213	8.6177388	.001562500
641	410881	263374721	25.3179778	8.6222248	.001560062
642	412164	264609288	25.3377189	8.6267063	.001557632
643	413449	265847707	25.3574447	8.6311830	.001555210
644	414736	267089984	25.3771551	8.6356551	.001552795
645	416125	268336125	25.3968502	8.6401226	.001550388
646	417516	269585136	25.4165302	8.6445855	.001547988
647	418609	270840023	25.4361947	8.6490437	.001545595
648	419904	272097792	25.4558441	8.6534974	.001543210
649	421201	273359449	25.4754784	8.6579465	.001540832
650	422500	274625000	25.4950976	8.6623911	.001538462
651	423801	275894451	25.5147013	8.6668310	.001536098
652	425104	277167808	25.5342907	8.6712665	.001533742
653	426409	278445077	25.5538647	8.6756974	.001531394
654	427716	279726264	25.5734237	8.6801237	.001529052
655	429025	281011375	25.5929678	8.6845456	.001526718
656	430336	282300416	25.6124969	8.6889630	.001524390
657	431639	283593393	25.6320112	8.6933759	.001522070
658	432964	284890312	25.6515107	8.6977843	.001519751
659	434281	286191179	25.6709953	8.7021882	.001517451
660	435600	287496000	25.6904652	8.7065877	.001515152
661	436921	288804781	25.7099203	8.7109827	.001512859
662	438244	290117528	25.7293607	8.7153734	.001510574
663	439569	291434247	25.7487864	8.7197596	.001508296
664	440896	292754944	25.7681975	8.7241414	.001506024
665	442225	294079625	25.7875939	8.7285187	.001503759
666	443556	295408296	25.8069758	8.7328918	.001501502
667	444899	296740963	25.8263431	8.7372604	.001499250
668	446224	298077632	25.8456960	8.7416246	.001497006
669	447561	299418309	25.8650343	8.7459846	.001494768
670	448900	300763000	25.8843582	8.7503401	.001492537
671	450241	302111711	25.9036677	8.7546913	.001490313
672	451584	303464448	25.9229628	8.7590383	.001488095
673	452929	304821217	25.9422435	8.7633809	.001485884
674	454276	306182024	25.9615100	8.7677192	.001483680
675	455625	307546875	25.9807621	8.7720532	.001481481
676	456976	308915776	26.0000000	8.7763830	.001479290

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
677	458329	310288733	26.0192237	8.7807084	.001477105
678	459684	311665752	26.0384331	8.7856296	.001474926
679	461041	313046839	26.0576284	8.7893466	.001472754
680	462400	314432000	26.0768096	8.7936593	.001470588
681	463761	315821241	26.0959767	8.7979679	.001468429
682	465124	317214568	26.1151297	8.8022721	.001466276
683	466489	318611987	26.1342687	8.8065722	.001464129
684	467856	320013504	26.1533937	8.8108681	.001461988
685	469225	321419125	26.1725047	8.8151598	.001459854
686	470596	322828856	26.1916017	8.8194474	.001457726
687	471969	324242703	26.2106848	8.8237307	.001455604
688	473344	325660672	26.2297541	8.8280099	.001453488
689	474721	327082769	26.2488095	8.8322850	.001451379
690	476100	328509000	26.2678511	8.8365559	.001449275
691	477481	329939371	26.2868789	8.8408227	.001447178
692	478864	331373888	26.3058929	8.8450854	.001445087
693	480249	332812557	26.3248932	8.8493440	.001443001
694	481636	334255384	26.3438797	8.8535985	.001440922
695	483025	335702375	26.3628527	8.8578489	.001438849
696	484416	337153536	26.3818119	8.8620952	.001436782
697	485809	338608873	26.4007576	8.8663375	.001434720
698	487204	340068392	26.4196896	8.8705757	.001432665
699	488601	341532099	26.4386081	8.8748099	.001430615
700	490000	343000000	26.4575131	8.8790460	.001428571
701	491401	344472101	26.4764046	8.8832661	.001426534
702	492804	345948408	26.4952826	8.8874882	.001424501
703	494209	347428927	26.5141472	8.8917063	.001422475
704	495616	348913664	26.5329983	8.8959204	.001420455
705	497025	350402625	26.5518361	8.9001304	.001418440
706	498436	351895816	26.5706605	8.9043366	.001416431
707	499849	353393243	26.5894716	8.9085387	.001414427
708	501264	354894912	26.6082694	8.9127369	.001412429
709	502681	356400829	26.6270539	8.9169311	.001410437
710	504100	357911000	26.6458252	8.9211214	.001408451
711	505521	359425431	26.6645833	8.9253078	.001406470
712	506944	360944128	26.6833281	8.9294902	.001404494
713	508369	362467097	26.7020598	8.9336687	.001402525
714	509796	363994344	26.7207784	8.9378433	.001400560
715	511225	365525875	26.7394839	8.9420140	.001398601
716	512656	367061696	26.7581763	8.9461809	.001396648
717	514089	368601813	26.7768557	8.9503438	.001394700
718	515524	370146232	26.7955220	8.9545029	.001392758
719	516961	371694959	26.8141754	8.9586581	.001390821
720	518400	373248000	26.8328157	8.9628095	.001388889
721	519841	374805361	26.8514432	8.9669570	.001386963
722	521284	376367048	26.8700577	8.9711007	.001385042
723	522729	377933067	26.8886593	8.9752406	.001383126
724	524176	379503424	26.9072481	8.9793766	.001381215
725	525625	381078125	26.9258240	8.9835089	.001379310
726	527076	382657176	26.9443872	8.9876373	.001377410
727	528529	384240583	26.9629375	8.9917620	.001375516
728	529984	385828352	26.9814751	8.9958899	.001373626

Number.	Squares.	Cubes.	√ Roots.	³ √ Roots.	Reciprocals.
729	531441	387420489	27·0000000	9·0000000	·001371742
730	532900	389017000	27·0185122	9·0041134	·001369863
731	534361	390617891	27·0370117	9·0082229	·001367989
732	535824	392223168	27·0554985	9·0123288	·001366120
733	537289	393832837	27·0739727	9·0164309	·001364256
734	538756	395446904	27·0924344	9·0205293	·001362398
735	540225	397065375	27·1108834	9·0246239	·001360544
736	541696	398688256	27·1293199	9·0287149	·001358696
737	543169	400315553	27·1477149	9·0328021	·001356852
738	544644	401947272	27·1661554	9·0368857	·001355014
739	546121	403583419	27·1845544	9·0409655	·001353180
740	547600	405224000	27·2029140	9·0450419	·001351351
741	549081	406869021	27·2213152	9·0491142	·001349528
742	550564	408518488	27·2396769	9·0531831	·001347709
743	552049	410172407	27·2580263	9·0572482	·001345895
744	553536	411830784	27·2763634	9·0613098	·001344086
745	555025	413493625	27·2946881	9·0653677	·001342282
746	556516	415160936	27·3130006	9·0694220	·001340483
747	558009	416832723	27·3313007	9·0734726	·001338688
748	559504	418508992	27·3495887	9·0775197	·001336898
749	561001	420189749	27·3678644	9·0815631	·001335113
750	562500	421875000	27·3861279	9·0856030	·001333333
751	564001	423564751	27·4043792	9·0896352	·001331558
752	565504	425259008	27·4226184	9·0936719	·001329787
753	567009	426957777	27·4408455	9·0977010	·001328021
754	568516	428661064	27·4590604	9·1017265	·001326260
755	570025	430368875	27·4772633	9·1057485	·001324503
756	571536	432081216	27·4954542	9·1097669	·001322751
757	573049	433798093	27·5136330	9·1137818	·001321004
758	574564	435519512	27·5317998	9·1177931	·001319261
759	576081	437245479	27·5499546	9·1218010	·001317523
760	577600	438976000	27·5680975	9·1258053	·001315789
761	579121	440711081	27·5862284	9·1298061	·001314060
762	580644	442450728	27·6043475	9·1338034	·001312336
763	582169	444194947	27·6224546	9·1377971	·001310616
764	583696	445943744	27·6405499	9·1417874	·001308901
765	585225	447697125	27·6586334	9·1457742	·001307190
766	586756	449455096	27·6767050	9·1497576	·001305483
767	588289	451217663	27·6947648	9·1537375	·001303781
768	589824	452984832	27·7128129	9·1577139	·001302083
769	591361	454756609	27·7308492	9·1616869	·001300390
770	592900	456533000	27·7488739	9·1656565	·001298701
771	594441	458314011	27·7668868	9·1696225	·001297017
772	595984	460099618	27·7848880	9·1735852	·001295337
773	597529	461889917	27·8028775	9·1775445	·001293661
774	599076	463684824	27·8208555	9·1815003	·001291990
775	600625	465484375	27·8388218	9·1854527	·001290323
776	602176	467288576	27·8567766	9·1894018	·001288660
777	603729	469097433	27·8747197	9·1933474	·001287001
778	605284	470910952	27·8926514	9·1972897	·001285347
779	606841	472729139	27·9105715	9·2012286	·001283697
780	608400	474552000	27·9284801	9·2051641	·001282051

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
781	609961	476379541	27·9463772	9·2090962	·001280410
782	611524	478211768	27·9642629	9·2130250	·001278772
783	613089	480048687	27·9821372	9·2169505	·001277139
784	614656	481890304	28·0000000	9·2208726	·001275510
785	616225	483736625	28·0178515	9·2247914	·001273885
786	617796	485587656	28·0356915	9·2287068	·001272265
787	619369	487443403	28·0535203	9·2326189	·001270648
788	620944	489303872	28·0713377	9·2365277	·001269036
789	622521	491169069	28·0891438	9·2404333	·001267427
790	624100	493039000	28·1069386	9·2443355	·001265823
791	625681	494913671	28·1247222	9·2482344	·001264223
792	627264	496793088	28·1424946	9·2521300	·001262626
793	628849	498677257	28·1602557	9·2560224	·001261034
794	630436	500566184	28·1780056	9·2599114	·001259446
795	632025	502459875	28·1957444	9·2637973	·001257862
796	633616	504358336	28·2134720	9·2676798	·001256281
797	635209	506261573	28·2311884	9·2715592	·001254705
798	636804	508169592	28·2488938	9·2754352	·001253133
799	638401	510082399	28·2665881	9·2793081	·001251364
800	640000	512000000	28·2842712	9·2831777	·001250000
801	641601	513922401	28·3019434	9·2870444	·001248439
802	643204	515849608	28·3196045	9·2909072	·001246883
803	644809	517781627	28·3372546	9·2947671	·001245330
804	646416	519718464	28·3548938	9·2986239	·001243781
805	648025	521660125	28·3725219	9·3024775	·001242236
806	649636	523606616	28·3901391	9·3063278	·001240695
807	651249	525557943	28·4077454	9·3101750	·001239157
808	652864	527514112	28·4253408	9·3140190	·001237624
809	654481	529475129	28·4429253	9·3178599	·001236094
810	656100	531441000	28·4604989	9·3216975	·001234568
811	657721	533411731	28·4780617	9·3255320	·001233046
812	659344	535387328	28·4956137	9·3293634	·001231527
813	660969	537367797	28·5131549	9·3331916	·001230012
814	662596	539353144	28·5306852	9·3370167	·001228501
815	664225	541343375	28·5482048	9·3408386	·001226994
816	665856	543338496	28·5657137	9·3446575	·001225499
817	667489	545338513	28·5832119	9·3484731	·001223990
818	669124	547343432	28·6006993	9·3522857	·001222494
819	670761	549353259	28·6181760	9·3560952	·001221001
820	672400	551368000	28·6356421	9·3599016	·001219512
821	674041	553387661	28·6530976	9·3637049	·001218027
822	675684	555412248	28·6705424	9·3675051	·001216545
823	677329	557441767	28·6879716	9·3713022	·001215067
824	678976	559476224	28·7054002	9·3750963	·001213592
825	680625	561515625	28·7228132	9·3788873	·001212121
826	682276	563559976	28·7402157	9·3826752	·001210654
827	683929	565609283	28·7576077	9·3864600	·001209190
828	685584	567663552	28·7749891	9·3902419	·001207729
829	687241	569722789	28·7923601	9·3940206	·001206273
830	688900	571787000	28·8097206	9·3977964	·001204819
831	690561	573856191	28·8270706	9·4015691	·001203369
832	692224	575930368	28·8444102	9·4053387	·001201923

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
833	693889	578009537	28.8617394	9.4691054	.001200480
834	695556	580093704	28.8790582	9.47128690	.001199041
835	697225	582182875	28.8963666	9.47466297	.001197605
836	698896	584277056	28.9136646	9.47803873	.001196172
837	700569	586376253	28.9309523	9.48141420	.001194743
838	702244	588480472	28.9482297	9.48478936	.001193317
839	703921	590589719	28.9654967	9.48816423	.001191895
840	705600	592704000	28.9827535	9.49153800	.001190476
841	707281	594823321	29.0000000	9.49491307	.001189061
842	708964	596947688	29.0172363	9.49828704	.001187648
843	710649	599077107	29.0344623	9.50166072	.001186240
844	712336	601211584	29.0516781	9.50503410	.001184834
845	714025	603351125	29.0688837	9.50840719	.001183432
846	715716	605495736	29.0860791	9.51177999	.001182033
847	717409	607645423	29.1032644	9.51515249	.001180638
848	719104	609800192	29.1204396	9.51852470	.001179245
849	720801	611960049	29.1376046	9.52189661	.001177856
850	722500	614125000	29.1547595	9.52526824	.001176471
851	724201	616295051	29.1719043	9.52863957	.001175088
852	725904	618470208	29.1890390	9.53201061	.001173709
853	727609	620650477	29.2061637	9.53538136	.001172333
854	729316	622835864	29.2232784	9.53875182	.001170960
855	731025	625026375	29.2403830	9.54212200	.001169591
856	732736	627222016	29.2574777	9.54549188	.001168224
857	734449	629422793	29.2745623	9.54886147	.001166861
858	736164	631628712	29.2916370	9.55223078	.001165501
859	737881	633839779	29.3087018	9.55559980	.001164144
860	739600	636056000	29.3257566	9.55896854	.001162791
861	741321	638277381	29.3428015	9.56233699	.001161440
862	743044	640503928	29.3598365	9.56570515	.001160093
863	744769	642735647	29.3768616	9.56907303	.001158749
864	746496	644972544	29.3938769	9.57244063	.001157407
865	748225	647214625	29.4108823	9.57580794	.001156069
866	749956	649461896	29.4278779	9.57917497	.001154734
867	751689	651714363	29.4448637	9.58254172	.001153403
868	753424	653972032	29.4618397	9.58590818	.001152074
869	755161	656234909	29.4788059	9.58927437	.001150748
870	756900	658503000	29.4957624	9.59264027	.001149425
871	758641	660776311	29.5127091	9.59600589	.001148106
872	760384	663054848	29.5296461	9.59937123	.001146789
873	762129	665338617	29.5465734	9.60273630	.001145475
874	763876	667627624	29.5634910	9.60610108	.001144165
875	765625	669921875	29.5803989	9.60946559	.001142857
876	767376	672221376	29.5972972	9.61282782	.001141553
877	769129	674526133	29.6141858	9.61619377	.001140251
878	770884	676836152	29.6310648	9.61955745	.001138952
879	772641	679151439	29.6479342	9.62292085	.001137656
880	774400	681472000	29.6647939	9.62628397	.001136364
881	776161	683797841	29.6816442	9.62964682	.001135074
882	777924	686128968	29.6984848	9.63300937	.001133787
883	779689	688465387	29.7153159	9.63637169	.001132503
884	781456	690807104	29.7321375	9.63973373	.001131222

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	707932288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998328	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117818
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111
901	811801	731432701	30.0166621	9.6584684	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103753
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550825	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188351	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083423
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.001076426
930	864900	804357000	30.4959014	9.7610001	.001075269
931	866761	806954491	30.5122926	9.7644974	.001074114
932	868624	809557568	30.5286750	9.7679922	.001072961
933	870489	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784616	.001069519
936	876096	820025856	30.5941171	9.7829466	.001068376

Number.	Squares	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
937	877969	822656953	30.6104557	9.7854288	.001067236
938	879844	825293672	30.6267857	9.7889087	.001066098
939	881721	827936019	30.6431069	9.7923861	.001064963
940	883600	830584000	30.6594194	9.7958611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896888	30.6920185	9.8028036	.001061571
943	889249	838561807	30.7083051	9.8062711	.001060445
944	891136	841232384	30.7245830	9.8097362	.001059322
945	893025	843908625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896808	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382879	9.8339238	.001051525
952	906304	862801408	30.8544972	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192477	9.8511280	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838668	9.8648483	.001041667
961	923521	887503681	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056347	31.0322413	9.8751135	.001038422
964	929296	895841344	31.0483494	9.8785305	.001037344
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805405	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287648	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330048	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2409987	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966168	31.3368792	9.9396363	.001018330
983	966289	949862087	31.3528308	9.9430092	.001017294
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146

Number.	squares.	Cubes.	$\sqrt{\text{roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
989	978121	967361669	31.4483704	9.9631981	.001011122
990	980100	970299000	31.4642654	9.9665549	.001010101
991	982081	973242271	31.4801525	9.9699055	.001009082
992	984064	976191488	31.4960315	9.9732619	.001008065
993	986049	979146657	31.5119025	9.9766120	.001007049
994	988036	982107784	31.5277655	9.9799599	.001006036
995	990025	985074875	31.5436206	9.9833055	.001005025
996	992016	988047936	31.5594677	9.9866488	.001004016
997	994009	991026973	31.5753068	9.9899900	.001003009
998	996004	994011992	31.5911380	9.9933289	.001002004
999	998001	997002999	31.6069613	9.9966656	.001001001
1000	1000000	1000000000	31.6227766	10.0000000	.001000000
1001	1000201	1003003001	31.6385840	10.0033222	.0009990010
1002	1004004	1006012008	31.6543866	10.0066622	.0009980040
1003	1006009	1009027027	31.6701752	10.0099899	.0009970090
1004	1008016	1012048064	31.6859590	10.0133155	.0009960159
1005	1010025	1015075125	31.7017349	10.0166389	.0009950249
1006	1010036	1018108216	31.7175030	10.0199601	.0009940358
1007	1014049	1021147343	31.7332633	10.0232791	.0009930487
1008	1016064	1024192512	31.7490157	10.0265958	.0009920635
1009	1018081	1027243729	31.7647603	10.0299104	.0009910803
1010	1020100	1030301000	31.7804972	10.0332228	.0009900990
1011	1020121	1033364331	31.7962262	10.0365330	.0009891197
1012	1024144	1036433728	31.8119474	10.0398410	.0009881423
1013	1026169	1039509197	31.8276609	10.0431469	.0009871668
1014	1028196	1042590744	31.8433666	10.0464506	.0009861933
1015	1030225	1045678375	31.8590646	10.0497521	.0009852217
1016	1032256	1048772096	31.8747549	10.0530514	.0009842520
1017	1034289	1051871913	31.8904374	10.0563485	.0009832842
1018	1036324	1054977832	31.9061123	10.0596435	.0009823183
1019	1038361	1058089859	31.9217794	10.0629364	.0009813543
1020	1040400	1061208000	31.9374388	10.0662271	.0009803922
1021	1042441	1064332261	31.9530906	10.0695156	.0009794319
1022	1044484	1067462648	31.9687347	10.0728020	.0009784736
1023	1046529	1070599167	31.9843712	10.0760863	.0009775171
1024	1048576	1073741824	32.0000000	10.0793684	.0009765625
1025	1050625	1076890625	32.0156212	10.0826484	.0009756098
1026	1052676	1080045576	32.0312348	10.0859262	.0009746589
1027	1054729	1083206683	32.0468407	10.0892019	.0009737098
1028	1056784	1086373952	32.0624391	10.0924755	.0009727626
1029	1058841	1089547389	32.0780298	10.0957469	.0009718173
1030	1060900	1092727000	32.0936131	10.0990163	.0009708738
1031	1062961	1095912791	32.1091887	10.1022835	.0009699321
1032	1065024	1099104768	32.1247568	10.1055487	.0009689922
1033	1067089	1102302937	32.1403173	10.1088117	.0009680542
1034	1069156	1105507304	32.1558704	10.1120726	.0009671180
1035	1071225	1108717875	32.1714159	10.1153314	.0009661836
1036	1073296	1111934656	32.1869539	10.1185882	.0009652510
1037	1075369	1115157653	32.2024844	10.1218428	.0009643202
1038	1077444	1118386872	32.2180074	10.1250953	.0009633911
1039	1079521	1121622319	32.2335229	10.1283457	.0009624639
1040	1081600	1124864000	32.2490310	10.1315941	.0009615385

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1041	1083681	1128111921	32.2645316	10.1348403	.0009606148
1042	1085764	1131366088	32.2800248	10.1380845	.0009596929
1043	1087849	1134626507	32.2955105	10.1413266	.0009587738
1044	1089936	1137893184	32.3109888	10.1445667	.0009578544
1045	1092025	1141166125	32.3264598	10.1478047	.0009569378
1046	1094116	1144445336	32.3419233	10.1510406	.0009560229
1047	1096209	1147730823	32.3573794	10.1542744	.0009551098
1048	1098304	1151022592	32.3728281	10.1575062	.0009541985
1049	1100401	1154320649	32.3882695	10.1607359	.0009532888
1050	1102500	1157625000	32.4037035	10.1639636	.0009523810
1051	1104601	1160935651	32.4191301	10.1671893	.0009514748
1052	1106704	1164252608	32.4345495	10.1704129	.0009505703
1053	1108809	1167575877	32.4499615	10.1736344	.0009496676
1054	1110916	1170905464	32.4653662	10.1768539	.0009487666
1055	1113125	1174241375	32.4807635	10.1800714	.0009478673
1056	1115136	1177583616	32.4961536	10.1832868	.0009469697
1057	1117249	1180932193	32.5115364	10.1865002	.0009460738
1058	1119364	1184287112	32.5269119	10.1897116	.0009451796
1059	1121481	1187648379	32.5422802	10.1929209	.0009442871
1060	1123600	1191016000	32.5576412	10.1961283	.0009433962
1061	1125721	1194389981	32.5729949	10.1993336	.0009425071
1062	1127844	1197770328	32.5883415	10.2025369	.0009416196
1063	1129969	1201157047	32.6035807	10.2057382	.0009407338
1064	1132096	1204550144	32.6190129	10.2089375	.0009398496
1065	1134225	1207949625	32.6343377	10.2121347	.0009389671
1066	1136356	1211355496	32.6496554	10.2153300	.0009380863
1067	1138489	1214767763	32.6649659	10.2185233	.0009372071
1068	1140624	1218186432	32.6802693	10.2217146	.0009363296
1069	1142761	1221611509	32.6955654	10.2249039	.0009354537
1070	1144900	1225043000	32.7108544	10.2280912	.0009345794
1071	1147041	1228480911	32.7261363	10.2312766	.0009337068
1072	1149184	1231925248	32.7414111	10.2344599	.0009328358
1073	1151329	1235376017	32.7566787	10.2376413	.0009319664
1074	1153476	1238833224	32.7719392	10.2408207	.0009310987
1075	1155625	1242296875	32.7871926	10.2439981	.0009302326
1076	1157776	1245766976	32.8024398	10.2471735	.0009293680
1077	1159929	1249243533	32.8176782	10.2503470	.0009285051
1078	1162084	1252726552	32.8329103	10.2535186	.0009276438
1079	1164241	1256216039	32.8481354	10.2566881	.0009267841
1080	1166400	1259712000	32.8633535	10.2598557	.0009259259
1081	1168561	1263214441	32.8785644	10.2630213	.0009250694
1082	1170724	1266723368	32.8937684	10.2661850	.0009242144
1083	1172889	1270238787	32.9089653	10.2693467	.0009233610
1084	1175056	1273760704	32.9241553	10.2725065	.0009225092
1085	1177225	1277289125	32.9393382	10.2756644	.0009216590
1086	1179396	1280824056	32.9545141	10.2788203	.0009208103
1087	1181569	1284365503	32.9696830	10.2819743	.0009199632
1088	1183744	1287913472	32.9848450	10.2851264	.0009191176
1089	1185921	1291467969	33.0000000	10.2882765	.0009182736
1090	1188100	1295029000	33.0151480	10.2914247	.0009174312
1091	1190281	1298596571	33.0302891	10.2945709	.0009165903
1092	1192464	1302170688	33.0454233	10.2977153	.0009157509

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1093	1194649	1305751357	33·0605505	10·3008577	·0009149131
1094	1196836	1309338584	33·0756708	10·3039982	·0009140768
1095	1199025	1312932375	33·0907842	10·3071368	·0009132420
1096	1201216	1316532736	33·1058907	10·3102735	·0009124008
1097	1203409	1320139673	33·1209903	10·3134083	·0009115770
1098	1205604	1323753192	33·1360830	10·3165411	·0009107468
1099	1207801	1327373299	33·1511689	10·3196721	·0009099181
1100	1210000	1331000000	33·1662479	10·3228012	·0009090909
1101	1212201	1334633301	33·1813200	10·3259284	·0009082652
1102	1214404	1338273208	33·1963853	10·3290537	·0009074410
1103	1216609	1341919727	33·2114438	10·3321770	·0009066183
1104	1218816	1345572864	33·2266955	10·3352985	·0009057971
1105	1221025	1349232625	33·2415403	10·3384181	·0009049774
1106	1223236	1352899016	33·2565783	10·3415358	·0009041591
1107	1225449	1356572043	33·2716095	10·3446517	·0009033424
1108	1227664	1360251712	33·2866339	10·3477657	·0009025271
1109	1229881	13639338029	33·3016516	10·3508778	·0009017133
1110	1232100	1367631000	33·3166625	10·3539880	·0009009009
1111	1234321	1371330631	33·3316666	10·3570964	·0009000900
1112	1236544	1375036928	33·3466640	10·3602029	·0008992806
1113	1238769	1378749897	33·3616546	10·3633076	·0008984726
1114	1240996	1382469544	33·3766385	10·3664103	·0008976661
1115	1243225	1386195875	33·3916157	10·3695113	·0008968610
1116	1245456	1389928896	33·4065862	10·3726103	·0008960753
1117	1247689	1393668613	33·4215499	10·3757076	·0008952551
1118	1249924	1397415032	33·4365070	10·3788030	·0008944544
1119	1252161	1401168159	33·4514573	10·3818965	·0008936550
1120	1254400	1404928000	33·4664011	10·3849882	·0008928571
1121	1256641	1408694561	33·4813381	10·3880781	·0008920607
1122	1258884	1412467848	33·4962684	10·3911661	·0008912656
1123	1261129	1416247867	33·5111921	10·3942527	·0008904720
1124	1263376	1420034624	33·5261092	10·3973366	·0008896797
1125	1265625	1423828125	33·5410196	10·4004192	·0008888889
1126	1267876	1427628376	33·5559234	10·4034999	·0008880995
1127	1270129	1431435383	33·5708206	10·4065787	·0008873114
1128	1272384	1435249152	33·5857112	10·4096557	·0008865248
1129	1274641	1439069689	33·6005952	10·4127310	·0008857396
1130	1276900	1442897000	33·6154726	10·4158044	·0008849558
1131	1279161	1446731091	33·6303434	10·4188760	·0008841733
1132	1281424	1450571968	33·6452077	10·4219458	·0008833922
1133	1283689	1454419637	33·6600653	10·4250138	·0008826125
1134	1285956	1458274104	33·6749165	10·4280800	·0008818342
1135	1288225	1462135375	33·6897610	10·4311443	·0008810573
1136	1290496	1466003456	33·7045991	10·4342069	·0008802817
1137	1292769	1469878353	33·7174306	10·4372677	·0008795075
1138	1295044	1473760072	33·7340556	10·4403677	·0008787346
1139	1297321	1477648619	33·7490741	10·4433839	·0008779631
1140	1299600	1481544000	33·7638860	10·4464393	·0008771930
1141	1301881	1485446221	33·7786915	10·4494929	·0008764242
1142	1304164	1489355288	33·7934905	10·4525448	·0008756567
1143	1306449	1493271207	33·8082830	10·4555948	·0008748906
1144	1308736	1497193984	33·8230691	10·4586431	·0008741259

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
1145	1311025	1501123625	33.8378486	10.4616896	.0008733624
1146	1313316	1505060136	33.8526218	10.4647343	.0008726003
1147	1315609	1509003523	33.8673884	10.4677773	.0008718396
1148	1317904	1512953792	33.8821487	10.4708158	.0008710801
1149	1320201	1516910949	33.8969025	10.4738579	.0008703220
1150	1322500	1520875000	33.9116499	10.4768955	.0008695652
1151	1324801	1524845951	33.9263909	10.4799314	.0008688097
1152	1327104	1528823808	33.9411255	10.4829656	.0008680556
1153	1329409	1532808577	33.9558537	10.4859980	.0008673027
1154	1331716	1536800264	33.9705755	10.4890286	.0008665511
1155	1334025	1540798875	33.9852910	10.4920575	.0008658009
1156	1336336	1544804416	34.0000000	10.4950847	.0008650519
1157	1338649	1548816893	34.0147027	10.4981101	.0008643042
1158	1340964	1552836312	34.0293990	10.5011337	.0008635579
1159	1343281	1556862679	34.0440890	10.5041556	.0008628128
1160	1345600	1560896000	34.0587727	10.5071757	.0008620690
1161	1347921	1564936281	34.0734501	10.5101942	.0008613244
1162	1350244	1568983528	34.0881211	10.5132109	.0008605852
1163	1352569	1573037749	34.0127858	10.5162259	.0008598452
1164	1354896	1577098944	34.1174442	10.5192391	.0008591065
1165	1357225	1581167125	34.1320963	10.5222506	.0008583691
1166	1359556	1585242296	34.1467422	10.5252604	.0008576329
1167	1361889	1589324463	34.1613817	10.5282685	.0008568980
1168	1364224	1593413632	34.1760150	10.5312749	.0008561644
1169	1366561	1597509809	34.1906420	10.5342795	.0008554320
1170	1368900	1601613000	34.2052627	10.5372825	.0008547009
1171	1371241	1605723211	34.2198773	10.5402837	.0008539710
1172	1373584	1609840448	34.2344855	10.5432832	.0008532423
1173	1375929	1613964717	34.2490875	10.5462810	.0008525149
1174	1378276	1618096024	34.2636834	10.5492771	.0008517888
1175	1380625	1622234375	34.2782730	10.5522715	.0008510638
1176	1382976	1626379776	34.2928564	10.5552612	.0008503401
1177	1385329	1630532233	34.3074336	10.5582552	.0008496177
1178	1387684	1634691752	34.3220046	10.5612445	.0008488964
1179	1390041	1638858339	34.3365694	10.5642322	.0008481764
1180	1392400	1643032000	34.3511281	10.5672181	.0008474576
1181	1394761	1647212741	34.3656805	10.5702024	.0008467401
1182	1397124	1651400568	34.3802268	10.5731849	.0008460237
1183	1399489	1655595487	34.3947670	10.5761658	.0008453085
1184	1401856	1659797504	34.4093011	10.5791449	.0008445946
1185	1404225	1664006625	34.4238289	10.5821225	.0008438819
1186	1406596	1668222856	34.4383507	10.5850983	.0008431703
1187	1408969	1672446203	34.4528663	10.5880725	.0008424600
1188	1411344	1676676672	34.4673759	10.5910450	.0008417508
1189	1413721	1680914629	34.4818793	10.5940158	.0008410429
1190	1416100	1685159000	34.4963766	10.5969850	.0008403361
1191	1418481	1689410871	34.5108678	10.5999525	.0008396306
1192	1420864	1693669888	34.5253530	10.6029184	.0008389262
1193	1423249	1697936057	34.5398321	10.6058826	.0008382230
1194	1425636	1702209384	34.5543051	10.6088451	.0008375209
1195	1428025	1706489875	34.5687720	10.6118060	.0008368201
1196	1430416	1710777536	34.5832329	10.6147652	.0008361204

Number	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1197	1432809	1715072373	34.5976879	10.6177228	.0008354219
1198	1435204	1719374392	34.6121366	10.6206783	.0008347245
1199	1437601	1723683599	34.6265794	10.6236331	.0008340284
1200	1440000	1728000000	34.6410162	10.6265857	.0008333333
1201	1442401	1732323601	34.6554469	10.6295367	.0008326395
1202	1444804	1736654408	34.6698716	10.6324860	.0008319468
1203	1447209	1740992427	34.6842904	10.6354338	.0008312552
1204	1449616	1745337664	34.6987031	10.6383799	.0008305648
1205	1452025	1749690125	34.7131099	10.6413244	.0008298755
1206	1454436	1754049816	34.7275107	10.6442672	.0008291874
1207	1456849	1758416743	34.7419055	10.6472085	.0008285004
1208	1459264	1762790912	34.7562944	10.6501480	.0008278146
1209	1461681	1767172329	34.7706773	10.6530860	.0008271299
1210	1464100	1771561000	34.7850543	10.6560223	.0008264463
1211	1466521	1775956931	34.7994253	10.6589570	.0008257638
1212	1468944	1780360128	34.8137904	10.6618902	.0008250825
1213	1471369	1784770597	34.8281495	10.6648217	.0008244023
1214	1473796	1789188344	34.8425023	10.6677516	.0008237232
1215	1476225	1793613375	34.8568501	10.6706799	.0008230453
1216	1478656	1798045696	34.8711915	10.6736066	.0008223684
1217	1481089	1802485313	34.8855271	10.6765317	.0008216927
1218	1483524	1806932232	34.8998567	10.6794552	.0008210181
1219	1485961	1811386459	34.9141805	10.6823771	.0008203445
1220	1488400	1815848000	34.9284984	10.6852973	.0008196721
1221	1490841	1820316861	34.9428104	10.6882160	.0008190008
1222	1493284	1824793048	34.9571166	10.6911331	.0008183306
1223	1495729	1829276567	34.9714169	10.6940486	.0008176615
1224	1498176	1833764247	34.9857114	10.6969625	.0008169935
1225	1500625	1838265625	35.0000000	10.6998748	.0008163265
1226	1503276	1842771176	35.0142823	10.7027855	.0008156607
1227	1505529	1847284083	35.0285598	10.7056947	.0008149959
1228	1507984	1851804352	35.0428309	10.7086023	.0008143322
1229	1510441	1856331989	35.0570963	10.7115083	.0008136696
1230	1512900	1860867000	35.0713558	10.7144127	.0008130081
1231	1515361	1865409391	35.0856096	10.7173155	.0008123477
1232	1517824	1869959168	35.0998575	10.7202168	.0008116883
1233	1520289	1874516337	35.1140997	10.7231165	.0008110300
1234	1522756	1879080904	35.1283361	10.7260146	.0008103728
1235	1525225	1883652875	35.1425568	10.7289112	.0008097166
1236	1527696	1888232256	35.1567917	10.7318062	.0008090615
1237	1530169	1892819053	35.1710108	10.7346997	.0008084074
1238	1532644	1897413272	35.1852242	10.7375916	.0008077544
1239	1535121	1902014919	35.1994318	10.7404819	.0008071025
1240	1537600	1906624000	35.2136337	10.7433707	.0008064516
1241	1540081	1911240521	35.2278229	10.7462579	.0008058018
1242	1542564	1915864488	35.2420204	10.7491436	.0008051530
1243	1545049	1920495907	35.2562051	10.7520277	.0008045052
1244	1547536	1925134784	35.2703842	10.7549103	.0008038585
1245	1550025	1929781125	35.2845575	10.7577913	.0008032129
1246	1552521	1934434936	35.2987252	10.7606708	.0008025682
1247	1555009	1939096223	35.3128872	10.7635488	.0008019246
1248	1557504	1943764992	35.3270435	10.7664252	.0008012821

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1249	1560001	1948441249	35.3411941	10.7693001	.0008006405
1250	1562500	1953125000	35.3553391	10.7721735	.0008000000
1251	1565001	1957816251	35.3694784	10.7750453	.0007993605
1252	1567504	1962515008	35.3836120	10.7779156	.0007987220
1253	1570009	1967221277	35.3977400	10.7807843	.0007980846
1254	1572516	1971935064	35.4118624	10.7836516	.0007974482
1255	1575025	1976656375	35.4259792	10.7865173	.0007968127
1256	1577536	1981385216	35.4400903	10.7893815	.0007961783
1257	1580049	1986121593	35.4541958	10.7922441	.0007955449
1258	1582564	1990865512	35.4682957	10.7951053	.0007949126
1259	1585081	1995616979	35.4823900	10.7979649	.0007942812
1260	1587600	2000376000	35.4964787	10.8008230	.0007936508
1261	1590121	2005142581	35.5105618	10.8036797	.0007930214
1262	1592644	2009916728	35.5246393	10.8065348	.0007923930
1263	1595166	2014698447	35.5387113	10.8093884	.0007917656
1264	1597696	2019487744	35.5527777	10.8122404	.0007911392
1265	1600225	2024284625	35.5668385	10.8150909	.0007905138
1266	1602756	2029089096	35.5808937	10.8179400	.0007898894
1267	1605289	2033901163	35.5949434	10.8207876	.0007892660
1268	1607824	2038720832	35.6089876	10.8236336	.0007886435
1269	1610361	2043548109	35.6230262	10.8264782	.0007880221
1270	1612900	2048383000	35.6370593	10.8293213	.0007874016
1271	1615441	2053225511	35.6510869	10.8321629	.0007867821
1272	1617984	2058075648	35.6651090	10.8350030	.0007861635
1273	1620529	2062933417	35.6791255	10.8378416	.0007855460
1274	1623076	2067798824	35.6931366	10.8406788	.0007849294
1275	1625625	2072671875	35.7071421	10.8435144	.0007843127
1276	1628176	2077552576	35.7211422	10.8463485	.0007836991
1277	1630729	2082440933	35.7351367	10.8491812	.0007830854
1278	1633284	2087336952	35.7491258	10.8520125	.0007824726
1279	1635841	2092240639	35.7631095	10.8548422	.0007818608
1280	1638400	2097152000	35.7770876	10.8576704	.0007812500
1281	1640961	2102071841	35.7910603	10.8604972	.0007806401
1282	1643524	2106997768	35.8050276	10.8633225	.0007800312
1283	1646089	2111932187	35.8189894	10.8661454	.0007794232
1284	1648656	2116874304	35.8329457	10.8689687	.0007788162
1285	1651225	2121824125	35.8468966	10.8717897	.0007782101
1286	1653796	2126781656	35.8608421	10.8746091	.0007776050
1287	1656369	2131746903	35.8747822	10.8774271	.0007770008
1288	1658944	2136719872	35.8887169	10.8802436	.0007763975
1289	1661521	2141700569	35.9026461	10.8830587	.0007757952
1290	1664100	2146689000	35.9165699	10.8858723	.0007751938
1291	1666681	2151685171	35.9304884	10.8886845	.0007745933
1292	1669264	2156689088	35.9444015	10.8914952	.0007739938
1293	1671849	2161700757	35.95833092	10.8943044	.0007733952
1294	1674436	2166720184	35.9722115	10.8971123	.0007727975
1295	1677025	2171747375	35.9861084	10.8999186	.0007722008
1296	1679616	2176782336	36.0000000	10.9027235	.0007716049
1297	1682209	2181825073	36.0138862	10.9055269	.0007710100
1298	1684804	2186875592	36.0277671	10.9083290	.0007704160
1299	1687401	2191933899	36.0416426	10.9111296	.0007698229
1300	1690000	2197000000	36.0555128	10.9139287	.0007692308

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1301	1692601	2202073901	36.0693776	10.9167265	.0007686395
1302	1695204	2207155608	36.0832371	10.9195228	.0007680492
1303	1697809	2212245127	36.0970913	10.9223177	.0007674579
1304	1700416	2217342464	36.1109402	10.9251111	.0007668712
1305	1703025	2222447625	36.1247837	10.9279031	.0007662835
1306	1705636	2227560616	36.1386220	10.9306937	.0007656968
1307	1708249	2232681443	36.1524550	10.9334829	.0007651109
1308	1710864	2237810112	36.1662826	10.9362706	.0007645260
1309	1713481	2242946629	36.1801050	10.9390569	.0007639419
1310	1716100	2248091000	36.1939221	10.9418418	.0007633588
1311	1718721	2253243231	36.2077340	10.9446253	.0007627765
1312	1721344	2258403328	36.2215406	10.9475074	.0007621951
1313	1723969	2263571297	36.2353419	10.9501880	.0007616146
1314	1726596	2268747144	36.2491379	10.9529673	.0007610350
1315	1729225	2273930875	36.2629287	10.9557451	.0007604563
1316	1731856	2279122196	36.2767143	10.9585215	.0007598784
1317	1734489	2284322013	36.2904246	10.9612965	.0007593011
1318	1737124	2289529432	36.3041697	10.9640701	.0007587253
1319	1739761	2294744759	36.3180396	10.9668423	.0007581501
1320	1742400	2299968000	36.3318042	10.9696131	.0007575758
1321	1745041	2305199161	36.3455637	10.9723825	.0007570023
1322	1747684	2310438248	36.3593179	10.9751505	.0007564297
1323	1750329	2315685267	36.3730670	10.9779171	.0007558579
1324	1752976	2320940224	36.3868108	10.9806823	.0007552870
1325	1755625	2326203125	36.4005494	10.9834462	.0007547170
1326	1758276	2331473976	36.4142829	10.9862086	.0007541478
1327	1760929	2336752783	36.4280112	10.9889696	.0007535795
1328	1763584	2342039552	36.4417343	10.9917293	.0007530120
1329	1766241	2347334289	36.4554523	10.9944876	.0007524454
1330	1768900	2352637000	36.4691650	10.9972445	.0007518797
1331	1771561	2357947691	36.4828727	11.0000000	.0007513148
1332	1774224	2363266368	36.4965752	11.0027541	.0007507508
1333	1776889	2368593037	36.5102725	11.0055069	.0007501875
1334	1779556	2373927704	36.5239647	11.0082583	.0007496252
1335	1782225	2379270375	36.5376518	11.0110082	.0007490637
1336	1784896	2384621056	36.5513388	11.0137569	.0007485030
1337	1787569	2389979753	36.5650106	11.0165041	.0007479432
1338	1790244	2395346472	36.5786823	11.0192500	.0007473842
1339	1792921	2400721219	36.5923489	11.0219945	.0007468260
1340	1795600	2406104000	36.6060104	11.0247377	.0007462687
1341	1798281	2411494821	36.6196668	11.0274795	.0007457122
1342	1800964	2416893688	36.6333181	11.0302199	.0007451565
1343	1803649	2422300607	36.6469144	11.0329590	.0007446016
1344	1806336	2427715584	36.6606056	11.0356967	.0007440476
1345	1809025	2433138625	36.6742416	11.0384320	.0007434941
1346	1811716	2438569736	36.6878726	11.0411680	.0007429421
1347	1814409	2444008923	36.7014986	11.0439017	.0007423905
1348	1817104	2449456192	36.7151195	11.0466339	.0007418398
1349	1819801	2454911549	36.7287353	11.0493649	.0007412898
1350	1822500	2460375000	36.7423461	11.0520945	.0007407407
1351	1825201	2465846551	36.7559519	11.0548227	.0007401921
1352	1827904	2471326208	36.7695526	11.0575497	.0007396450

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1353	1830609	2476813977	36.7831483	11.0602752	.0007390983
1354	1833316	2482309864	36.7967390	11.0629994	.0007385524
1355	1836025	2487813875	36.8103246	11.0657222	.0007380074
1356	1838736	2493326016	36.8239053	11.0684437	.0007374631
1357	1841449	2498846293	36.8374809	11.0711639	.0007369197
1358	1844164	2504374712	36.8510515	11.0738828	.0007363770
1359	1846881	2509911279	36.8646172	11.0766003	.0007358352
1360	1849600	2515456000	36.8781778	11.0793165	.0007352941
1361	1852321	2521008881	36.8917335	11.0820314	.0007347539
1362	1855044	2526569928	36.9052842	11.0847449	.0007342144
1363	1857769	2532139147	36.9188299	11.0874571	.0007336757
1364	1860496	2537716544	36.9323706	11.0901679	.0007331378
1365	1863225	2543302125	36.9459064	11.0928775	.0007326007
1366	1865956	2548895896	36.9594372	11.0955857	.0007320644
1367	1868689	2554497863	36.9729631	11.0982926	.0007315289
1368	1871424	2560108032	36.9864840	11.1009982	.0007309942
1369	1874161	2565726409	37.0000000	11.1037025	.0007304602
1370	1876900	2571353000	37.0135110	11.1064054	.0007299270
1371	1879641	2576987811	37.0270172	11.1091070	.0007293946
1372	1882384	2582630848	37.0405184	11.1118073	.0007288630
1373	1885129	2588282117	37.0540146	11.1145064	.0007283321
1374	1887876	2593941624	37.0675060	11.1172041	.0007278020
1375	1890625	2599609375	37.0899924	11.1199004	.0007272727
1376	1893376	2605285376	37.0944740	11.1225955	.0007267442
1377	1896129	2610969633	37.1079506	11.1252893	.0007262164
1378	1898884	2616662152	37.1214224	11.1279817	.0007256894
1379	1901641	2622362939	37.1348893	11.1306729	.0007251632
1380	1904400	2628072000	37.1483512	11.1333628	.0007246377
1381	1907161	2633789341	37.1618084	11.1360514	.0007241130
1382	1909924	2639514968	37.1752606	11.1387386	.0007235890
1383	1912689	2645248887	37.1887079	11.1414246	.0007230658
1384	1915456	2650991104	37.2021505	11.1441093	.0007225434
1385	1918225	2656741625	37.2155881	11.1467926	.0007220217
1386	1920996	2662500456	37.2290209	11.1494747	.0007215007
1387	1923769	2668267603	37.2424489	11.1521555	.0007209805
1388	1926544	2674043072	37.2558720	11.1548350	.0007204611
1389	1929321	2679826869	37.2692903	11.1575133	.0007199424
1390	1932100	2685619000	37.2827037	11.1601903	.0007194245
1391	1934881	2691419471	37.2961124	11.1628659	.0007189073
1392	1937664	2697228288	37.3095162	11.1655403	.0007183908
1393	1940449	2703045457	37.3229152	11.1682134	.0007178751
1394	1943236	2708870984	37.3363094	11.1708852	.0007173601
1395	1946025	2714704875	37.3496988	11.1735558	.0007168459
1396	1948816	2720547136	37.3630834	11.1762250	.0007163324
1397	1951609	2726397773	37.3764632	11.1788930	.0007158196
1398	1954404	2732256792	37.3898382	11.1815598	.0007153076
1399	1957201	2738124199	37.4032084	11.1842252	.0007147963
1400	1960000	2744000000	37.4165738	11.1868894	.0007142857
1401	1962801	2749884201	37.4299345	11.1895523	.0007137759
1402	1965604	2755776808	37.4432904	11.1922139	.0007132668
1403	1968409	2761677827	37.4566416	11.1948743	.0007127584
1404	1971216	2767587264	37.4699880	11.1975334	.0007122507

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1405	1974025	2773505123	37.4833296	11.2001913	.0007117438
1406	1976836	2779431416	37.4966665	11.2028479	.0007112376
1407	1979649	2785366143	37.5099987	11.2055032	.0007107321
1408	1982464	2791309312	37.5233261	11.2081573	.0007102273
1409	1985281	2797260929	37.5366487	11.2108101	.0007097232
1410	1988100	2803221000	37.5499667	11.2134617	.0007092199
1411	1990921	2809189531	37.5632799	11.2161120	.0007087172
1412	1993744	2815166528	37.5765885	11.2187611	.0007082153
1413	1996569	2821151997	37.5898922	11.2214089	.0007077141
1414	1999396	2827145944	37.6031913	11.2240054	.0007072136
1415	2002225	2833148375	37.6164857	11.2267007	.0007067138
1416	2005056	2839159296	37.6297754	11.2293448	.0007062147
1417	2007889	2845178713	37.6430604	11.2319876	.0007057163
1418	2010724	2851206632	37.6563407	11.2346292	.0007052186
1419	2013561	2857243059	37.6696164	11.2372696	.0007047216
1420	2016400	2863288000	37.6828874	11.2399087	.0007042254
1421	2019241	2869341461	37.6961536	11.2425465	.0007037298
1422	2022084	2875403448	37.7094153	11.2451831	.0007032349
1423	2024929	2881473967	37.7226722	11.2478185	.0007027407
1424	2027776	2887553024	37.7359245	11.2504527	.0007022472
1425	2030625	2893640625	37.7491722	11.2530856	.0007017544
1426	2033476	2899736776	37.7624152	11.2557173	.0007012623
1427	2036329	2905841483	37.7756535	11.2583478	.0007007708
1428	2039184	2911954752	37.7888873	11.2609770	.0007002801
1429	2042041	2918076589	37.8021163	11.2636050	.0006997901
1430	2044900	2924207000	37.8153408	11.2662318	.0006993007
1431	2047761	2930345991	37.8285606	11.2688573	.0006988120
1432	2050624	2936493568	37.8417759	11.2714816	.0006983240
1433	2053489	2942649737	37.8549864	11.2741047	.0006978367
1434	2056356	2948814504	37.8681924	11.2767266	.0006973501
1435	2059225	2954987875	37.8813938	11.2793472	.0006968641
1436	2062096	2961169856	37.8945906	11.2819666	.0006963788
1437	2064969	2967360453	37.9077828	11.2845849	.0006958942
1438	2067844	2973559672	37.9209704	11.2872019	.0006954103
1439	2070721	2979767519	37.9341538	11.2898177	.0006949270
1440	2073600	2985984000	37.9473319	11.2924323	.0006944444
1441	2076481	2992209121	37.9605058	11.2950457	.0006939625
1442	2079364	3008442888	37.9736751	11.2976579	.0006934813
1443	2082249	3004685307	37.9868398	11.3002688	.0006930007
1444	2085136	3010936384	38.0000000	11.3028786	.0006925208
1445	2088025	3017196125	38.0131556	11.3054871	.0006920415
1446	2080916	3023464536	38.0263067	11.3080945	.0006915629
1447	2093809	3029741623	38.0394532	11.3107006	.0006910850
1448	2096701	3036027392	38.0525952	11.3133056	.0006906078
1449	2099601	3042321849	38.0657326	11.3159094	.0006901312
1450	2102500	3048625000	38.0788655	11.3185119	.0006896552
1451	2105401	3054936851	38.0919939	11.3211132	.0006891799
1452	2108304	3061257408	38.1051178	11.3237134	.0006887052
1453	2111209	3067586777	38.1182371	11.3263124	.0006882312
1454	2114116	3073924664	38.1313519	11.3289102	.0006877579
1455	2117025	3080271375	38.1444622	11.3315067	.0006872852
1456	2119936	3086626816	38.1575681	11.3341022	.0006868132

Number.	Squares.	Cubes.	$\sqrt{\quad}$ Roots.	$\sqrt[3]{\quad}$ Roots.	Reciprocals.
1457	2122849	3092990993	38.1706693	11.3366964	.0006863412
1458	2125764	3099363912	38.1837662	11.3392894	.0006858711
1459	2128681	3105745579	38.1968585	11.3418813	.0006854010
1460	2131600	3112136000	38.2099463	11.3444719	.0006849315
1461	2134521	3118535181	38.2230297	11.3470614	.0006844627
1462	2137444	3124943128	38.2361085	11.3496497	.0006839945
1463	2140369	3131359847	38.2491829	11.3522368	.0006835270
1464	2143296	3137785344	38.2622529	11.3548227	.0006830601
1465	2146225	3144219625	38.2753184	11.3574075	.0006825939
1466	2149156	3150662696	38.2883794	11.3599911	.0006821282
1467	2152089	3157114563	38.3014360	11.3625735	.0006816633
1468	2155024	3163575232	38.3144881	11.3651547	.0006811989
1469	2157961	3170044709	38.3275358	11.3677347	.0006807352
1470	2160900	3176523000	38.3405790	11.3703136	.0006802721
1471	2163841	3183010111	38.3536178	11.3728914	.0006798097
1472	2166784	3189506048	38.3666522	11.3754679	.0006793478
1473	2169729	3196010817	38.3796821	11.3780433	.0006788866
1474	2172676	3202524424	38.3927076	11.3806175	.0006784261
1475	2175625	3209046875	38.4057287	11.3831906	.0006779661
1476	2178576	3215578176	38.4187454	11.3857625	.0006775068
1477	2181529	3222118333	38.4317577	11.3883332	.0006770481
1478	2184484	3228667352	38.4447656	11.3909028	.0006765900
1479	2187441	3235225239	38.4577691	11.3934712	.0006761325
1480	2190400	3241792000	38.4707681	11.3960384	.0006756757
1481	2193361	3248367641	38.4837627	11.3986045	.0006752194
1482	2196324	3254952168	38.4967530	11.4011695	.0006747638
1483	2199289	3261545587	38.5097390	11.4037332	.0006743088
1484	2202256	3268147904	38.5227206	11.4062959	.0006738544
1485	2205225	3274759125	38.5356977	11.4088574	.0006734007
1486	2208196	3281379256	38.5486705	11.4114177	.0006729474
1487	2211169	3288008303	38.5616389	11.4139769	.0006724950
1488	2214144	3294646272	38.5746030	11.4165349	.0006720430
1489	2217121	3301293169	38.5875627	11.4190918	.0006715917
1490	2220100	3307949000	38.6005181	11.4206476	.0006711409
1491	2223081	3314613771	38.6134691	11.4222022	.0006706908
1492	2226004	3321287488	38.6264158	11.4227556	.0006702413
1493	2229049	3327970157	38.6393582	11.4293079	.0006697924
1494	2232036	3334661784	38.6522962	11.4318591	.0006693440
1495	2235025	3341362375	38.6652299	11.4344092	.0006688963
1496	2238016	3348071936	38.6781593	11.4369581	.0006684492
1497	2241009	3354790473	38.6910843	11.4395059	.0006680027
1498	2244004	3361517992	38.7040050	11.4420525	.0006675567
1499	2247001	3368254499	38.7169214	11.4445980	.0006671114
1500	2250000	3375000000	38.7298335	11.4471424	.0006666667
1501	2253001	3381754501	38.7427412	11.4496857	.0006662225
1502	2256004	3388518008	38.7556447	11.4522278	.0006657790
1503	2259009	3395290527	38.7685439	11.4547688	.0006653360
1504	2262016	3402072064	38.7814389	11.4573087	.0006648936
1505	2265025	3408862625	38.7943294	11.4598476	.0006644518
1506	2268036	3415662216	38.8072158	11.4623850	.0006640106
1507	2271049	3422470843	38.8200978	11.4649215	.0006635700
1508	2274064	3429288512	38.8329757	11.4674568	.0006631300

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1509	2277081	3436115229	38·8458491	11·4699911	·0006626905
1510	2280100	3442951000	38·8587184	11·4725242	·0006622517
1511	2283121	3449795831	38·8715834	11·4750562	·0006618134
1512	2286144	3456649728	38·8844442	11·4775871	·0006613757
1513	2289169	3463512697	38·8973006	11·4801169	·0006609385
1514	2292196	3470384744	38·9101529	11·4826455	·0006605020
1515	2295225	3477265875	38·9230009	11·4851731	·0006600660
1516	2298256	3484156096	38·9358447	11·4876995	·0006596306
1517	2301289	3491055413	33·9486841	11·4902249	·0006591958
1518	2304324	3597963832	38·9615194	11·4927491	·0006587615
1519	2307361	3504881359	38·9743505	11·4952722	·0006583278
1520	2310400	3511808000	38·9871774	11·4977942	·0006578947
1521	2313441	3518743761	39·0000000	11·5003151	·0006574622
1522	2316484	3525688648	39·0128184	11·5028348	·0006570302
1523	2319529	3532642667	39·0256326	11·5053535	·0006565988
1524	2322576	3539605824	39·0384426	11·5078711	·0006561680
1525	2325625	3546578125	39·0512483	11·5103876	·0006557377
1526	2328676	3553559576	39·0640499	11·5129030	·0006553080
1527	2331729	3560549552	39·0768473	11·5154173	·0006548788
1528	2334784	3567538183	39·0896406	11·5179305	·0006544503
1529	2337841	3574538889	39·1024296	11·5204425	·0006540222
1530	2340900	3581537000	39·1152144	11·5229535	·0006535948
1531	2343961	3588604291	39·1279951	11·5254634	·0006531679
1532	2347024	3595640768	39·1407716	11·5279722	·0006527415
1533	2350089	3602686437	39·1535439	11·5304799	·0006523157
1534	2353156	3609741304	39·1663120	11·5329865	·0006518905
1535	2356225	3616805375	39·1790760	11·5354920	·0006514658
1536	2359256	3623878656	39·1918359	11·5379965	·0006510417
1537	2362369	3630961153	39·2045915	11·5404998	·0006506181
1538	2365444	3638052872	39·2173431	11·5430021	·0006501951
1539	2368521	3645153819	39·2300905	11·5455033	·0006497726
1540	2371600	3652264000	39·2428337	11·5480034	·0006493506
1541	2374681	3659383421	39·2555728	11·5505025	·0006489293
1542	2377764	3666512088	39·2683078	11·5530004	·0006485084
1543	2380849	3673650007	39·2810387	11·5554972	·0006480881
1544	2383936	3680797184	39·2937654	11·5579931	·0006476684
1545	2387025	3687953625	39·3064880	11·5604878	·0006472492
1546	2390116	3695119336	39·3192065	11·5629815	·0006468305
1547	2393209	3702294323	39·3319208	11·5654740	·0006464124
1548	2396304	3709478592	39·3446311	11·5679655	·0006459948
1549	2399401	3716672149	39·3573373	11·5704559	·0006455778
1550	2402500	3723875000	39·3700394	11·5729453	·0006451613
1551	2405601	3731087151	39·3827373	11·5754336	·0006447453
1552	2408704	3738308608	39·3954312	11·5779208	·0006443299
1553	2411809	3745539377	39·4081210	11·5804069	·0006439150
1554	2414916	3752779464	39·4208067	11·5828919	·0006435006
1555	2418025	3760028875	39·4334883	11·5853759	·0006430868
1556	2421136	3767287616	39·4461658	11·5878588	·0006426735
1557	2424249	3774555693	39·4588393	11·5903407	·0006422608
1558	2427364	3781833112	39·4715087	11·5928215	·0006418485
1559	2430481	3789119879	39·4841740	11·5953013	·0006414368
1560	2433600	3796416000	39·4968353	11·5977799	·0006410256

Number	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1561	2436721	3803721481	39.5094925	11.6002576	.0006406150
1562	2439844	3811036328	39.5221457	11.6027342	.0006402049
1563	2442969	3818360547	39.5347948	11.6052097	.0006397953
1564	2446096	3825641444	39.5474399	11.6076841	.0006393862
1565	2449225	3833037125	39.5600809	11.6101575	.0006389776
1566	2452356	3840389496	39.5727179	11.6126299	.0006385696
1567	2455489	3847751263	39.5853508	11.6151012	.0006381621
1568	2458624	3855123432	39.5979797	11.6175715	.0006377551
1569	2461761	3862503009	39.6106046	11.6200407	.0006373486
1570	2464900	3869883000	39.6232255	11.6225088	.0006369427
1571	2468041	3877292411	39.6358424	11.6249759	.0006365372
1572	2471184	3884701248	39.6484552	11.6274420	.0006361323
1573	2474329	3892119157	39.6610640	11.6299070	.0006357279
1574	2477476	3899547224	39.6736688	11.6323710	.0006353240
1575	2480625	3906984375	39.6862696	11.6348339	.0006349206
1576	2483776	3914430976	39.6988665	11.6372957	.0006345178
1577	2486929	3921887033	39.7114593	11.6397566	.0006341154
1578	2490084	3929352552	39.7240481	11.6422164	.0006337136
1579	2493241	3936827539	39.7366329	11.6446751	.0006333122
1580	2496400	3944312000	39.7492138	11.6471329	.0006329114
1581	2499561	3951805941	39.7617907	11.6495895	.0006325111
1582	2502724	3959309368	39.7743636	11.6520452	.0006321113
1583	2505889	3966822287	39.7869325	11.6544998	.0006317119
1584	2509056	3974344704	39.7994976	11.6569534	.0006313131
1585	2512225	3981876625	39.8120585	11.6594059	.0006309148
1586	2515396	3989418056	39.8246155	11.6618574	.0006305170
1587	2518569	3996969003	39.8371686	11.6643079	.0006301197
1588	2521744	4004529472	39.8497177	11.6667574	.0006297229
1589	2524921	4012099469	39.8622628	11.6692058	.0006293266
1590	2528100	4014679000	39.8748040	11.6716532	.0006289308
1591	2531281	4027268071	39.8873413	11.6740996	.0006285355
1592	2534464	4034866688	39.8998747	11.6765449	.0006281407
1593	2537649	4042474857	39.9124041	11.6789892	.0006277464
1594	2540836	4050092584	39.9249295	11.6814325	.0006273526
1595	2544025	4057719875	39.9374511	11.6838748	.0006269592
1596	2547216	4065356736	39.9499687	11.6863161	.0006265664
1597	2550409	4073003173	39.9624824	11.6887563	.0006261741
1598	2553604	4080659192	39.9749922	11.6911955	.0006257822
1599	2556801	4088324799	39.9874980	11.6936337	.0006253909
1600	2560000	4096000000	40.0000000	11.6960709	.0006250000

To find the Square Root of Numbers exceeding 1600.

Example 4. Require the Square Root of 34698. In the column of Squares you will find,

$$\begin{array}{r} +34969 = 187^2, \\ -34698 = 186^2 + \dots \end{array}$$

271 divided by

$$\begin{array}{r} +34969 = 187^2, \\ -34596 = 186^2. \end{array}$$

373 = 000.727.

$$\sqrt{34698} = 186.727 \text{ nearly.}$$

When the number contains Integer and Decimals.

Example 5. Required the Square Root of 7845.45? In the column of *Squares* you will find,

$$\begin{array}{r} +7849.96 = 88.6^2, \\ -7845.45 = 88.5^2 \dots \\ \hline 451 \text{ divided by} \end{array}$$

$$\begin{array}{r} +7849.96 = 88.6^2, \\ -7832.25 = 88.5^2, \\ \hline 1771 = 00.0256. \end{array}$$

$$\sqrt{7845.45} = 88.5256 \text{ nearly.}$$

When the number of ciphers in the integer is *even*, the number of ciphers taken in the Square column must also be *even*; but when the number of ciphers in the integer is *odd*, the number taken in the Square column must also be *odd*.

To find the Cube Root of Numbers exceeding 1600.

Example 6. Required the Cube Root of 5694958? In the *Cube* column you will find,

$$\begin{array}{r} +5735339 = 179^3 \\ -5694958 = 178^3 \dots \\ \hline 40381 \text{ divided by} \end{array}$$

$$\begin{array}{r} +5735339 = 179^3. \\ -5639752 = 178^3. \\ \hline 95587 = 000.4225, \end{array}$$

$$\sqrt[3]{5694958} = 178.4225 \text{ nearly.}$$

When the number contains Integer and Decimals.

Example 7. Required the Cube Root of 4186.586? In the column of *Cubes* you will find,

$$\begin{array}{r} +4251.528 = 16.2^3 \\ -4186.585 = 16.1^3 \dots \\ \hline 64942 \end{array}$$

$$\begin{array}{r} 4251.528 = 16.2^3 \\ 4173.281 = 16.1^3 \\ \hline 78247 = 00.083 \end{array}$$

$$\sqrt[3]{4186.586} = 16.183 \text{ nearly.}$$

The following notice must be particularly attended to, when extracting *Cube Root* of numbers with decimals.

2 ciphers in the integer must be 5, 8, or 11 ciphers in the *Cube* column.

3	"	"	"	3, 6, or 9	"	"
4	"	"	"	4, or 7	"	"
5	"	"	"	5, or 8	"	"
6	"	"	"	6, or 9	"	"
7	"	"	"	7, or 10	"	"

Example 8. Required the Cube Root of 61358.75? In the *Cube* column and 8 ciphers you will find,

$$\begin{array}{r} +61629.875 = 395^3 \\ -61358.750 = 394^3 \dots \\ \hline 271.125 \text{ divided by} \end{array}$$

$$\begin{array}{r} +61629875 = 39.5^3 \\ -61162984 = 39.4^3 \\ \hline 466891 = 00.05807 \end{array}$$

$$\sqrt[3]{61358.75} = 39.45807.$$

To find the Fourth Root.

Rule. Extract the Square Root of the number as before described, and of that root extract the Square Root again, then the last is the Fourth root of the number.

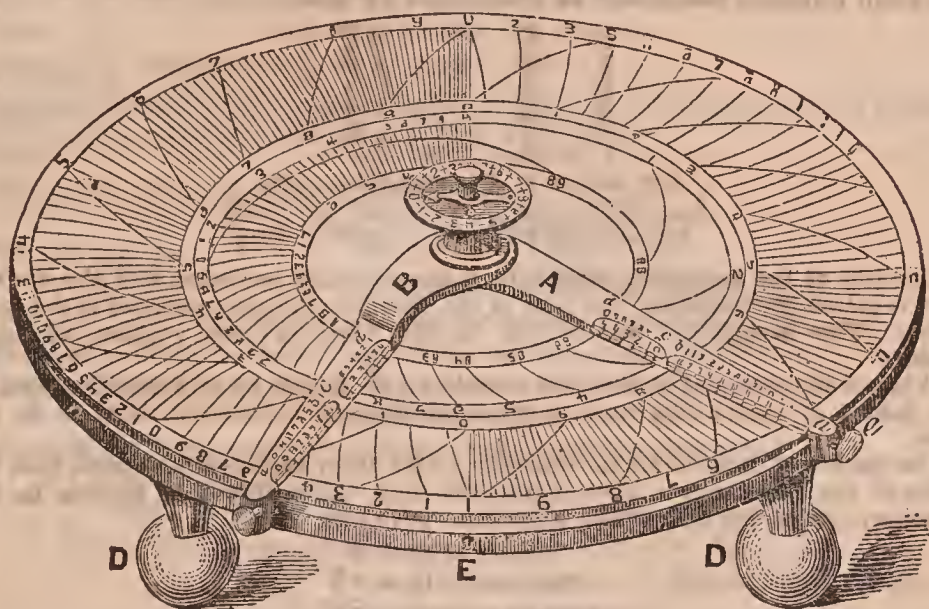
Example 9. Required the fourth root of 2469781?

$$\sqrt[4]{2469781} = \sqrt{\sqrt{2469781}} = \sqrt{1571.4463} = 39.6467, \text{ the answer.}$$

To find the Sixth Root.

Rule. Find the Cube Root of the number as before described, and of that root extract the Cube Root again, and then the last is the Sixth root of the number.

NYSTROM'S CALCULATOR.



ALL calculations in this POCKET BOOK have been computed by this Instrument. It consists of a silvered brass plate on which are fixed two moveable arms, extending from the centre to the periphery. On the plate are engraved a number of curved lines in such form and divisions, that by their intersection with the arms, numbers are read and problems solved.

The arrangement for trigonometrical calculations is such that it is not necessary to notice *sine*, *cosine*, *tan*, &c., &c., operating only by the angles themselves expressed in degrees and minutes. This makes trigonometrical solutions so easy, that any one who understands Simple Arithmetic, will be able to solve trigonometrical questions.

Calculations are performed by it almost instantly, no matter how complicated they may be, while there is nothing intricate or difficult in its use. The author of this book, who is the inventor, has in the last five years thoroughly tested its practical utility.

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THE attention of Engineers, Ship builders, and all whose business requires frequent and extensive calculations, is called to Nystrom's Calculator. Price \$15. To be obtained with description by applying to JOHN W. NYSTROM, Philadelphia. Communications will be promptly attended to.

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GEORGE THORESTED, *Manufacturer*,
23 Hammersly Street,
New York.

LOGARITHMS.

A Logarithm is an exponent of a power to which 10 must be raised to give a certain number, which will be understood by this

Number,...	Logarithm,	Exponent, Base,.....
log. 100 = 2	because	$10^2 = 100.$
log. 10000 = 4	"	$10^4 = 10000$
log. 5012 = 3.7	"	$10^{3.7} = 5012.$

The *unit* of the logarithm is called the *characteristic* or *index*, and the *decimal part* is called the *mantissa*, the sum of the *characteristic* and *mantissa* is the *Logarithm*. The invariable number 10 is the *base* for the system of Logarithms.

It is not necessary that the *base* should be 10, it can be any number, but all the tables of logarithms now in common use, are calculated with 10 to the base.

The nature of logarithms in connection with their numbers are such, that the *index* of the logarithm is always one less than the *number of figures* in the number, (when the base of the logarithm is 10,) as,

$$\begin{aligned} \text{index } 5012 &= 3 \\ \text{mantissa } 5012 &= 0.7 \\ \text{logarithm } 5012 &= 3.7. \end{aligned}$$

Let 10 be raised to any power x , and
the power of $10^x = a$ or $\log. a = x$,
the power of $10^z = b$ or $\log. b = z$.

Let the product of $ab = c$ and the quotient $\frac{a}{b} = c$.

$$10^x \times 10^z = 10^{x+z} = ab = c \quad \text{or} \quad \log. c = x+z.$$

$$\frac{10^x}{10^z} = 10^{x-z} = \frac{a}{b} = d \quad \text{or} \quad \log. d = x-z.$$

$$a = m^z \quad \text{or} \quad \log. m = z \times \log. a.$$

$$\sqrt[3]{a} = n \quad \text{or} \quad \log. n = \log. a : 3.$$

Any number represented by the letters a, b, c , or d , can be a power of 10, which exponent is the logarithm for the number. Logarithms are calculated for every number with three figures in the accompanying Table, by which any operation in Multiplication, Division, Involution and Evolution can be performed by simple Addition or Subtraction of Logarithms. Tables of Logarithms are commonly more extensive, and calculated for any number of four or five figures, which would occupy too much room in this book; but by the proportional parts, the logarithm can be found by this Table, to four or five figures. The *index* of the logarithms do not appear in the Table, only the *mantissa*. It is easily remembered that the *index* is one less, than the *number of figures in the number*; then when the number is only one figure, the index is 0; and when the number is a fraction, the index is negative.

When the logarithm is to be found for a fraction, we commonly have the fraction expressed in a decimal; and then the negative index is equal to the number of ciphers before the first figure, and commonly marked after the *mantissa*; thus explained in whole numbers and fractions:

$$\begin{aligned} \log. 365 &= 2.56229. & \log. 0.365 &= .56229-1. \\ \log. 46.7 &= 1.66931 & \log. 0.0467 &= .66931-2. \\ \log. 7.59 &= 0.88024 & \log. 0.00759 &= .88024-3. \end{aligned}$$

In the accompanying Table of Logarithms, for the trigonometrical lines the negative index is marked thus,

$$\log. \sin. 35^\circ 40' = \log. 0.58306 = 1.76572.$$

To find the Logarithm of Numbers.

Example 1. Find the logarithm of 45?

To 45 in the first column of the Table, answers 65321 in the next column, which is the mantissa; index = 1 because 45 is two figures.

Then, $\log. 45 = 1.65321$, the answer.

Example 2. Find the logarithm of 768?

Opposite 76 in the first column, answers 88536 in the column marked 8 on the top or bottom. Index = 2 because 768 is three figures.

Then, $\log. 768 = 2.88536$.

Example 3. Find the Logarithm of 6846?

$$\log. 6840 = 3.83505$$

$$\text{Proportional part, } 64 \times 0.6 = \underline{384}$$

$$\log. 6846 = 3.835434 \text{ the answer.}$$

To find the number for a given Logarithm.

Example 1. What number answers to the logarithm 3.87157?

In the Table you will find in the column of logarithms, that

$$\log. 7440 = 3.87157.$$

Example 2. What number answers to the logarithm 3.801884?

$$\text{Given logarithm } 3.801884,$$

$$\text{Subt. nearest table log. } 3.801400 = \log. 6330,$$

$$\text{Divided by proportional part, } 69 \overline{)484} \text{ ----- } 7, \\ 6337 \text{ the req. numb.}$$

Multiplication by Logarithms.

Rule. Add together the logarithms of the factors, and the sum is the logarithm of the product.

Example 1. Multiply 425 by 48.

$$\text{To } \log. 425 = 2.62839,$$

$$\text{Add } \log. 48 = 1.68124,$$

$$\text{The product, } \log. 20400 = \underline{4.30963}.$$

Example 2. Multiply 79600 by 0.435.

$$\text{To } \log. 79600 = 4.90091,$$

$$\text{Add, } \log. 0.435 = \underline{.63848-1},$$

$$\text{The product } \log. 34690 = \underline{4.53939}.$$

Division by Logarithms.

Rule. From the logarithm of the dividend subtract the logarithm of the divisor, and the difference is the logarithm of the quotient.

Example 1. Divide 43800 by 368.

$$\text{From } \log. 43800 = 4.64147,$$

$$\text{Subtract } \log. 368 = 2.56584,$$

$$\text{The quotient } \log. 119 = \underline{2.07563}.$$

Example 2. Divide 36 by 0.625.

$$\text{From } \log. 36 = 1.55636,$$

$$\text{Subtract, } \log. 0.625 = \underline{.79588-1}.$$

$$\text{The quotient, } \log. 57.6 = \underline{1.76048}.$$

A negative index follows an opposite operation of its mantissa, as if the mantissa is subtracted, add the negative index, and vice versa.

Evolution by Logarithms.

Rule. Multiply the logarithm of the number by its exponent, and the product is the logarithm of the power of the number.

Involution by Logarithms.

Rule. Divide the logarithm of the number by the index of the root, and the quotient is the logarithm of the root of the number.

LOGARITHM OF NUMBERS, FROM

0 TO 1000.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01702	02118	02530	02938	03342	03742	115
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13355	13672	13987	14301	323
14	14612	14921	15228	15535	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23044	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32221	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36172	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39449	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44715	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46239	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53147	53275	53402	53521	53655	53781	53907	54033	54157	54282	126
35	54406	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62117	62221	104
42	62324	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63346	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66275	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67209	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69019	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72427	72509	72591	72672	72754	72836	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74771	78
No.	0	1	2	3	4	5	6	7	8	9	Prop.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84751	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87906	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92992	93044	93095	93146	93196	93247	93298	93348	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95984	95951	95999	96047	96094	96142	96189	96236	96284	96331	48
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	56
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181	56
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	55
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	55
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	54
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	54
No.	0	1	2	3	4	5	6	7	8	9	Prop.

Examples. Find the Logarithm of

Log. 3 ?	-	-	-	-	-	-	0.47712, the answer.
Log. 54 ?	-	-	-	-	-	-	1.73239, "
Log. 867 ?	-	.	.	.	-	-	2.93802, "

Deg	0'	10'	20'	30'	40'	50'	60'	
0	0:00000	3:46372	3:26475	3:94084	2:06577	2:16268	2:24185	89
1	2:24185	2:30879	2:36677	2:41791	2:46366	2:50504	2:54281	88
2	2:54281	2:57756	2:60973	2:63968	2:66768	2:69399	2:71880	87
3	2:71880	2:74225	2:76451	2:78567	2:80585	2:82513	2:84258	86
4	2:84358	2:86128	2:87828	2:89464	2:91040	2:92560	2:94029	85
5	2:94029	2:95449	2:96824	2:98157	2:99449	1:00704	1:01923	84
6	1:01923	1:03108	1:04262	1:05385	1:06480	1:07548	1:08589	83
7	1:08589	1:09606	1:10599	1:11569	1:12518	1:13447	1:14355	82
8	1:14355	1:15249	1:16116	1:16970	1:17807	1:18628	1:19433	81
9	1:19433	1:20223	1:20999	1:21760	1:22509	1:23244	1:23967	80
10	1:23967	1:24677	1:25376	1:26063	1:26739	1:27404	1:28059	79
11	1:28059	1:28704	1:29339	1:29965	1:30581	1:31189	1:31787	78
12	1:31787	1:32378	1:32959	1:33533	1:34099	1:34657	1:35208	77
13	1:35208	1:35752	1:36288	1:36818	1:37341	1:37857	1:38367	76
14	1:38367	1:38871	1:39368	1:39860	1:40345	1:40825	1:41299	75
15	1:41299	1:41768	1:42231	1:42689	1:43142	1:43590	1:44033	74
16	1:44033	1:44472	1:44905	1:45334	1:45758	1:46178	1:46593	73
17	1:46593	1:47004	1:47411	1:47814	1:48212	1:48607	1:48998	72
18	1:48998	1:49385	1:49768	1:50147	1:50523	1:50895	1:51264	71
19	1:51264	1:51629	1:51991	1:52349	1:52704	1:53056	1:53405	70
20	1:53405	1:53750	1:54093	1:54432	1:54768	1:55102	1:55432	69
21	1:55432	1:55760	1:56085	1:56407	1:56726	1:57043	1:57357	68
22	1:57357	1:57668	1:57977	1:58284	1:58587	1:58889	1:59187	67
23	1:59187	1:59484	1:59778	1:60070	1:60359	1:60646	1:60931	66
24	1:60931	1:61214	1:61494	1:61772	1:62048	1:62322	1:62594	65
25	1:62594	1:62864	1:63132	1:63398	1:63662	1:63924	1:64184	64
26	1:64184	1:64442	1:64698	1:64952	1:65205	1:65455	1:65704	63
27	1:65704	1:65951	1:66197	1:66440	1:66682	1:66922	1:67160	62
28	1:67160	1:67397	1:67632	1:67866	1:68098	1:68328	1:68557	61
29	1:68557	1:68784	1:69009	1:69233	1:69456	1:69677	1:69897	60
30	1:69897	1:70115	1:70331	1:70546	1:70760	1:70773	1:71183	59
31	1:71183	1:71393	1:71601	1:71808	1:72014	1:72218	1:72421	58
32	1:72421	1:72622	1:72822	1:73021	1:73219	1:73415	1:73610	57
33	1:73610	1:73804	1:73997	1:74188	1:74379	1:74568	1:74756	56
34	1:74756	1:74942	1:75128	1:75312	1:75496	1:75678	1:75859	55
35	1:75859	1:76039	1:76217	1:76395	1:76572	1:76747	1:76921	54
36	1:76921	1:77095	1:77267	1:77438	1:77609	1:77778	1:77946	53
37	1:77946	1:78113	1:78279	1:78444	1:78608	1:78772	1:78934	52
38	1:78934	1:79095	1:79255	1:79415	1:79573	1:79730	1:79887	51
39	1:79887	1:80042	1:80197	1:80351	1:80503	1:80655	1:80806	50
40	1:80806	1:80956	1:81106	1:81254	1:81401	1:81548	1:81694	49
41	1:81694	1:81839	1:81983	1:82126	1:82268	1:82410	1:82551	48
42	1:82551	1:82691	1:82830	1:82968	1:83105	1:83242	1:83378	47
43	1:83378	1:83513	1:83647	1:83781	1:83914	1:84045	1:84177	46
44	1:84177	1:84307	1:84437	1:84566	1:84694	1:84821	1:84948	45
45	1:84948	1:85074	1:85199	1:85324	1:85448	1:85571	1:85693	44
	60'	50'	40'	30'	20'	10'	0'	Deg

LOGARITHM COSINE.

The negative index is noted by two points, and must always follow an opposite operation to that of the mantissa. If the mantissa is added, subtract the index, and vice versa.

Deg.	0'	10'	20'	30'	40'	50'	60'	
46	1:85693	1:85815	1:85936	1:86056	1:86175	1:86294	1:86412	43
47	1:86412	1:86530	1:86647	1:86763	1:86878	1:86993	1:87107	42
48	1:87107	1:87220	1:87333	1:87445	1:87557	1:87667	1:87778	41
49	1:87778	1:87887	1:87996	1:88104	1:88212	1:88319	1:88425	40
50	1:88425	1:88531	1:88636	1:88740	1:88844	1:88947	1:89050	39
51	1:89050	1:89152	1:89253	1:89354	1:89454	1:89554	1:89653	38
52	1:89653	1:89751	1:89849	1:89946	1:90043	1:90139	1:90234	37
53	1:90234	1:90329	1:90424	1:90517	1:90611	1:90703	1:90795	36
54	1:90795	1:90887	1:90978	1:91068	1:91158	1:91247	1:91336	35
55	1:91336	1:91424	1:91512	1:91599	1:91685	1:91771	1:91857	34
56	1:91857	1:91942	1:92026	1:92110	1:92194	1:92276	1:92359	33
57	1:92359	1:92440	1:92522	1:92602	1:92683	1:92762	1:92842	32
58	1:92842	1:92920	1:92998	1:93076	1:93153	1:93230	1:93306	31
59	1:93306	1:93382	1:93457	1:93532	1:93606	1:93679	1:93753	30
60	1:93753	1:93825	1:93898	1:93969	1:94040	1:94111	1:94181	29
61	1:94181	1:94251	1:94321	1:94389	1:94458	1:94526	1:94593	28
62	1:94593	1:94660	1:94726	1:94792	1:94858	1:94923	1:94988	27
63	1:94988	1:95052	1:95115	1:95179	1:95241	1:95304	1:95366	26
64	1:95366	1:95427	1:95488	1:95548	1:95608	1:95668	1:95727	25
65	1:95727	1:95786	1:95844	1:95902	1:95959	1:96016	1:96073	24
66	1:96073	1:96129	1:96184	1:96239	1:96294	1:96348	1:96402	23
67	1:96402	1:96456	1:96509	1:96561	1:96613	1:96665	1:96716	22
68	1:96716	1:96767	1:96817	1:96867	1:96917	1:96966	1:97015	21
69	1:97015	1:97063	1:97111	1:97158	1:97205	1:97252	1:97298	20
70	1:97298	1:97344	1:97389	1:97434	1:97479	1:97523	1:97567	19
71	1:97567	1:97610	1:97653	1:97695	1:97737	1:97779	1:97820	18
72	1:97820	1:97861	1:97901	1:97942	1:97981	1:98020	1:98059	17
73	1:98059	1:98098	1:98136	1:98173	1:98210	1:98247	1:98284	16
74	1:98284	1:98320	1:98355	1:98391	1:98425	1:98460	1:98494	15
75	1:98494	1:98528	1:98561	1:98594	1:98626	1:98658	1:98690	14
76	1:98690	1:98721	1:98752	1:98783	1:98813	1:98843	1:98872	13
77	1:98872	1:98901	1:98930	1:98958	1:98986	1:99013	1:99040	12
78	1:99040	1:99067	1:99093	1:99119	1:99144	1:99169	1:99194	11
79	1:99194	1:99219	1:99243	1:99266	1:99289	1:99312	1:99335	10
80	1:99335	1:99357	1:99378	1:99400	1:99421	1:99441	1:99462	9
81	1:99462	1:99481	1:99501	1:99520	1:99539	1:99557	1:99575	8
82	1:99575	1:99592	1:99610	1:99626	1:99643	1:99659	1:99675	7
83	1:99675	1:99690	1:99705	1:99719	1:99734	1:99748	1:99761	6
84	1:99761	1:99774	1:99787	1:99799	1:99811	1:99823	1:99834	5
85	1:99834	1:99845	1:99855	1:99865	1:99875	1:99885	1:99894	4
86	1:99894	1:99902	1:99911	1:99918	1:99926	1:99933	1:99940	3
87	1:99940	1:99946	1:99952	1:99958	1:99964	1:99968	1:99973	2
88	1:99973	1:99977	1:99981	1:99985	1:99988	1:99991	1:99993	1
89	1:99993	1:99995	1:99997	1:99998	1:99999	1:99999	1:99999	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

LOGARITHM COSINE.

Examples. Find the Logarithms,

Log:sine $35^{\circ} 40'$? - - - - - = 1:76572, the answer.
 Log:cosine $18^{\circ} 20'$? - - - - - = 1:97737, "

Deg.	0'	10'	20'	30'	40'	50'	60'	Deg.
0	3:00000	3:46372	3:76476	3:94085	2:06580	2:16272	2:24192	89
1	2:24192	2:30888	2:36689	2:41806	2:46384	2:50526	2:54308	88
2	2:54308	2:57787	2:61009	2:64009	2:66816	2:69452	2:71939	87
3	2:71939	2:74292	2:76524	2:78648	2:80674	2:82610	2:84464	86
4	2:84464	2:86243	2:87952	2:89598	2:91184	2:92715	2:94195	85
5	2:94195	2:95626	2:97013	2:98357	2:99662	1:00929	1:02162	84
6	1:02162	1:03360	1:04528	1:05665	1:06775	1:07857	1:08914	83
7	1:08914	1:09946	1:10955	1:11942	1:12908	1:13854	1:14780	82
8	1:14780	1:15687	1:16577	1:17449	1:18305	1:19146	1:19971	81
9	1:19971	1:20781	1:21578	1:22360	1:23130	1:23887	1:24631	80
10	1:24631	1:25364	1:26086	1:26796	1:27496	1:28185	1:28863	79
11	1:28865	1:29534	1:30195	1:30846	1:31488	1:32122	1:32747	78
12	1:32747	1:33364	1:33973	1:34575	1:35169	1:35756	1:36336	77
13	1:36336	1:36909	1:37475	1:38035	1:38588	1:39136	1:39677	76
14	1:39677	1:40212	1:40741	1:41265	1:41784	1:42297	1:42805	75
15	1:42805	1:43308	1:43805	1:44298	1:44787	1:45270	1:45749	74
16	1:45749	1:46224	1:46694	1:47160	1:47622	1:48080	1:48533	73
17	1:48533	1:48983	1:49429	1:49872	1:50310	1:50746	1:51177	72
18	1:51177	1:51605	1:52030	1:52452	1:52870	1:53285	1:53697	71
19	1:53697	1:54106	1:54511	1:54914	1:55314	1:55712	1:56106	70
20	1:56106	1:56498	1:56887	1:57273	1:57657	1:58038	1:58417	69
21	1:58417	1:58794	1:59168	1:59539	1:59909	1:60276	1:60641	68
22	1:60641	1:61003	1:61364	1:61722	1:62078	1:62433	1:62785	67
23	1:62785	1:63135	1:63483	1:63830	1:64174	1:64517	1:64858	66
24	1:64858	1:65197	1:65534	1:65870	1:66204	1:66536	1:66867	65
25	1:66867	1:67196	1:67523	1:67849	1:68174	1:68496	1:68818	64
26	1:68818	1:69138	1:69456	1:69773	1:70089	1:70403	1:70716	63
27	1:70716	1:71028	1:71338	1:71647	1:71955	1:72262	1:72567	62
28	1:72567	1:72871	1:73174	1:73476	1:73777	1:74076	1:74375	61
29	1:74374	1:74672	1:74968	1:75264	1:75558	1:75851	1:76143	60
30	1:76143	1:76435	1:76725	1:77014	1:77303	1:77590	1:77877	59
31	1:77877	1:78163	1:78447	1:78731	1:79015	1:79293	1:79578	58
32	1:79578	1:79859	1:80139	1:80418	1:80697	1:80974	1:81251	57
33	1:81251	1:81527	1:81803	1:82078	1:82352	1:82625	1:82898	56
34	1:82898	1:83170	1:83442	1:83713	1:83983	1:84253	1:84522	55
35	1:84522	1:84791	1:85059	1:85326	1:85593	1:85860	1:86126	54
36	1:86126	1:86391	1:86656	1:86920	1:87184	1:87448	1:87711	53
37	1:87711	1:87974	1:88236	1:88498	1:88759	1:89020	1:89281	52
38	1:89281	1:89541	1:89801	1:90060	1:90319	1:90578	1:90836	51
39	1:90836	1:91095	1:91352	1:91610	1:91867	1:92121	1:92381	50
40	1:92381	1:92637	1:92894	1:93149	1:93405	1:93661	1:93916	49
41	1:93916	1:94171	1:94427	1:94680	1:94935	1:95189	1:95442	48
42	1:95443	1:95697	1:95952	1:96205	1:96458	1:96712	1:96965	47
43	1:96965	1:97218	1:97471	1:97725	1:97978	1:98230	1:98482	46
44	1:98485	1:98736	1:98989	1:99242	1:99494	1:99747	1:00000	45
Deg.	60'	50'	40'	30'	20'	10'	0'	Deg.

LOGARITHM COTANGENT.

The negative index is noted by two points, and must always follow an opposite operation to that of the mantissa. If the mantissa is added, subtract the index, and *vice versa*.

Deg	0'	10'	20'	30'	40'	50'	60'	
45	0.00000	0.00252	0.00505	0.00758	0.01010	0.01263	0.01516	44
46	0.01516	0.01769	0.02022	0.02275	0.02528	0.02781	0.03034	43
47	0.03034	0.03287	0.03541	0.03794	0.04048	0.04302	0.04556	42
48	0.04556	0.04810	0.05064	0.05319	0.05573	0.05828	0.06083	41
49	0.06083	0.06339	0.06594	0.06850	0.07106	0.07362	0.07618	40
50	0.07618	0.07875	0.08132	0.08389	0.08647	0.08904	0.09162	39
51	0.09163	0.09421	0.09680	0.09939	0.10199	0.10458	0.10716	38
52	0.10719	0.10979	0.11240	0.11502	0.11763	0.12025	0.12288	37
53	0.12288	0.12551	0.12815	0.13079	0.13343	0.13608	0.13873	36
54	0.13873	0.14139	0.14406	0.14673	0.14940	0.15208	0.15477	35
55	0.15477	0.15746	0.16016	0.16286	0.16557	0.16829	0.17101	34
56	0.17101	0.17374	0.17647	0.17921	0.18196	0.18472	0.18748	33
57	0.18748	0.19025	0.19302	0.19581	0.19860	0.20140	0.20421	32
58	0.20421	0.20702	0.20984	0.21268	0.21552	0.21836	0.22122	31
59	0.22122	0.22409	0.22696	0.22985	0.23274	0.23564	0.23856	30
60	0.23856	0.24148	0.24441	0.24735	0.25031	0.25327	0.25624	29
61	0.25624	0.25923	0.26222	0.26523	0.26825	0.27128	0.27432	28
62	0.27432	0.27737	0.28044	0.28352	0.28661	0.28971	0.29283	27
63	0.29283	0.29596	0.29910	0.30226	0.30543	0.30861	0.31181	26
64	0.31181	0.31503	0.31826	0.32150	0.32476	0.32803	0.33132	25
65	0.33132	0.33463	0.33795	0.34129	0.34465	0.34802	0.35141	24
66	0.35141	0.35482	0.35825	0.36169	0.36516	0.36864	0.37214	23
67	0.37214	0.37567	0.37921	0.38277	0.38635	0.38996	0.39359	22
68	0.39359	0.39723	0.40090	0.40460	0.40831	0.41205	0.41582	21
69	0.41582	0.41961	0.42342	0.42726	0.43112	0.43501	0.43893	20
70	0.43893	0.44287	0.44685	0.45085	0.45488	0.45893	0.46302	19
71	0.46302	0.46714	0.47129	0.47548	0.47969	0.48394	0.48822	18
72	0.48822	0.49254	0.49689	0.50127	0.50570	0.51016	0.51466	17
73	0.51466	0.51919	0.52377	0.52839	0.53305	0.53775	0.54250	16
74	0.54250	0.54729	0.55213	0.55701	0.56194	0.56692	0.57194	15
75	0.57194	0.57702	0.58215	0.58734	0.59258	0.59787	0.60322	14
76	0.60322	0.60864	0.61411	0.61964	0.62524	0.63090	0.63663	13
77	0.63663	0.64243	0.64830	0.65424	0.66026	0.66635	0.67252	12
78	0.67252	0.67877	0.68511	0.69153	0.69804	0.70465	0.71133	11
79	0.71133	0.71814	0.72503	0.73203	0.73913	0.74635	0.75368	10
80	0.75368	0.76112	0.76869	0.77639	0.78422	0.79218	0.80028	9
81	0.80028	0.80853	0.81694	0.82550	0.83422	0.84312	0.85219	8
82	0.85219	0.86145	0.87091	0.88057	0.89044	0.90053	0.91085	7
83	0.91085	0.92142	0.93224	0.94334	0.95471	0.96639	0.97838	6
84	0.97838	0.99070	1.00337	1.01642	0.02986	1.04373	1.05804	5
85	1.05804	1.07284	1.08815	1.10401	1.12047	1.13756	1.15535	4
86	1.15535	1.17389	1.19325	1.21351	1.23475	1.25707	1.28060	3
87	1.28060	1.30547	1.33184	1.35990	1.38990	1.42212	1.45691	2
88	1.45691	1.49473	1.53615	1.58193	1.63310	1.69111	1.75807	1
89	1.75807	1.83727	1.93419	2.05914	1.23523	2.53627	0.00000	0
	60'	50'	40'	30'	20'	10'	0'	Deg

LOGARITHM COTANGENT.

Example. Find the Logarithms,

Log.tan. $36^{\circ} 40'$? - - - = 1.87185, the answer.

Log.tan. $58^{\circ} 50'$? - - - = 0.21836, "

ARITHMETICAL PROGRESSION.

Arithmetical Progression is a series of numbers, as 2, 4, 6, 8, 10, 12, &c., or 18, 15, 12, 9, 6, 3, in which every successive term is increased or diminished by a constant number.

Letters will denote,

a = the first term of the series.

b = any other term whose number from a is n .

n = number of terms within a and b .

δ = the difference between the terms.

S = the sum of all the terms.

In the series, 2, 5, 8, 11, $a = 2$, $b = 11$, $n = 4$, $\delta = 3$, and $S = 26$.

When the series is decreasing, take the first term = b and the last term = a .

The accompanying Table contains all the formulas or questions in Arithmetical Progressions, and the nature of the question will tell which formula is to be used.

Formulas for Arithmetical Progressions.

$$a = b - \delta (n-1), \quad . \quad . \quad . \quad 1,$$

$$a = \frac{2S}{n} - b, \quad . \quad . \quad . \quad 2,$$

$$a = \frac{S}{n} - \frac{\delta}{2}(n-1), \quad . \quad . \quad . \quad 3,$$

$$b = a + \delta (n-1), \quad . \quad . \quad . \quad 4,$$

$$b = \frac{2S}{n} + a, \quad . \quad . \quad . \quad 5,$$

$$b = \frac{S}{n} + \frac{\delta}{2}(n-1), \quad . \quad . \quad . \quad 6,$$

$$n = \frac{b-a}{\delta} + 1, \quad . \quad . \quad . \quad 7,$$

$$n = \frac{2S}{a+b}, \quad . \quad . \quad . \quad 8,$$

$$\delta = \frac{b-a}{n-1}, \quad . \quad . \quad . \quad 9,$$

$$\delta = \frac{(b+a)(b-a)}{2S-a-b}, \quad . \quad . \quad . \quad 10,$$

$$\delta = \frac{2(S-an)}{n(n-1)}, \quad . \quad . \quad . \quad 11,$$

$$\delta = \frac{2(bn-S)}{n(n-1)}, \quad . \quad . \quad . \quad 12,$$

$$S = \frac{n(a+b)}{2}, \quad . \quad . \quad . \quad 13,$$

$$S = \frac{(a+b)(b+\delta-a)}{2\delta}, \quad . \quad . \quad . \quad 14,$$

$$S = n \left[a + \frac{\delta}{2}(n-1) \right] \quad 15,$$

$$S = n \left[b - \frac{\delta}{2}(n-1) \right] \quad . \quad . \quad . \quad 16,$$

$$a = \frac{\delta}{2} \pm \sqrt{\left(b + \frac{\delta}{2}\right)^2 - 2\delta S}, \quad . \quad . \quad . \quad 17,$$

$$b = \frac{\delta}{2} \pm \sqrt{\left(a - \frac{\delta}{2}\right)^2 + 2\delta S}, \quad . \quad . \quad . \quad 18,$$

$$n = \frac{1}{2} - \frac{a}{\delta} \pm \sqrt{\left(\frac{1}{2} - \frac{a}{\delta}\right)^2 + \frac{2S}{\delta}}, \quad . \quad . \quad . \quad 19,$$

$$n = \frac{1}{2} + \frac{b}{\delta} \pm \sqrt{\left(\frac{1}{2} + \frac{b}{\delta}\right)^2 - \frac{2S}{\delta}}, \quad . \quad . \quad . \quad 20,$$

Example 1. A man was engaged to dig a well at one dollar (\$1) for the first foot of the depth of the well, \$1.84 for the second, and 84 cents more per every successive foot in depth, until he reached the water, which was found at a depth of 25 feet. How much money is due to the man?

This will be answered by the formula 15, in which $a=1$, $d=0.84$, and $n=25$, then the sum,

$$S = 25 \left[1 + \frac{0.84}{2}(25 - 1) \right] = \$277 \text{ the answer.}$$

Example 2. A Propeller ship which is to run between Philadelphia and Charleston, cost \$116500, of which the company agreed to pay on account \$14075 at her first trip to Charleston; and per every successive trip, they paid \$650 less than the former. How many trips must the vessel make until she is fully paid?

This will be answered by the formula 20, in which $b = \$14075$, $d = 650$, and $S = 116500$.

$$n = \frac{1}{2} + \frac{14075}{650} - \sqrt{\left(\frac{14075}{650} + \frac{1}{2} \right)^2 - \frac{2 \times 116500}{650}} = 10.6 \text{ or } 11 \text{ trips.}$$

Arithmetical Progressions of a Higher Order.

Arithmetical Progressions are of the first order, when the difference δ is a constant number, but when the difference δ progresses itself with a constant number, the Progression is of the *second order*.

When the difference δ progresses in a second order, the Progression is of the *third order*, &c., &c., and is thus explained:

1, 2, 3, 4, 5, 6, n , - - - Arith. Prog., first order.

1, 3, 6, 10, 15, 21, $\frac{n(n+1)}{2}$ - - - 2d. order.

1, 4, 10, 20, 35, 56, $\frac{n(n+1)(n+2)}{2 \times 3}$, - - - 3d. order.

1, 5, 15, 35, 70, 126, $\frac{n(n+1)(n+2)(n+3)}{2 \times 3 \times 4}$, - - - 4th. order.

Here you will discover that the sum of n terms in one order, is equal to the same n th term in the next higher order. Arithmetical Progressions of the *first*, *second*, and *third orders*, are applied to

PILES OF BALLS AND SHELLS.

Triangular Piling.

Example 1. A complete triangular pile of balls has $n = 12$ balls in each side. Require how many balls in the base, and how many in the whole pile?

In the base, . . . = $\frac{12(12+1)}{2} = 73$ balls, - - - 2d. order.

Whole pile, . . . = $\frac{12(12+1)(12+2)}{2 \times 3} = 364$ balls, - - - 3d. order.

Square Piling.

1, 4, 9, 16, 25, 36, n^2 - - - 2d. order.

1, 5, 14, 30, 55, 91, $\frac{n(n+1)(2n+1)}{2 \times 3}$, - - - 3d. order.

[See Examples 2 and 3 on page 67.]

GEOMETRICAL PROGRESSION.

Geometrical Progression is a series of numbers, as $2:4:8:16:32: \&c.$, or $729:243:81:27:9: \&c.$, in which every successive term is multiplied or divided by a constant factor.

Letters will denote,

a = the first term of the series.

b = any other term whose number from a is n .

n = number of terms within a and b .

r = ratio, or the factor by which the terms are multiplied or divided.

S = Sum of the terms.

In the series $1:3:9:27: a = 1, b = 27, n = 4, r = 3, S = 40$.

The accompanying Table contains all the formulas or questions in Geometrical Progressions. The nature of the question will tell which formula is to be used.

Formulas for Geometrical Progressions.

$$a = \frac{b}{r^{n-1}}, \quad - \quad - \quad - \quad 1,$$

$$a = S - r(S - b), \quad - \quad - \quad 2,$$

$$a = S \frac{r-1}{r^n-1}, \quad - \quad - \quad 3,$$

$$r = \sqrt[n-1]{\frac{b}{a}}, \quad - \quad - \quad - \quad 7,$$

$$r = \frac{S-a}{S-b}, \quad - \quad - \quad - \quad 8,$$

$$ar^n + S - rS - a = 0, \quad - \quad 9,$$

$$b = ar^{n-1}, \quad - \quad - \quad - \quad 4,$$

$$b = S - \frac{S-a}{r}, \quad - \quad - \quad 5,$$

$$b = S \left(\frac{r-1}{r^n-1} \right) r^{n-1}, \quad 6,$$

$$S = \frac{br-a}{r-1}, \quad - \quad - \quad - \quad 10,$$

$$S = \frac{a(r^n-1)}{1-r}, \quad - \quad - \quad - \quad 11,$$

$$S = \frac{b(r^n-1)}{(r-1)r^{n-1}}, \quad - \quad - \quad 12,$$

$$n = 1 + \frac{\log b - \log a}{\log r}, \quad - \quad - \quad - \quad - \quad 13,$$

$$n = 1 + \frac{\log b - \log a}{\log(S+a) - \log(S-b)}, \quad - \quad - \quad - \quad - \quad 14,$$

$$n = \frac{\log[a + S(r-1)] - \log a}{\log r}, \quad - \quad - \quad - \quad - \quad 15,$$

$$n = 1 + \frac{\log b - \log[br - S(r-1)]}{\log r}, \quad - \quad - \quad - \quad - \quad 16,$$

$$S = \frac{b \sqrt[n-1]{b} - a \sqrt[n-1]{a}}{\sqrt[n-1]{b} - \sqrt[n-1]{a}}, \quad - \quad - \quad - \quad - \quad 17,$$

Example 1. Required the 10th term in the Geometrical Progression $4:12:36\dots$?

Given $a = 4$, $n = 10$, and $r = 3$. We have,

Formula 4. $b = ar^{n-1} = 4 \times 3^9 = 78732$, the tenth term.

Example 2. Required the sum of the 10 terms in the preceding example?

Formula 11, $S = \frac{a(r^n - 1)}{r - 1} = \frac{4(3^{10} - 1)}{2} = 118096$, the sum.

Example 3. Insert 6 proportional terms between 3 and 384?

Given $a = 3$, $b = 384$, and $n = 6 + 2 = 8$.

Formula 7, $r = \sqrt[n-1]{\frac{b}{a}} = \sqrt[7]{\frac{384}{3}} = 2$,

then $3:6:12:24:48:96:192:384$, the answer.

Example 4. A man had 16 twenty dollar gold pieces, which he agreed to exchange for copper in such a way, that he gets one cent on the first \$20, two on the second, four on the third, and eight on the fourth, &c., &c.; until the sixteen \$20 pieces were covered. How many cents will come on the sixteenth gold piece, and what will be the whole amount of copper on the gold?

In the progression $1:2:4:8:\&c.$, we have,

Given $n = 16$, $r = 2$, and $a = 1$, then,

Formula 4. $b = 1 \times 2^{16-1} = \frac{2^{16}}{2} = \frac{48}{2} = \frac{16^4}{2} = \frac{256^2}{2} = 32768$ cents, on the sixteenth piece.

The total sum of cents will be found by the

Formula 10. $S = \frac{32768 \times 2 - 1}{2 - 1} = 65535$ cents = \$655.35.

Piling of Balls and Shells.—[From page 64.]

Example 2. How many balls are contained in a complete square pile, $n = 10$ rows?

$$\frac{10(10+1)(2 \times 10 + 1)}{2 \times 3} = \frac{10 \times 11 \times 21}{6} = 385 \text{ balls.}$$

Rectangular Piling.

Let m be the number of balls on the top of the complete pile, and $n =$ number of rows in the same, then the number of balls in the whole pile will be,

$$\frac{n(n+1)(2n+3m-2)}{2 \times 3}, \quad \text{3d. order.}$$

The number of balls in the longest bottom side will be $= m + n - 1$.

Example 3. The rectangular pile having 15 rows and 23 balls on the the top, how many in the whole pile?

$$\frac{15(15+1)(2 \times 15 + 3 \times 23 - 2)}{2 \times 3} = \frac{15 \times 16 \times 67}{6} = 2680 \text{ balls.}$$

COMPOUND INTEREST.

Compound Interest is when the Interest is added to the Capital for each year, and the sum is the Capital for the following year.

Amount, $a = c(1+p)^n$, 1,

Capital, $c = \frac{a}{(1+p)^n}$, 2,

Per centage, $p = \sqrt[n]{\frac{a}{c}} - 1$, 3,

Number of years, $n = \frac{\log a - \log c}{\log(1+p)}$, 4,

In these formulas p must be expressed in a fraction of 100.

Example 1. A capital $c = \$650$ standing with Compound Interest at $p = 5$ per cent. what will it amount to in $n = 9$ years.

Amount $a = \$650 (1.05)^9 = \13419 dollars.

Example 2. A man commenced business with $c = 300$ dollars, after $n = 5$ years he had $a = \$875$ dollars. At what rate did his money increase, and how soon will he have a fortune of 50000 dollars?

The first question, or the per centage, will be answered by the formula 3.

$$p = \sqrt[5]{\frac{875}{300}} - 1 = \sqrt[5]{2.29166} - 1 = 0.87, \text{ or } 87 \text{ per cent.}$$

The time from the commencement of business until the fortune is completed, will be answered from the formula 4.

$$n = \frac{\log 50000 - \log 300}{\log 1.87} = \frac{4.69897 - 2.47712}{0.2720048} = 8.169 \text{ years,}$$

or 8 years and 2 months.

ANNUITIES.

Annuity is a certain sum of money to be paid at regular intervals.

A yearly payment or annuity b , is standing for n years, to find the whole amount a at p per cent. Interest.

Amount, $a = bn \left[1 + \frac{p}{2} (n+1) \right]$ Simple Int., 1,

Amount, $a = \frac{b}{p} [(1+p)^n - 1]$ Comp. Int., 2,

A yearly payment or annuity b , is to be paid for n years. to find the present worth, or the amount a , which would pay it in full, at the beginning of the time n , deducting p per cent. Interest.

Amount, $a = bn \left[1 - \frac{p}{2} \left(\frac{n+1}{1+pn} \right) \right]$ Simp. Int., 3,

Amount, $a = \frac{b}{p} \left[1 - \frac{1}{(1+p)^n} \right]$ Comp. Int., 4,

A debt D , standing for Interest, is diminished yearly by a sum b ; to find the debt d after n years, and the time n when it is fully paid?

The debt d after n years will be,

$$d = \frac{(Dp - b)(1+p)^n + b}{p} \quad \text{Comp. Int.,} \quad - \quad - \quad - \quad 5.$$

The time n until fully paid will be,

$$n = \frac{\log b - \log.(b - Dp)}{\log.(1+p)} \quad - \quad - \quad - \quad - \quad - \quad 6.$$

If $b = Dp$ then $n = \infty$, or the debt D will never be paid. If $b < Dp$, the debt D will be increased.

To find the yearly annuity b , which will pay a debt D in n years, at p per cent. Compound Interest?

$$b = \frac{Dp(1+p)^n}{(1+p)^n - 1} \quad - \quad - \quad - \quad - \quad - \quad 7.$$

PAPER.

1 Ream = 20 quires = 480 sheets.

1 quire = 24 sheets.

Drawing Paper.

Cap, - - -	13×16 inches.	Columbier, - -	34×23 inches.
Demy, - - -	20×15 "	Atlas, - - -	33×26 "
Medium, - -	22×17 "	Theorem, - -	34×28 "
Royal, - - -	24×19 "	Double Elephant,	40×26 "
Super Royal, -	27×19 "	Antiquarian, -	52×31 "
Imperial, - -	30×21 "	Emperor, - -	40×60 "
Elephant, - -	28×22 "	Uncle Sam, - -	48×120 "

Continuous Colossal Drawing Paper, No. A, and No. B, 56 inches wide, and of any required length. No. A, of this paper is excellent for mechanical drawings. Price from 40 to 50 cents per yard.

Tracing Paper.

Double Crown,	30 by 20 inches.	} Glazed or Crystal, Yellow or Blue Wove.
Double Double Crown,	40 " 30 "	
Double Double Double Crown,	60 " 40 "	

Finest French Vegetable Tracing Paper.

Grand Raisin (or Royal) 24 in. by 18. Grand Aigle 40 in. by 27.

Mounted Tracing Paper.

This paper is mounted on cloth, and is still transparent; it will take ink and water colours. It is 38 inches wide, and of any required length.

Vellum Writing Cloth,

Adapted for every description of tracing; it is transparent, durable, and strong. It is 18 to 38 inches wide, and of any required length.

SELECTION OF WATER COLOURS.

Blue. Real Ultramarine.
" French Blue.
" Indigo.
" Cobalt Blue.

Green. Olive Green.

Yellow. Cadmium.

" Gamboge.

" Ochre.

Red. Carmine.

" Crimson Lake.

Red. Rose Madder.

" Light Red.

Brown. Vandyke.

" Brown Madder.

Black. India Ink.

" Blue Black.

" Ivory Black.

" Lamp Black.

White. Chinese White.

UNITED STATES' STANDARD MEASURES AND WEIGHTS.

MEASURE OF LENGTH.

The *Standard Measure of Length* is a brass rod = 1 yard at the temperature of 32° Fahrenheit. The length of a pendulum vibrating seconds in vacuo, at Philadelphia is 1·08614 yards, at + 32° Fahrenheit.

The *Surveying Chain* is = 22 yards = 66 feet. It consists of 100 links, and each link = 7·92 inches.

ROPES AND CABLES.

1 Cable length = 120 fathoms = 720 feet.

1 fathom = 6 feet.

GEOGRAPHICAL AND NAUTICAL MEASURES.

1 Degree of the great circle of the Earth round the Equator = 69·545 statute miles = 60 Nautical miles.

1 Statute mile = 5280 feet = 0·86275 Nautical miles.

1 Nautical mile = 6120 feet = 1·159 Statute miles.

LOG LINE.

The *Log Line* should be about 150 fathoms long, and 10 fathoms from the Log to the first knot on the line. If half a minute glass is used, it will be 51 feet between each succeeding knot. For 28 seconds glass it will be 47·6 feet = 7·93 fathoms per knot. This is the length of knot by calculation, but practically it is shortened to 7·5 fathoms per knot for 28 seconds glass.

MEASURE OF CAPACITY.

Gallon. The standard Gallon measures 231 cubic inches, and contains 8·338822 pounds Avoirdupois = 58372·1757 grains Troy, of distilled water, at its maximum density 39·83° Fahrenheit, and 30 inches barometer height.

Bushel. The standard Bushel measures 2150·42 cubic inches = 77·627413 pounds Avoirdupois of distilled water at 39·83° Fahrenheit, barometer 30 inches. Its dimensions are 18½ inches inside diameter, 19½ inches outside, and 8 inches deep; and when heaped, the cone must not be less than 6 inches high, equal 2747·70 cubic inches for a true cone.

Pound. The standard Pound Avoirdupois is the weight of 27·7015 cubic inches of distilled water, at 39·83° Fahrenheit, barometer 30 inches, and weighed in the air.

MEASURE OF LENGTH.

Miles.	Furlongs.	Chains.	Rods.	Yards.	Feet.	Inches.
1	8	80	320	1760	5280	63360
0·125	1	10	40	220	660	7920
0·0125	0·1	1	4	22	66	792
0·003125	0·025	0·25	1	5·5	16·5	198
0·0005318	0·0045454	0·045454	0·181818	1	3	36
0·0001893	0·00151515	0·01515151	0·060606	0·33333	1	12
0·000015783	0·000126262	0·00126262	0·00505050	0·0277777	0·083333	1

MEASURE OF SURFACE.

Sq. Miles.	Acres	S.Chains.	Sq. Rods.	Sq. Yards.	Sq. Feet.	Sq. Inches.
1	640	6400	102400	3072000	27878400	4014489600
0·001562	1	10	160	4840	43560	696960
0·0001562	0·1	1	16	484	4356	69696
0·000003764	0·00325	0·0325	1	30·25	272·25	39204
0·000000323	0·0002066	0·002066	0·0330	1	9	1296
0·0000000358	0·00002230	0·0002230	0·00357	0·1111111	1	144
0·00000000025	0·000000143	0·00000143	0·00002552	20·0007716	0·003944	1

MEASURE OF CAPACITY.

Cub. Yard.	Barrels.	Bushels.	Cub. Feet.	Pecks.	Gallons.	Cub. Inch.
1	5.6103	25.2467	27	100.887	201.974	46656
0.1782	1	4.5	4.8125	18	36	8316
0.03961	0.2222	1	1.2438	4	8	2150.42
0.037037	0.2078	0.804	1	3.73809	7.47619	1728
0.60402	0.05555	0.25	0.26738	1	2	412
0.60451	0.02777	0.125	0.12369	0.5	1	231
0.00002143	0.0001202	0.000465	0.0005787	0.0021645	0.004329	1

MEASURE OF LIQUIDS.

Gallon.	Quarts.	Pints.	Gills.	Cub. inch.
1	4	8	32	231
0.25	1	2	8	57.75
0.125	0.5	1	4	28.875
0.03125	0.125	0.25	1	7.2175
0.004329	0.017315	0.03463	0.13858	1

MEASURES OF WEIGHTS.

AVOIRDUPOIS.

Ton.	Cwt.	Pounds.	Ounces.	Drams.
1	20	2240	35840	573440
0.05	1	112	1792	28672
0.00044642	0.0089285	1	16	256
0.00002730	0.000558	0.0025	1	16
0.00000174	0.0000348	0.0016	0.0025	1

TROY.

Pounds.	Ounces.	Dwt.	Grains.	Pound Avoir.
1	12	240	5760	0.822861
0.083333	1	20	480	0.068571
0.004166	0.05000	1	24	0.003425
0.0001736	0.00208333	0.0416666	1	0.00020571
1.215275	14.58333	29.16666	4861	1

APOTHECARIES'.

Pounds.	Ounces.	Drams.	Scruples.	Grains.
1	12	96	988	5760
0.08333	1	8	24	480
0.01041666	0.125	1	3	60
0.0034722	0.0416666	0.3333	1	20
0.00017361	0.020833	0.16666	0.05	1

DIAMOND.

Carat.	Grain.	Parts.	Grains. Troy.
1	4	64	32
0.25	1	16	0.8
0.015625	0.0625	1	0.05
0.3125	1.25	20	1

GOLD COINS. U. S. STANDARD WEIGHT.

Name of the Coins	Dollars.	WEIGHT TROY.	
		Grains.	Ounces.
Double Eagle	\$ 20	516	1.075
Eagle	\$ 10	258	0.5375
Half Eagle	\$ 5	129	0.26875
Three Dollar piece	\$ 3	77.4	0.16125
Quarter Eagle	\$ 2.50	64.5	0.134375
Dollar piece	\$ 1	25.8	0.05375
Value per Grain	\$ 0.0387596	1	0.00208333
Value per Ounce	\$ 18.0046	480	1

SILVER COINS. U. S. STANDARD WEIGHT.

Name of the Coins.	Cents.	WEIGHT TROY.	
		Grains.	Ounces.
One Dollar	100	384	0.8
Half Dollar or five Dimes	50	192	0.4
Quarter Dollar or 2½ Dimes	25	96	0.2
One Dime	10	38.4	0.08
Half Dime	5	19.2	0.04
Three Cents piece	3	11.52	0.024
Value per Grain	0.26041668	1	0.00208333
Value per Ounce	125	480	1
Copper Cent	1	168	0.35
Half Cent	0.5	84	0.175
Value per Grain	0.00595238	1	0.00208333
Value per Ounce	2.8571424	480	1

The Standard fineness of Gold and Silver Coins is *one weight of alloy to nine weights of pure metal*. The alloy for Gold Coin is Silver and Copper, and Copper for Silver Coin.

Relative value of Foreign Gold and Silver Coins, fixed by the law of the United States.

1 Pound Sterling of Great Britain	\$ 4.84
1 Shilling	0.242
1 Pound Sterling of Nova Scotia, New Brunswick, Newfoundland and Canada	4.00
1 Dollar of Mexico, Peru, and Central America	1.00
1 Pagoda of India	1.84
1 Real Vellon of Spain	0.05
1 Real Plate of Spain	0.10
1 Rupee Company	0.44½
1 Rupee of British India	0.44½
1 Franc of France and Belgium	0.18½
1 Specie Dollar of Sweden and Norway	1.06
1 Ducat of Sweden	2.15
1 Specie Dollar of Denmark	1.05
1 Florin of Netherland	0.40
1 Florin of Southern States of Germany	0.40
1 Guilder of Netherland	0.40
1 Livre Tournoise of France	0.18½

1 <i>Livre</i> of the Lombardy Venitian Kingdom	\$3.16
1 <i>Livre</i> of Tuscany	0.16
1 <i>Livre</i> of Sardinia	0.18 ⁹ / ₁₀
1 <i>Milrea</i> of Portugal	1.12
1 <i>Milrea</i> of Azores	0.83 ¹ / ₂
1 <i>Marc Banco</i> of Hamburg	0.35
1 <i>Rix Dollar</i> or <i>Thaler</i> of Prussia and the Northern States of Germany, .	0.69
1 <i>Rix Dollar</i> of Bremen	0.78 ² / ₃
1 <i>Rouble Silver</i> of Russia	0.75
1 <i>Florin</i> of Austria	0.48 ¹ / ₂
1 <i>Ducat</i> of Naples	0.80
1 <i>Ounce</i> of Sicily	2.40
1 <i>Tael</i> of China	1.43
1 <i>Livre</i> of Leghorn	0.16

FOREIGN MEASURES OF LENGTH COMPARED WITH AMERICAN.

<i>Places.</i>	<i>Measures.</i>	<i>Inches.</i>	<i>Places.</i>	<i>Measures.</i>	<i>Inches.</i>
Amsterdam	Foot	11.14	Malta . .	Foot	11.17
Antwerp .	"	11.24	Moscow .	"	13.17
Bavaria . .	"	11.42	Naples . .	Palmo	10.38
Berlin . .	"	12.19	Prussia .	Foot	12.36
Bremen . .	"	11.38	Persia . .	Arish	38.27
Brussels .	"	11.45	Rhineland	Foot	12.35
China . .	" Mathematic	13.12	Riga . .	"	10.79
" . .	" Builder's	12.71	Rome . .	"	11.60
" . .	" Tradesman's	13.32	Russia . .	"	13.75
" . .	" Surveyor's	12.58	Sardinia .	Palmo	9.78
Copenhagen	"	12.35	Sicily . .	"	9.53
Dresden .	"	11.14	Spain . .	Foot	11.03
England .	"	12.00	" . .	Toesas	66.72
Florence .	Braccio	21.60	" . .	Palmo	8.34
France . .	Pied de Roi	12.79	Strasburgh	Foot	11.39
" . .	Metre	39.381	Sweden .	"	11.69
Geneva . .	Foot	19.20	Turin . .	"	12.72
Genoa . .	Palmo	9.72	Venice . .	"	13.40
Hamburgh .	Foot	11.29	Vienna . .	"	12.45
Hanover .	"	11.45	Zurich . .	"	11.81
Leipsic . .	"	11.11	Utrecht .	"	10.74
Lisbon . .	"	12.96	Warsaw .	"	14.03
" . .	Palmo	8.64			

ENGLISH AND FRENCH MEASURES OF LENGTH.

BRITISH. Yard is referred to a natural standard, which is the length of a pendulum vibrating seconds in vacuo in London, at the level of the sea; measured on a brass rod, at the temperature of 62° Fahrenheit, = 39.1393 *Imperial inches*.

FRENCH. Old System.—1 Line = 12 points . = 0.08884 U. S. inches.
 1 Inch = 12 lines . = 1.06604 "
 1 Foot = 12 inches . = 12.7925 "
 1 Toise = 6 feet . = 76.755 "
 1 League = 2280 toises . (common.)
 1 League = 2000 toises . (post.)
 1 Fathom = 5 feet.

New System.—1 Millimetre . . . = .03939 U. S. inches.
 1 Centimetre . . . = .39380 "
 1 Decimetre . . . = 3.93809 "
 1 Metre . . . = 39.38091 "
 1 Decametre . . . = 393.80917 "
 1 Hecatometre . . . = 3938.09171 "

FOREIGN ROAD MEASURES COMPARED WITH AMERICAN.

Places.	Measures	Yards.	Places.	Measures.	Yards.
Arabia . .	Mile	2148	Hungary .	Mile	9113
Bohemia .	"	10137	Ireland .	"	3038
China . .	Li	629	Netherlands	"	1093
Denmark .	Mile	8244	Persia . .	Parasang	6086
England .	" Statute	1760	Poland . .	Mile, long	8101
" . .	" Geographical	2025	Portugal .	League	6760
Flanders .	"	6869	Prussia . .	Mile	8468
France . .	League, marine	6075	Rome . .	"	2025
" . .	" common	4861	Russia . .	Verst	1167
" . .	" post	4264	Scotland .	Mile	1984
Germany .	Mile, long	10126	Spain . .	League, common	7416
Hamburgh .	"	8244	Sweden . .	Mile	11700
Hanover .	"	11559	Switzerland	"	9153
Holland .	"	6395	Turkey . .	Berri	1826

MEASURES OF SURFACE.

FRENCH.	<i>Old System.</i> —1 Square Inch . .	= 1·1364 U. S. inches.
	1 Arpent (Paris) . .	= 900 square toises.
	1 Arpent (woodland) = 100 square royal perches.	
<i>New System.</i> —	1 Arc	= 100 square metres.
	1 Decare	= 10 ares.
	1 Hectare	= 100 ares.
	1 Square Metre . . .	= 1550·85 square inches, or 10·7698 square feet.
	1 Arc	= 1076·98 "

FOREIGN MEASURES OF SURFACE COMPARED WITH AMERICAN.

Places.	Measures..	Sq. Yds.	Places.	Measures.	Sq. Yds.
Amsterdam	Morgen	9722	Portugal .	Geira	6970
Berlin . .	" great	6786	Prussia . .	Morgen	3653
" . .	" small	3054	Rome . .	Pezza	3158
Canary Isles	Fanegada	2422	Russia . .	Dessetina	13066·6
England . .	Acre	4840	Scotland .	Acre	6150
Geneva . .	Arpent	6179	Spain . .	Fanegada	5500
Hamburgh .	Morgen	11545	Sweden . .	Tunneland	5900
Hanover . .	"	3100	Switzerland	Faux	7855
Ireland . .	Acre	7840	Vienna . .	Joch	6889
Naples . .	Moggia	3998	Zurich . .	Common acre	3875·0

FOREIGN MEASURES OF CAPACITY.

BRITISH.	The <i>Imperial gallon</i> measures 277·274 cubic inches, containing 10 lbs. Avoirdupois of distilled water, weighed in air, at the temperature of 62° degrees, the barometer at 30 inches.				
	<i>For Grain.</i> 8 bushels = 1 quarter.				
	1 quarter = 10·2694 cubic feet.				
	<i>Coal, or heaped measure.</i> 3 bushels = 1 sack.				
	12 sacks = 1 chaldron.				
FRENCH.	<i>Imperial bushel</i> = 2218·192 cubic inches.				
	* <i>Heaped bushel.</i> 19½ ins. diam., cone 6 ins. high = 2812·4872 cubic ins.				
	1 chaldron = 58·658 cubic feet, and weighs 3136 pounds.				
	1 chaldron (Newcastle) = 5936 pounds.				
	<i>New System.</i> —1 <i>Litre</i> = 1 cub. decimetre, or 61·074 U. S. cubic inches.				
SPANISH.	<i>Old System.</i> —1 <i>Poiseau</i> = 13 litres = 793·964 cub. ins., or 3·43 galls.				
	1 <i>Pinte</i> = 0·931 litres, or 56·817 cubic inches.				
	1 <i>Wine Arroba</i> = 4·2455 gallons.				
	1 <i>Fanega</i> (common measure) = 1·593 bushels.				

* When heaped in the form of a true cone.

FOREIGN LIQUID MEASURES COMPARED WITH AMERICAN.

<i>Places.</i>	<i>Measures.</i>	<i>Cub. In.</i>	<i>Places.</i>	<i>Measures.</i>	<i>Cub. In.</i>
Amsterdam .	Anker	2331	Naples . . .	Wine Barille	2544
" .	Stoop	146	" . . .	Oil Stajo	1133
Antwerp . .	"	194	Oporto . . .	Almude	1555
Bordeaux . .	Barrique	14033	Rouie . . .	Wine Barille	2560
Bremen . . .	Stubgens	194.5	" . . .	Oil "	2240
Canaries . .	Arrobas	949	" . . .	Boccali	80
Constantinople	Almud	319	Russia . . .	Weddras	752
Copenhagen .	Anker	2355	" . . .	Kunkas	94
Florence . . .	Oil Barille	1946	Scotland . .	Pint	103.5
" . . .	Wine "	2427	Sicily . . .	Oil Caffiri	662
France . . .	Litre	61.07	Spain . . .	Azumbres	22.5
Geneva . . .	Setier	2760	" . . .	Quartillos	30.5
Genoa . . .	Wine Barille	4530	Sweden . . .	Eimer	4794
" . . .	Pinte	90.5	" . . .	Kanna	159.57
Hamburgh . .	Stubgen	221	Trieste . . .	Orne	4007
Hanover . . .	"	231	Tripoli . . .	Mattari	1376
Hungary . . .	Eimer	4474	Tunis . . .	Oil "	1157
Leghorn . . .	Oil Barille	1942	Venice . . .	Secchio	628
Lisbon . . .	Almude	1040	Vienna . . .	Eimer	3452
Malta . . .	Caffiri	1270	" . . .	Maas	86.33

FOREIGN DRY MEASURES COMPARED WITH AMERICAN.

<i>Places.</i>	<i>Measures.</i>	<i>Cub. In.</i>	<i>Places.</i>	<i>Measures.</i>	<i>Cub. In.</i>
Alexandria .	Rebele	9587	Malta . . .	Salme	16930
" . . .	Kislos	10418	Marseilles .	Charge	9411
Algiers . . .	Tarrie	1219	Milan . . .	Moggi	8444
Amsterdam .	Mudde	6596	Naples . . .	Tomoli	3122
" . . .	Sack	4947	Oporto . . .	Alquiere	1051
Antwerp . . .	Viertel	4705	Persia . . .	Artaba	4013
Azores . . .	Alquiere	731	Poland . . .	Zorzec	3120
Berlin . . .	Scheffel	3180	Riga . . .	Loop	3078
Bremen . . .	"	4339	Rome . . .	Rubbio	16904
Candia . . .	Charge	9288	" . . .	Quarti	4226
Constantinople	Kislos	2023	Rotterdam .	Sach	6361
Copenhagen .	Toende	8489	Russia . . .	Chetwert	12448
Corsica . . .	Stajo	6014	Sardinia . .	Starelli	2988
Florence . . .	Stari	1449	Scotland . .	Firlot	2197
Geneva . . .	Coupes	4739	Sicily . . .	Salme gros	21014
Genoa . . .	Mina	7382	" . . .	" generale	16886
Greece . . .	Medimni	2390	Smyrna . . .	Kislos	2141
Hamburgh . .	Scheffel	6426	Spain . . .	Catrize	41269
Hanover . . .	Malter	6868	Sweden . . .	Tunna	8940
Leghorn . . .	Stajo	1501	Trieste . . .	Stari	4521
" . . .	Sacco	4503	Tripoli . . .	Caffiri	19780
Lisbon . . .	Alquiere	817	Tunis . . .	"	21855
" . . .	Fanega	3268	Venice . . .	Stajo	4945
Madeira . . .	Alquiere	684	Vienna . . .	Metzen	3753
Malaga . . .	Fanaga	3783			

FRENCH MEASURES OF SOLIDITY.

FRENCH.	1 Cubic Foot	=	2093.470 U. S. inches.
	Decistree	=	3.5375 cubic feet.
	Stere (a cubic metre)	=	25.375 "
	Decastere	=	353.75 "
	1 Stere	=	61074.664 cubic inches.

For the *Square* and *Cubic Measures* of other countries, take the length of the measure in Table, page 72, and square or cube it as required.

ENGLISH AND FRENCH MEASURES OF WEIGHT.

BRITISH.	1 troy Grain	=	·003961 cubic inches of distilled water.	
	1 troy Pound	=	22·815689 cubic inches of water.	
FRENCH.	<i>Old System.</i> —1 Grain	. . . =	0·8188 grains troy.	
	1 Gros	. . . =	58·9548 “	
	1 Ounce	. . . =	1·0780 oz. avoirdupois.	
	1 Livre	. . . =	1·0780 lbs. “	
	<i>New System.</i> —Milligramme	. . . =	·01543 troy grains.	
	Centigramme	. . . =	·15433 “	
	Decigramme	. . . =	1·54331 “	
	Gramme	. . . =	15·43315 “	
	Decagramme	. . . =	154·33159 “	
	Hecagramme	. . . =	1543·3159 “	
	1 Millier	= 1000 Kilogrammes	=	1 ton sea weight.
	1 Kilogramme	. . . =	2·204737 lbs. avoirdupois.	
	1 Pound avoirdupois	. . . =	0·4535685 Kilogramme	
	1 Pound troy	. . . =	0·3732223 “	

NOTE.—In the new French system, the values of the base of each measure, viz., Metre, Litre, Stere, Are, and Gramme, are decreased or increased by the following words prefixed to them. Thus,

Milli	expresses the 1000th part.	Hecato	expresses 100 times the value.
Centi	“ 100th “	Chilio	“ 1000 “
Deci	“ 10th “	Myrio	“ 1000 “
Deca	“ 10 times the value.		

FOREIGN WEIGHTS COMPARED WITH AMERICAN.

Places.	Weights.	Number equal to 100 avoirdupois pounds.	Places.	Weights.	Number equal to 100 avoirdupois pounds.
Aleppo	Rottoli	20·46	Hanover	Pound	93·20
“	Oke	35·80	Japan	Catty	76·92
Alexandria	Rottoli	107·	Leghorn	Pound	133·56
Algiers	“	84·	Leipsic	“ (common)	97·14
Amsterdam	Pound	91·8	Lyons	“ (silk)	98·81
Antwerp	“	96·75	Madeira	“	143·20
Barcelona	“	112·6	Mocha	Maund	33·33
Batavia	Catty	76·78	Morea	Pound	90·79
Bengal	Seer	53·57	Naples	Rottoli	50·91
Berlin	Pound	96·8	Rome	Pound	133·69
Bologna	“	125·3	Rotterdam	“	91·80
Bremen	“	90·93	Russia	“	110·86
Brunswick	“	97·14	Sicily	“	142·85
Cairo	Rottoli	105·	Smyrna	Oke	36·51
Candia	“	85·9	Sumatra	Catty	35·56
China	Catty	75·45	Sweden	Pound	106·67
Constantinople	Oke	35·55	“	“	120·68
Copenhagen	Pound	90·80	Tangiers	“ (miner's)	94·27
Corsica	“	131·72	Tripoli	Rottoli	89·28
Cyprus	Rottoli	19·07	Tunis	“	90·09
Damascus	“	25·28	Venice	Pound (heavy)	94·74
Florence	Pound	133·56	“	“ (light)	150·
Geneva	“ (heavy)	82·35	Vienna	“	81·
Genoa	“	92·86	Warsaw	“	112·25
Hamburgh	“	93·63			

PROVISIONS.

Component parts of the Navy Ration.	Pounds.						Ounces						Fractions of a Pint.			
	Beef.	Pork.	Flour.	Raisins or dried fruit.	Pickles.	Rice.	Biscuits.	Sugar.	Either: Coffee. Cocoa.	Tea.	Butter.	Cheese.	Beans.	Molasses.	Vinegar.	Spirits.
Sunday.	1		$\frac{1}{2}$	$\frac{1}{2}$			14	2	1	1	1					$\frac{1}{2}$
Monday.		1					14	2	1	1	1					$\frac{1}{2}$
Tuesday.	1					$\frac{1}{2}$	14	2	1	1	1	2				$\frac{1}{2}$
Wednesday.		1			$\frac{1}{2}$		14	2	1	1	1		$\frac{1}{2}$			$\frac{1}{2}$
Thursday.	1		$\frac{1}{2}$	$\frac{1}{2}$			14	2	1	1	1	2				$\frac{1}{2}$
Friday.	1					$\frac{1}{2}$	14	2	1	1	1			$\frac{1}{2}$		$\frac{1}{2}$
Saturday.		1			$\frac{1}{2}$		14	2	1	1	1				$\frac{1}{2}$	$\frac{1}{2}$
Total Rations per week.	4	3	1	$\frac{1}{2}$	$\frac{1}{2}$	1	98	14	7	7	4	4	12	$\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$

GEOMETRY.

DEFINITIONS.

Demonstration is a course of reasoning by which a truth is established. It consists of,

Thesis, the truth to be established, and,

Hypothesis, the foundation for the demonstration.

Axiom is that which is self-evident and requires no demonstration.

Theorem is something to be proved by demonstration.

Postulate is something to be done, but is self evident and requires no demonstration.

Problem is something proposed to be done, and requires demonstration.

Proposition is either a Theorem or a Problem.

Corollary is an obvious consequence deduced from something that has gone before.

Scolium is a remark on preceding propositions, commonly demonstrated by algebraical formulæ.

Lemma is something premised for a following demonstration.

Geometrical Quantities.

Point is a position, but no magnitude.

A Line is length, without breadth or thickness.

A Straight Line is the shortest distance between two points.

Curved line is a length which in every point changes its direction.

Superficies, Surface, Area, is that which has length and breadth, but no thickness.

Plane surface is a plane which coincides with a straight line in every direction.

Curved surface is a plane which coincides with a curved line.

Solid has length, breadth and thickness.

Circle.

Circle, Circumference, Periphery, is a curved line drawn on a plane surface, and bounded at a common distance from one point in the plane, (centre.)

Radius is a line* drawn from the centre in a circle to the periphery.

Diameter is a line drawn through the centre to the periphery, or the longest line in a circle.

Chord is any line extending its both ends to the periphery of a circle. and does not go through the centre.

Arc is a part of a periphery.

Circle plane, is a plane surface bounded within a circumference.

Sector is a part of a circle-plane bounded within an arc and two radii.

Segment is a part of a circle plane bounded within a chord and an arc.

Zone is a part of a circle included between two parallel chords.

Lune is the space between the intersecting arcs of two eccentric circles.

Oval is a round figure having one long and one short diameter at right angles to one another.

Semicircle is a half circle.

Quadrant is a quarter of a circle.

Angles.

Angle is the opening or inclination of two lines which meet in one point.

If two radii being drawn from the extremities of a circle arc, to the centre; the arc, is a measure of the angle at the centre.

Right angle is when the opening is a quarter of a circle.

Acute angle is less than a right angle.

Obtuse angle is greater than a right angle.

* *Line* by itself means a straight line.

Triangles.

Triangle is a figure of three sides.

Equilateral Triangle has all its sides equal.

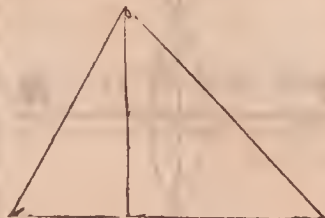
Isosceles Triangle has two of its sides equal.

Scalene Triangle has all its sides unequal.

Right-angled triangle has one right angle.

Obtuse-angled triangle, has one obtuse angle.

Acute-angled triangle has all its angles acute.



Quadrangles.

Quadrangle is a figure of four sides.

Parallelogram having its opposite sides parallel, and the opposite angles equal.

Square, having its four sides equal, and four right angles.

Rectangle, having its opposite sides equal, and four right angles.

Rhombus, having four equal sides, and opposite angles equal but not right.

Rhomboid, same as a parallelogram.

Trapezium, having four unequal sides.

Trapezoid, having only two opposite sides parallel.

Gnomon is the space included between the lines forming two similar parallelograms, of which the smaller is inscribed in the larger, so as to have one common angle.

Polygons.

Polygons are plane and rightlined figures.

Regular Polygons are plane figures which inscribe, or circumscribe a circle, and their sides being equal. Polygons are named according to their number of sides, thus,

Trigon has three sides.

Tetragon " four "

Pentagon " five "

Hexagon " six "

Heptagon " seven "

Octagon has eight sides.

Nonagon " nine "

Decagon " ten "

Undecagon " eleven "

Dodecagon " twelve "

For properties of Polygons see page 103.

Solids.

Sphere is a solid bounded within a half circle rotating round its diameter.

Spherical segment, (zone) is a part of a sphere cut off by a plane.

Spheroid is a sphere flatted or longed at two opposite sides; as our earth is flatted at the poles, and having one diameter shortest; an egg is longed, and having one diameter longest.

Spindle is a solid bounded within a curved line rotating round its base.

Cylinder is a solid bounded within a rectangle rotating round one of its sides, (axis.) A cylinder has a circle plane to its base.

Cone is bounded within a right-angled triangle rotating round one of its sides that forms the right angle.

Ungula is the bottom part of a Cone or Cylinder, cut off by a plane passing obliquely through the base.

Cube is bounded within six squares.

Parallelopiped is bounded within six parallelograms.

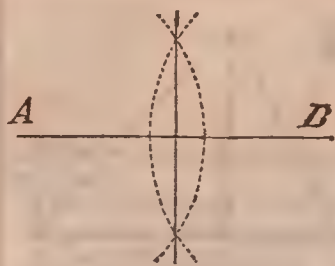
Prism is a solid described by a rightlined plane moving in a straight line, so that the plane forms an angle to its direction line.

Prismoid is a prism cut obliquely at the ends.

Pyramid is bounded between a rightlined plane, and one point at a distance from the plane. The sides of the rightlined plane, are bases of triangles terminating at the aforesaid point, (vertex.)

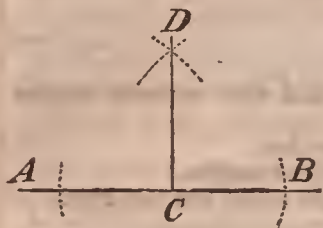
Perimeter is the sum of all the sides in a figure, plane or solid.

Polyhedrons. See page 95, for properties and names of the five regular polyhedrons



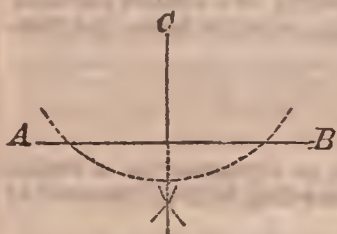
1.

To divide a given line AB into two equal parts; and to erect a perpendicular through the middle.



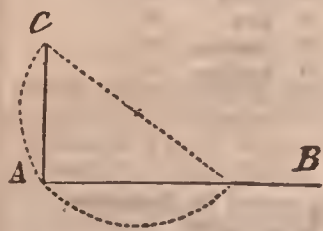
2.

At a given point C on the line AB erect a perpendicular CD .



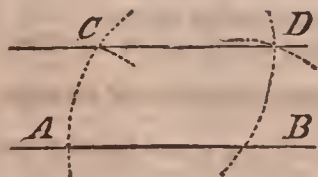
3.

From a given point C , at a distance from the line AB , draw a perpendicular to the line.



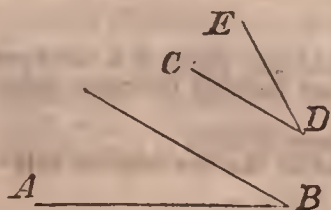
4.

At the end A of a given line AB , erect a perpendicular AC .



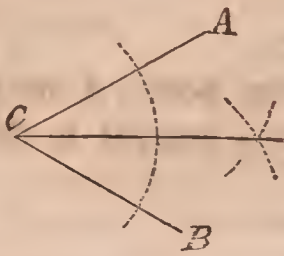
5.

Through a given point C , at a distance from the line AB , draw a line CD parallel to AB .



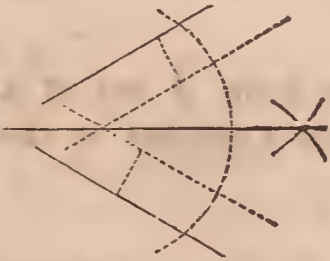
6.

On the given line AB , and at the point B , construct an angle, equal to the angle CDE .



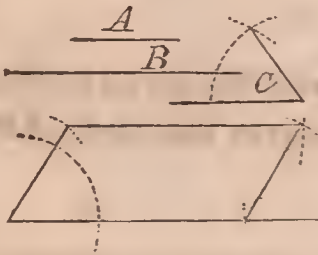
7.

Divide the angle ACB into two equal parts.



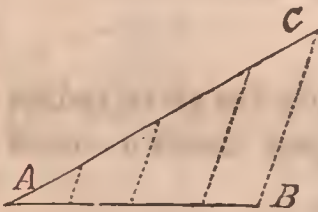
8.

Divide an angle into two equal parts, when the lines do not extend until they meet.



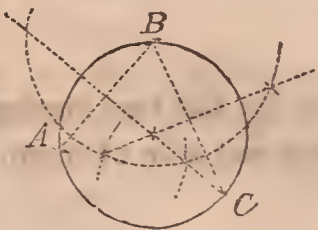
9.

To construct a parallelogram, with two sides A and B , the including angle C is given.



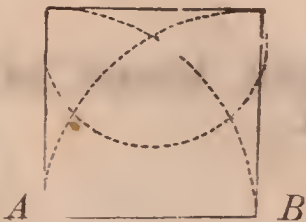
10.

To divide the line AB , in the same proportions, as the line AC is divided.



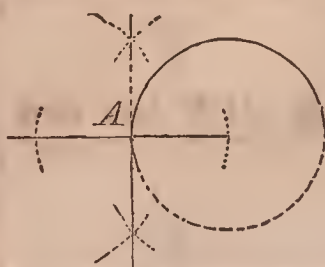
11.

Through three given points A, B , and C , draw a circle.



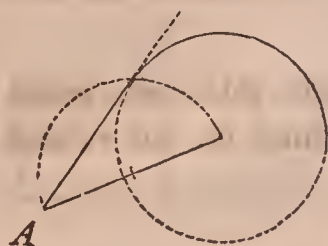
12.

Construct a square upon the given line AB .



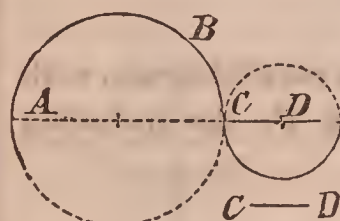
13.

Through a given point A in a circumference, draw a tangent to the circle.



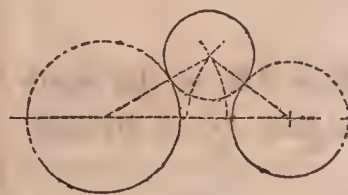
14.

From a given point A out of a circumference, draw a tangent to the circle.



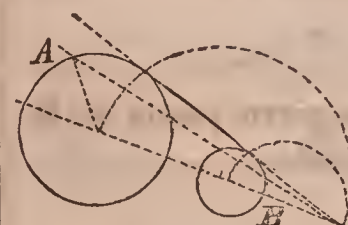
15.

Draw a circle with the given radius $C'D'$, that will tangent the circle AB at C



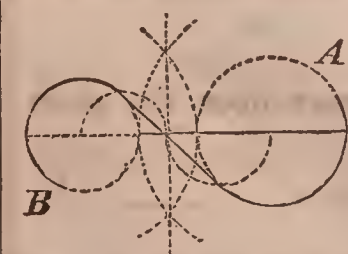
16.

Draw a circle with the given radius CD , that will tangent the two circles A and B .



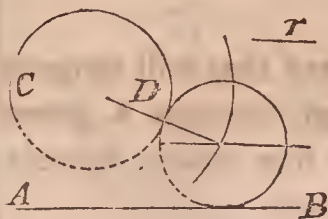
17.

Draw a tangent to the two circles A and B , that is on one side of them.



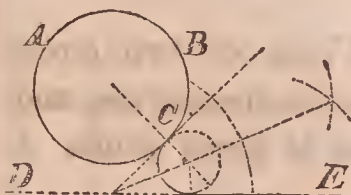
18.

Draw a tangent between two circles A and B .



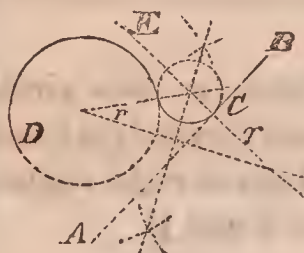
19.

With a given radius r , draw a circle that will *tangent* the given line AB and the given circle CD .



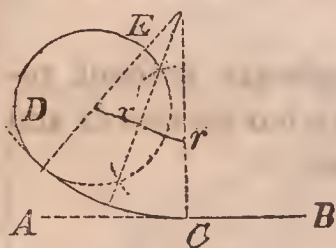
20.

To find the centre and radius of a circle that will *tangent* the given circle AB at C , and the line DE .



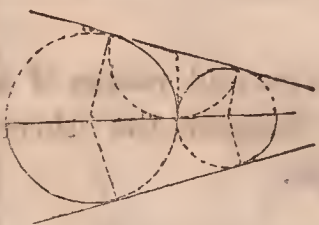
21.

To find the centre and radius of a circle, that will *tangent* the given line AB at C , and the circle DE .



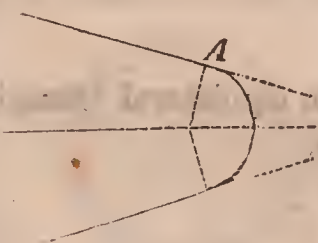
22.

To find the centre and radius of a circle that will *tangent* the given line AB at C , and the circle DE .



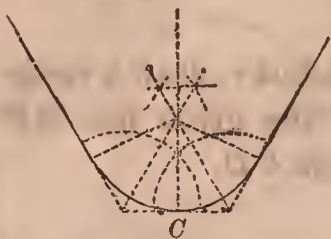
23.

Between two lines, draw two circles that will *tangent* themselves and the lines.



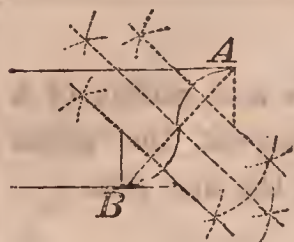
24.

Draw a circle-arc, that will *tangent* two lines inclined to one another, and the one tangencing point A being given.



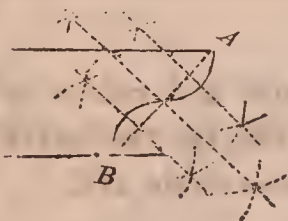
25.

Draw a circle-arc that will tangent two lines, and go through a given point *C*, on the line which bisects the angle of the lines.



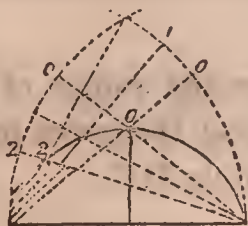
26.

To draw a *Cyma*, or two circle arcs that will *tangent* themselves, and two parallel lines at given points *A* and *B*.



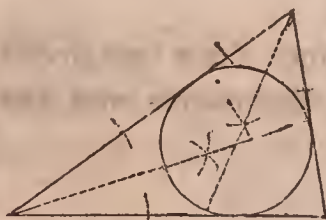
27.

To draw a *Talon*, or two circle arcs that will *tangent* themselves; and meet two parallel lines at right angles, in the given points *A* and *B*.



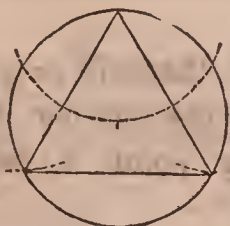
28.

To draw a circle-arc without recourse to its centre but its chords and height being given.



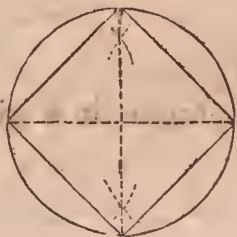
29.

To find the centre and radius of a circle that will tangent the three sides in a triangle.



30.

To inscribe an equilateral triangle in a given circle.



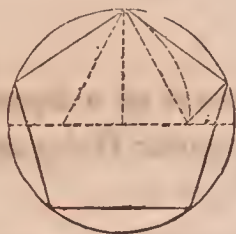
31.

To inscribe a square in a given circle.



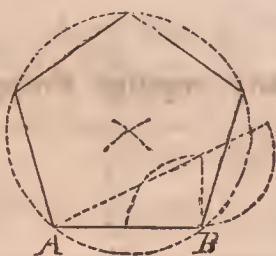
32.

To describe a square about a given circle.



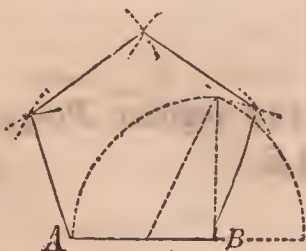
33.

To inscribe a *Pentagon* in a given circle.



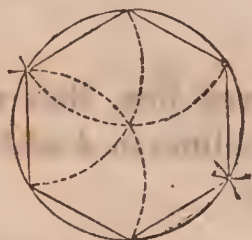
34.

To construct a *Pentagon* on a given line *AB*.



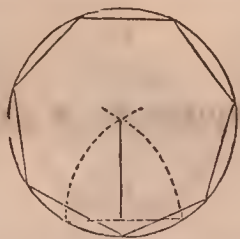
35.

To construct a *Pentagon* on a given line *AB*, without recourse to its centre.



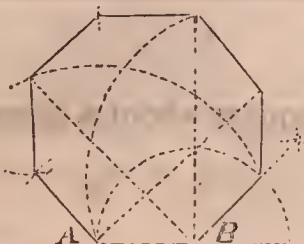
36.

To construct a *Hexagon* in a given circle.



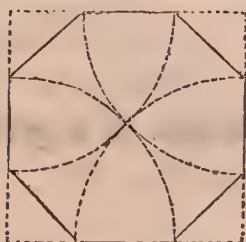
37.

To inscribe a *Heptagon* in a given circle.



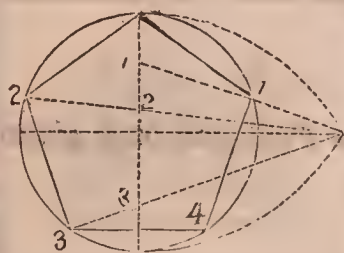
38.

To construct an *Octagon*, on a given line *AB*.



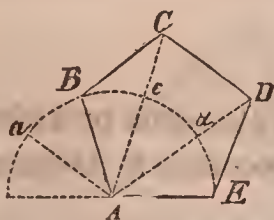
39.

To cut off the corners of a *Square*, so that it will be a regular *Octagon*.



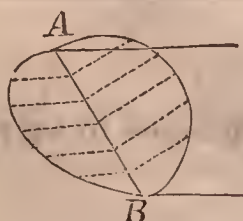
40.

To construct any regular *Polygon* in a given circle.



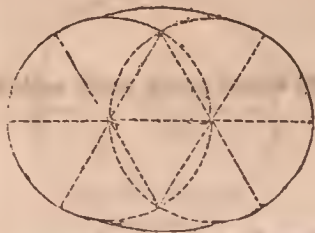
41.

To construct any regular *Polygon* on a given line *AE*.



42.

To draw a curved line, that will tangent two parallel lines in *A* and *B*.



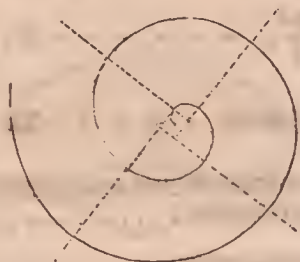
43.

To draw an *Ellipse* by compasses and four circle arcs.



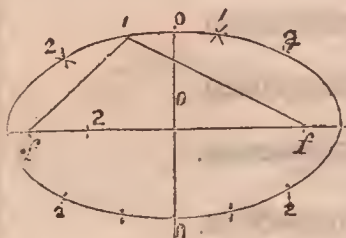
44.

To draw an *Ellipse* by compasses and six circle arcs. (*Isometric Ellipse.*)



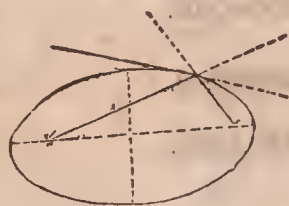
45.

To draw a *Spiral* by compasses and four centres.



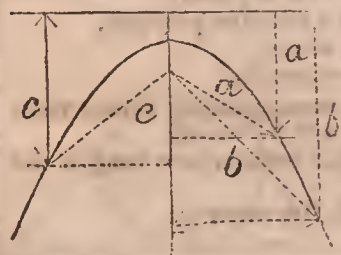
46.

To construct an *Ellipse*,



47.

To draw a *Tangent* to an *Ellipse*.



48.

To construct a *Parabola*.

CIRCLE.

THE periphery of a circle is divided into 360° (degrees) equal parts, each called a degree.

One degree = $60'$ (minutes.)

One minute = $60''$ (seconds.)

Half a circle (hemisphere) = 180° .

Quarter of a circle (quadrant) = 90° .

By the accompanying formula any part of the circle can be calculated.

Formula for the Circle.

$$p = \pi d = 3.14d, \dots 1,$$

$$p = 2\pi r = 6.28r, \dots 2,$$

$$p = 2\sqrt{\pi a} = 3.54\sqrt{a}, \dots 3,$$

$$d = \frac{p}{\pi} = \frac{p}{3.14}, \dots 4,$$

$$d = 2\sqrt{\frac{a}{\pi}} = 1.128\sqrt{a}, \dots 5,$$

$$r = \frac{p}{2\pi} = \frac{p}{6.28}, \dots 6,$$

$$r = \sqrt{\frac{a}{\pi}} = 0.564\sqrt{a}, \dots 7,$$

$$a = \frac{\pi d^2}{4} = 0.785d^2, \dots 8,$$

$$a = \pi r^2 = 3.14r^2, \dots 9,$$

$$a = \frac{p^2}{4\pi} = \frac{p^2}{12.56}, \dots 10,$$

$$a = \frac{pr}{2} = \frac{pd}{4}, \dots 11,$$

$$a = \frac{\pi rd}{2} = 1.57rd, \dots 12.$$

$\pi = 3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823066470938446. \dots$

$$2\pi = 6.28218530710000.$$

$$3\pi = 9.42477796070000.$$

$$4\pi = 12.5663706143000.$$

$$5\pi = 15.7079632679000.$$

$$6\pi = 18.8495559215000.$$

$$7\pi = 21.9911485751000.$$

$$8\pi = 25.1327412887000.$$

$$9\pi = 28.2743338823000.$$

$$\frac{1}{2}\pi = 1.57079632679000.$$

$$\frac{1}{4}\pi = 0.78539816339700.$$

$$\frac{1}{8}\pi = 1.04719755119600.$$

$$\frac{1}{6}\pi = 0.52359877559800.$$

$$\frac{1}{8}\pi = 0.39269908169800.$$

$$\frac{1}{12}\pi = 0.26179938779900.$$

$$\frac{1}{360}\pi = 0.00872667621060.$$

$$\frac{1}{\pi} = 0.31830988618370.$$

$$\frac{2}{\pi} = 0.63661977236740.$$

$$\frac{3}{\pi} = 0.95492965855110.$$

$$\frac{6}{\pi} = 1.90965931710220.$$

$$\frac{12}{\pi} = 3.81971863420440.$$

$$\frac{360}{\pi} = 114.591559026122.$$

$$\pi^2 = 9.86965000000000.$$

$$\sqrt{\pi} = 1.77245300000000.$$

$$\sqrt{\frac{1}{\pi}} = 0.56418900000000.$$

$$\frac{1}{4\pi} = 0.07957747154500.$$

Letters denote,

r = radius of the circle.

d = diameter.

p = periphery.

a = area of a circle, or part thereof.

b = circle-arc, length of.

c = chord of a segment, length of.

h = height of a segment.

s = side of a regular polygon.

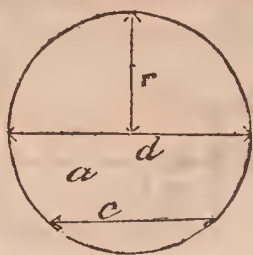
v = centre angle.

w = polygon angle.

Be careful to express all the dimensions by the same unit, as miles, rods, yards, feet, or inches, &c., &c., or else the calculation will be wrong.

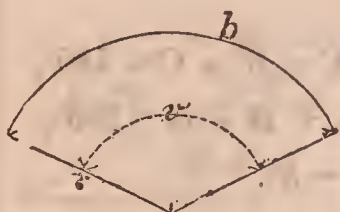
Example 1. Fig. 49. The diameter of a circle is 8 feet, 8 inches, how long is the circumference?

Formula 1. $p = \pi d = 3.14 \times 8.666 = 27.211$ feet, the answer.



49.

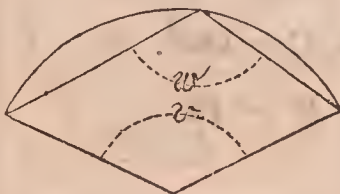
The periphery of a Circle is commonly expressed by the *Greek* letter $\pi = 3.14$ when the diameter $d = 1$ or the unit. For any other value of the diameter d , we will denote the periphery by the letter p , $r =$ radius, and $a =$ area of the circle. The periphery of a circle is equal to $\frac{3.14}{100}$ times its diameter, $c =$ chord.



50.

$$b = \frac{\pi r v}{180} = 0.0175 r v,$$

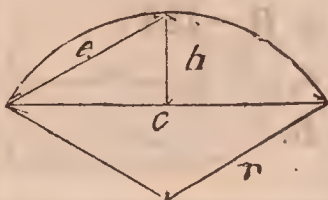
$$v = \frac{180 b}{\pi r} = 5.74 \frac{b}{r}.$$



51.

$$w = 180 - \frac{v}{2},$$

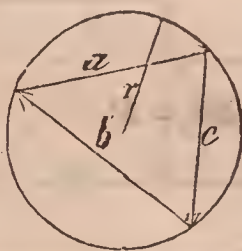
$$v = 2(180^\circ - w).$$



52.

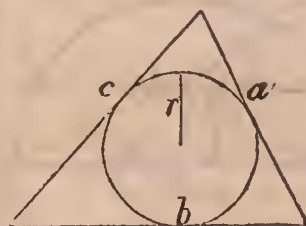
$$r = \frac{c^2 + 4h^2}{8h} = \frac{e^2}{2h},$$

$$c = 2 \sqrt{2hr - h^2}.$$



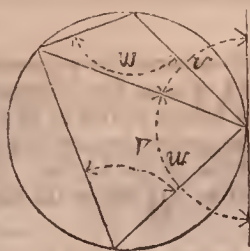
53.

$$r = \frac{ac}{2\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}}.$$



54.

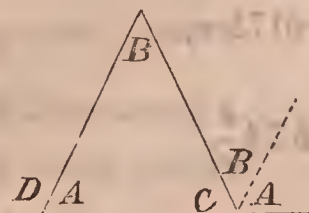
$$r = \frac{b\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}}{a + b + c}.$$



55.

$$v = v, \quad w = w,$$

$$w + v = 180^\circ, \quad w > v.$$

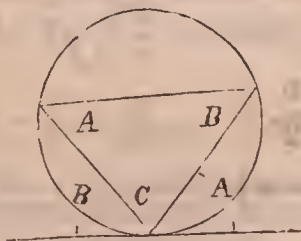


56.

$$D = B + C, \quad A' + B' + C = 180^\circ,$$

$$B = D - C, \quad A + B + C = 180^\circ,$$

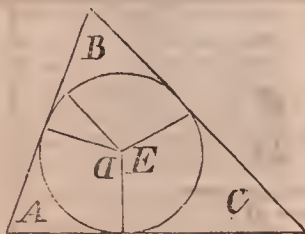
$$A' = A, \quad B' = B'.$$



57.

$$A + B + C = 180^\circ,$$

$$A' = A, \quad B' = B.$$

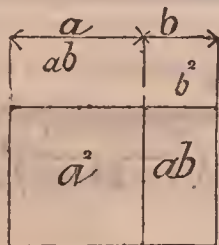


58.

$$E + C = A + D = 180^\circ,$$

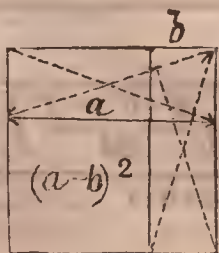
$$D = B + c,$$

$$E = A + B.$$



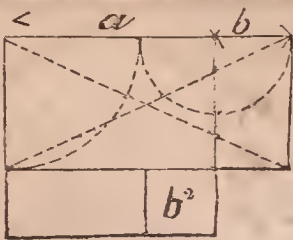
59.

$$(a + b)^2 = a^2 + 2ab + b^2.$$



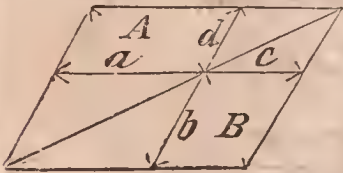
60.

$$(a - b)^2 = a^2 - 2ab + b^2.$$



61.

$$(a + b)(a - b) = a^2 - b^2.$$

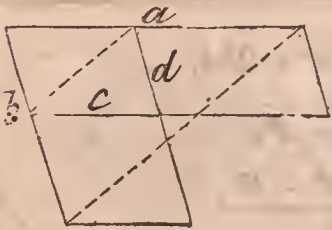


62.

$$a : b = c : d,$$

$$ad = bc,$$

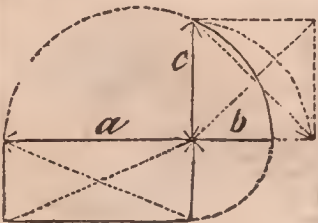
$$A = B.$$



63.

$$a : b = c : d,$$

$$ad = bc.$$

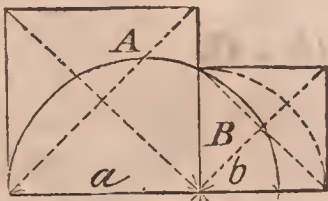


64.

$$a : c = c : b,$$

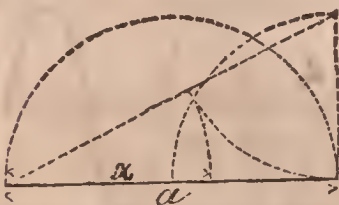
$$ab = c^2,$$

$$c = \sqrt{ab}.$$



65.

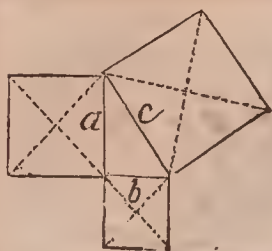
$$A : B = a : b.$$



66.

$$a : x = x : a - x,$$

$$x = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} - \frac{a}{2}.$$

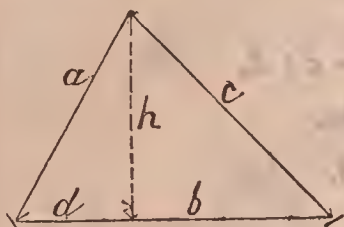


67.

$$c^2 = a^2 + b^2,$$

$$a^2 = c^2 - b^2,$$

$$b^2 = c^2 - a^2.$$

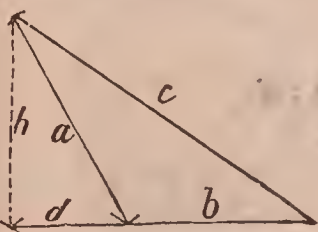


68.

$$c^2 = a^2 + b^2 - 2bd,$$

$$h = \sqrt{a^2 - d^2}.$$

$$d = \frac{a^2 + b^2 - c^2}{2b}.$$

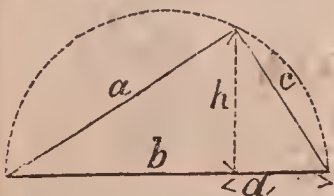


69.

$$c^2 = a^2 + b^2 + 2bd,$$

$$h^2 = \sqrt{a^2 - d^2},$$

$$d = \frac{c^2 - a^2 - b^2}{2b}.$$

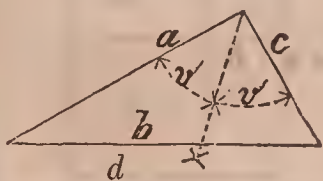


70.

$$a : b = h : c,$$

$$h = \frac{ac}{b} = \frac{ad}{c},$$

$$d = \frac{c^2}{b} = \frac{ch}{a}.$$

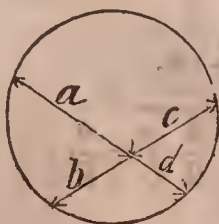


71.

$$a : c = d : (b - d),$$

$$d = \frac{ab}{c + a},$$

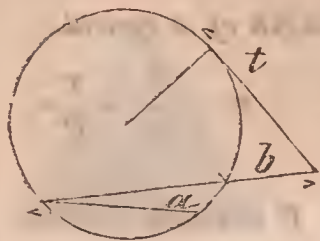
$$v = v.$$



72.

$$a : c = b : d,$$

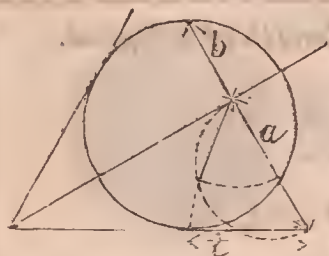
$$ad = bc.$$



73.

$$a : t = t : b,$$

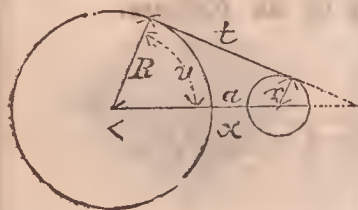
$$t^2 = ab.$$



74.

$$t^2 = (a + b)(a - b),$$

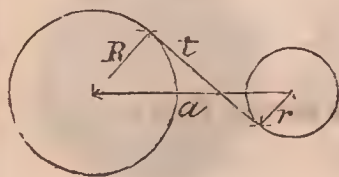
$$t = \sqrt{a^2 - b^2}.$$



75.

$$x = \frac{aR}{R - r}, \quad a = \sqrt{t^2 + (R - r)^2},$$

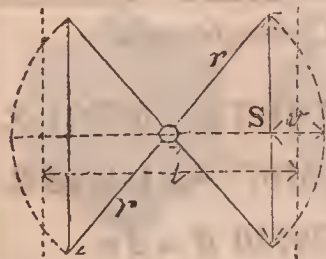
$$t = \sqrt{a^2 - (R - r)^2}, \quad \sin. v = \frac{t}{a}.$$



76.

$$t = \sqrt{a^2 - (R + r)^2},$$

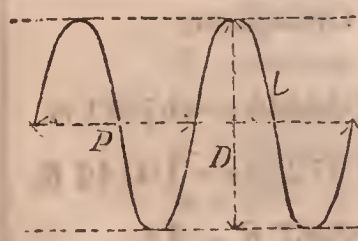
$$a = \sqrt{t^2 + (R + r)^2}.$$



77.

$$V = r - \sqrt{r^2 - \frac{S^2}{4}}, \quad l = 2r - V,$$

$$S = 2\sqrt{r^2 - (r - V)^2}, \quad r = \frac{1}{2}(l + V).$$

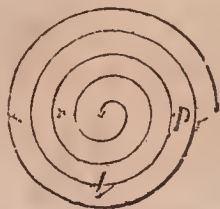


78.

$$P = \sqrt{\frac{l^2}{n^2} - \pi^2 d^2},$$

$$l = n \sqrt{\pi^2 d^2 + P^2},$$

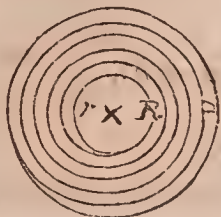
$$n = \frac{l}{\sqrt{\pi^2 d^2 + P^2}}.$$



79. To find the length of a Spiral.

$$l = \pi r n = \frac{\pi r^2}{P}, \quad n = \frac{l}{\pi r} = \frac{r}{P},$$

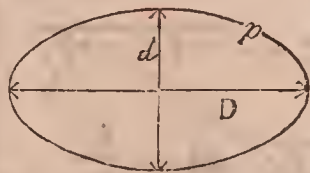
$$P = \frac{\pi r^2}{l} = \frac{r}{n}. \quad P = \text{Pitch.}$$



80. To find the length of a Spiral.

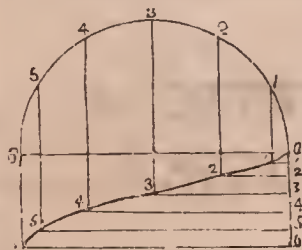
$$l = \pi n (R + r),$$

$$l = \frac{\pi}{P} (R^2 - r^2).$$



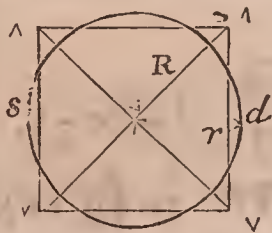
81. Periphery of an Ellipse.

$$p = 2 \sqrt{D^2 + 1.4674 d^2}.$$



82.

To construct a screw Helix.

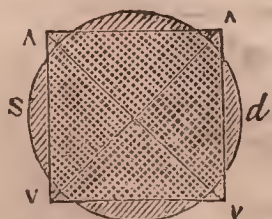


83. To square a Circumference.

$$R = 0.556355 d = 1.1127 r = 0.7071 S.$$

$$S = 0.785398 d = 1.57079 r = 1.4142 R.$$

$$d = 1.27322 S = 1.79740 R = 2 r.$$

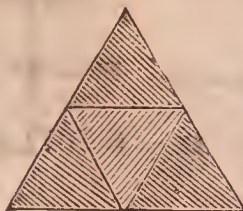
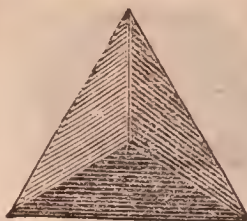


84. To square a Circleplane.

$$R = 0.650744 d = 1.30148 r = 0.7071 S.$$

$$S = 0.886226 d = 1.77245 r = 1.4142 R.$$

$$d = 1.12831 S = 1.5367 R = 2 r.$$

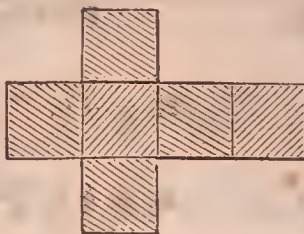
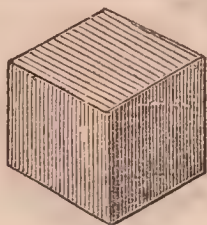
85. *Tetrahedron.*

$$r = 0.20413 s.$$

$$R = 0.60725 s.$$

$$a = 1.73205 s^2.$$

$$c = 0.11785 s^3.$$

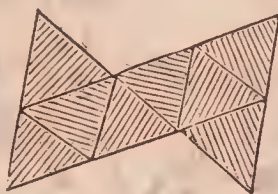
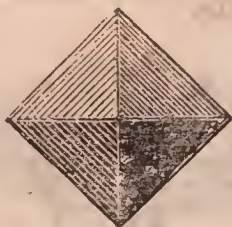
86. *Hexahedron.*

$$r = 0.50000 s.$$

$$R = 0.86602 s.$$

$$a = 6.00000 s^2.$$

$$c = 1.00000 s^3.$$

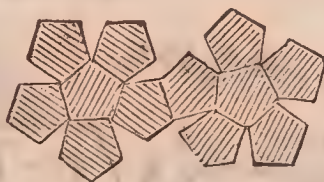
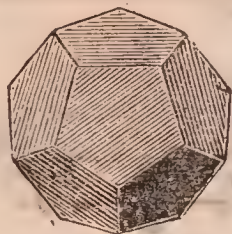
87. *Octahedron.*

$$r = 0.40721 s.$$

$$R = 0.71710 s.$$

$$a = 3.46410 s^2.$$

$$c = 0.47140 s^3.$$

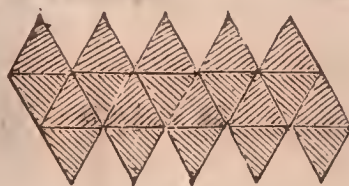
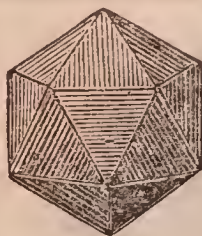
88. *Dodecahedron.*

$$r = 1.11350 s.$$

$$R = 1.36428 s.$$

$$a = 20.5457 s^2.$$

$$c = 7.66312 s^3.$$

89. *Icosahedron.*

$$r = 0.7558 s.$$

$$R = 0.9510 s.$$

$$a = 8.66025 s^2.$$

$$c = 2.18169 s^3.$$

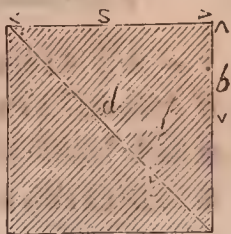
r = Radius of an inscribed Sphere.

R = Radius of circumscribed Sphere.

a = Area of the Polyhedrons.

c = Cubic contents of the Polyhedrons.

s = Side or edge of the Polyhedrons.



90.

Square.

$$a = s^2 = 4b^2.$$

$$a = 0.7071d^2 = 2.8284 c^2.$$

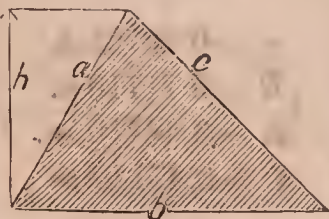


91.

Rectangle.

$$a = a b,$$

$$a = b \sqrt{d^2 - b^2}.$$

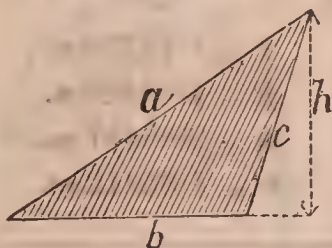


92.

Triangle.

$$a = \frac{b h}{2} = \frac{1}{2} b h,$$

$$a = \frac{b}{2} \sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b} \right)^2}.$$

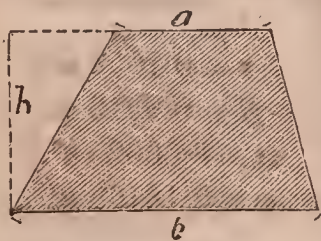


93.

Triangle.

$$a = \frac{1}{2} b h,$$

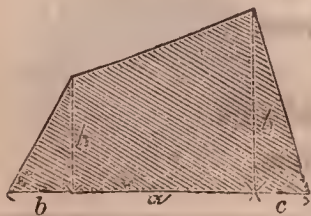
$$a = \frac{b}{2} \sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2b} \right)^2}.$$



94.

Quadrangle.

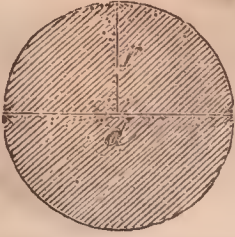
$$a = \frac{1}{2} h(a + b).$$



95.

Quadrangle.

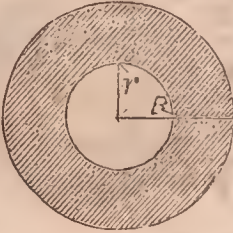
$$a = \frac{1}{2} (a [h + h'] + b h' + c h).$$



96. *Circle Plane.*

$$a = \pi r^2 = 0.785 d^2,$$

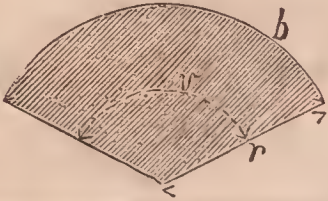
$$a = \frac{p r}{2} = 0.0794 P^2.$$



97. *Circle Ring.*

$$a = \pi(R^2 - r^2) = \pi(R + r)(R - r),$$

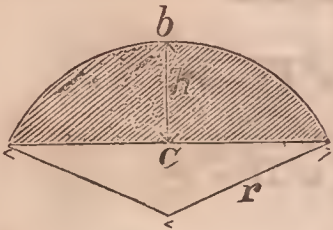
$$a = 0.785(D^2 - d^2).$$



98. *Sector.*

$$a = \frac{1}{2} b r,$$

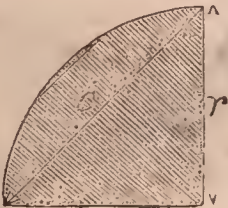
$$a = \frac{\pi r^2 v}{360} = \frac{r^2 v}{114.5}.$$



99. *Segment.*

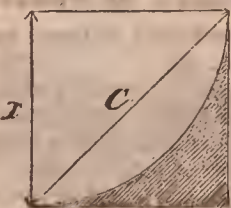
$$a = \frac{1}{2} [b r - c (r - h)],$$

$$a = \frac{\pi r^2 v}{360} + \frac{c}{2} (r - h).$$



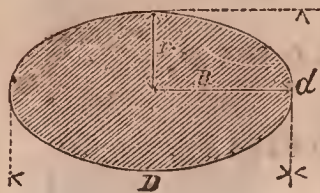
100. *Quadrant.*

$$a = 0.785 r^2 = 0.3916 c^2.$$



101.

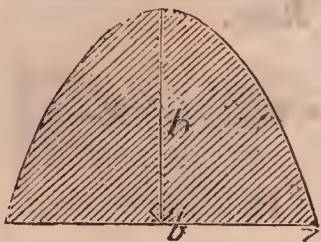
$$a = 0.215 r^2 = 0.1075 c^2.$$



102.

Ellipse.

$$a = \pi R r = 0.785 D d.$$

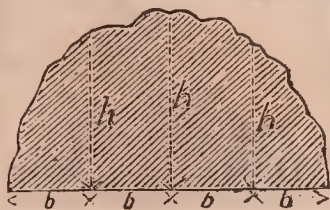


103.

Parabola.

$$a = \frac{2}{3} b h = \frac{1}{6} b^2,$$

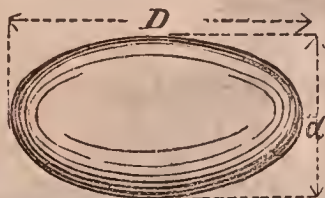
$$a = \frac{1}{3} h \sqrt{p h}.$$



104.

Irregular Figure.

$$a = b (h + h' + h'').$$

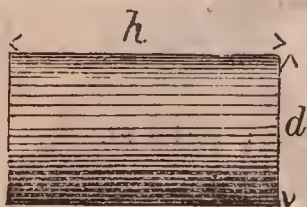


105.

Ellipsoid.

$$a = 8.88 r \sqrt{R^2 - r^2},$$

$$a = 2.22 d \sqrt{D^2 - d^2}.$$

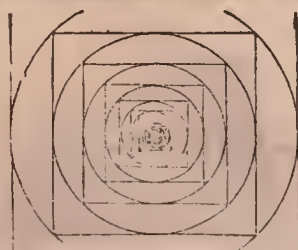


106.

Cylinder.

$$a = 2 \pi r h = \pi d h,$$

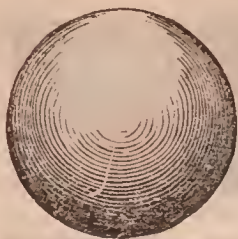
$$h = \frac{a}{2 \pi r} = \frac{a}{\pi d}.$$



The road to Extremity of Space.

1st. Draw a Circle and inscribe a Square, and in that Square a Circle, &c., &c., &c. The last figure that can be drawn, is one extremity of Space Required if the last one is a Circle or a Square?

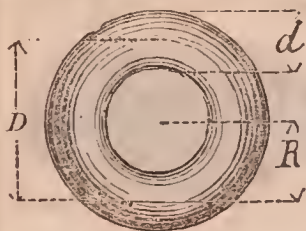
2nd. Draw a Circle and circumscribe a Square, and around that Square a Circle, &c., &c., &c. The last one that can be circumscribed is the other extremity of space. Required if the last figure is a Circle or a Square?



107.

Sphere.

$$a = 4 \pi r^2 = 12.56 r^2 = \pi d^2.$$

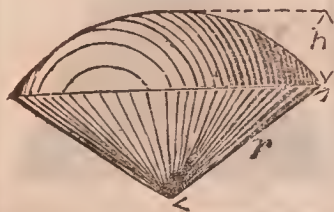


108.

Torus.

$$a = 4 \pi^2 R r = 39.44 R r,$$

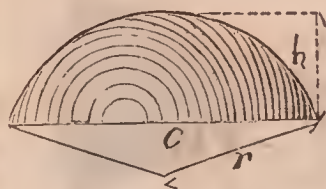
$$a = 9.86 D d.$$



109.

Sphere Sector.

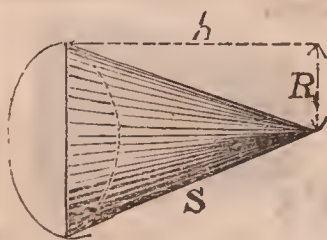
$$a = \frac{\pi r}{2} (4 h + c).$$



110.

Circle Zone.

$$a = 2 \pi r h = \frac{\pi}{4} (c^2 + h^2).$$

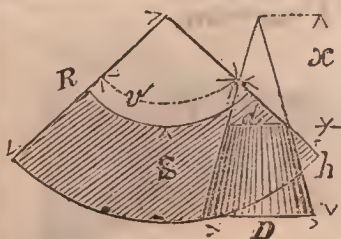


111.

Cone.

$$a = \pi R s,$$

$$a = \pi R \sqrt{R^2 + h^2}.$$



112.

Cone.

$$x = \frac{d h}{D - d}, \quad R = s + \frac{d s}{D - d},$$

$$a = \frac{\pi s}{2} (D + d),$$

$$v = \frac{180 D}{R} = \frac{180 (D - d)}{s}.$$

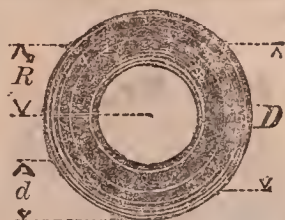


113.

Sphere.

$$C = \frac{4}{3} \pi r^3 = 4.189 r^3,$$

$$C = \frac{\pi d^3}{6} = 0.523 d^3.$$

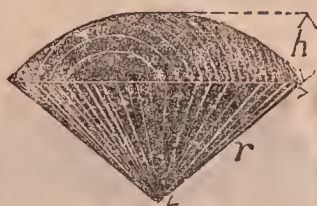


114.

Torus.

$$C = 2 \pi R r^2 = 19.72 R r^2,$$

$$C = 2.463 D d^2.$$

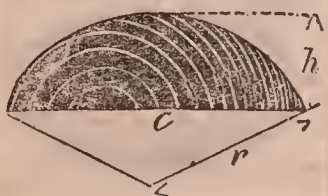


115.

Sphere Sector.

$$C = \frac{2}{3} \pi r^2 h = 2.0944 r^2 h,$$

$$C = \frac{2}{3} \pi r^2 \left(r \pm \sqrt{r^2 - \frac{1}{4} c^2} \right).$$

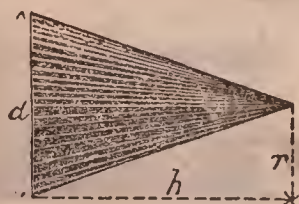


116.

Zone.

$$C = \pi h^2 \left(r - \frac{1}{3} h \right),$$

$$C = \pi h^2 \left(\frac{c^2 + 4h^2}{8r} - \frac{1}{3} h \right).$$

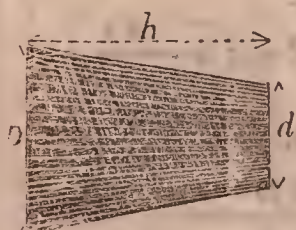


117.

Cone.

$$C = \frac{\pi r^2}{3} h = 1.046 r^2 h,$$

$$C = 0.2616 d^2 h.$$



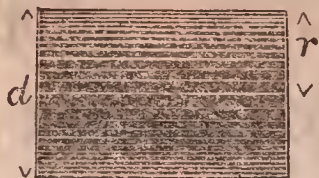
118.

Conic Frustrum.

$$C = \frac{1}{3} \pi h (R^2 + R r + r^2),$$

$$C = \frac{1}{12} \pi h (D^2 + D d + d^2)$$

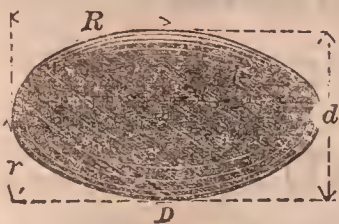
119.

Cylinder.

$$c = \pi r^2 h = 0.785 d^2 h,$$

$$c = \frac{p^2 h}{4 \pi} = 0.0796 p^2 h.$$

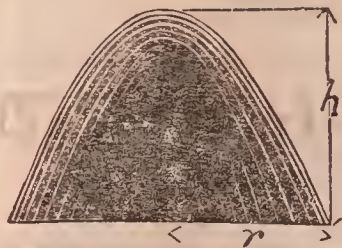
120.

Ellipsoid.

$$c = 0.424 \pi^2 R r^2 = 4.1847 R r^2,$$

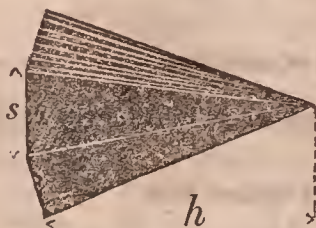
$$c = 0.053 \pi^2 D d^2 = 0.5231 D d^2.$$

121.

Paraboloid.

$$c = \frac{1}{2} \pi r^2 h = 1.5707 r^2 h.$$

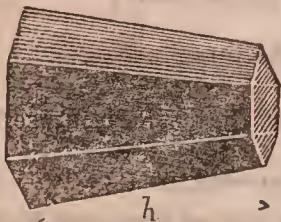
122.

Pyramid.

$$c = \frac{1}{6} a h,$$

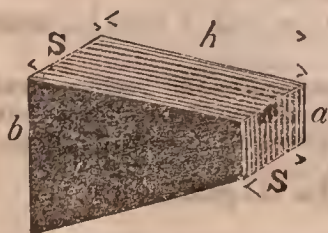
$$c = \frac{n s h}{6} \sqrt{r^2 - \frac{s^2}{4}}.$$

123.

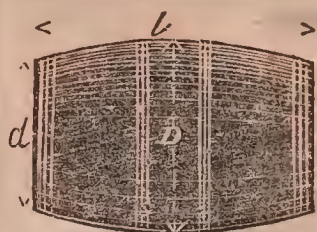
Pyramidal Frustrum.

$$c = \frac{h}{3} (A + a + \sqrt{A a}).$$

124.

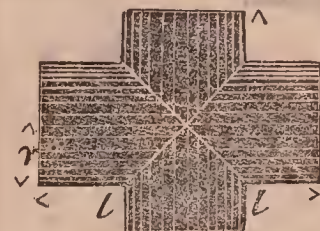
Wedge Frustrum.

$$c = \frac{h s}{2} (a + b).$$

125. *Cask.*

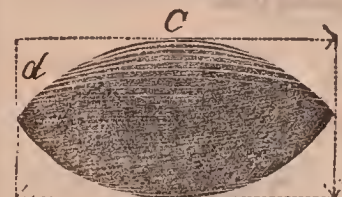
$$c = 1.0453 l (0.4 D^2 + 0.2 D d + 0.15 d^2),$$

$$\text{Gallon} = \frac{l}{2200} (4 D^2 + 2 D d + 1.5 d^2).$$

126. *Cylinder Sections.*

$$c = \pi r^2 (l + l' - \frac{2}{3} r),$$

$$c = \pi r^2 (l + l') - 2.1 r^3.$$

127. *Circular Spindle.*

$$c = \pi (\frac{1}{6} c^3 - 0.2 d [c + \frac{2}{3} \sqrt{c^2 + d^2}] \sqrt{d^2 + c^2})$$

Example 1. Fig. 92. The base of a Triangle is $b = 8$ feet, 3 inches, and the height, $h = 5$ feet, 6 inches. What is the area $a = ?$

$$a = \frac{b h}{2} = \frac{8.25 \times 5.5}{2} = 22.6875 \text{ square feet.}$$

Example 2. Fig. 98. A Circle Sector having an angle $v = 39^\circ$ and the radius $r = 67\frac{3}{4}$ inches. What is the area of the sector $a = ?$

$$a = \frac{\pi r^2 v}{360} = \frac{3.14 \times 67.75^2 \times 39^\circ}{360} = 1562.1 \text{ square feet.}$$

Example 3. Fig. 110. A Spherical Zone having its diameter $c = 18\frac{1}{2}$ inches and height $h = 7\frac{3}{4}$ inches. What is the convex surface of the Zone?

$$a = \frac{\pi}{4} (c^2 + h^2) = \frac{3.14}{4} (18.5^2 + 7.75^2) = 315.96 \text{ square inches.}$$

Example 4. Fig. 88. Require the radius R of a Sphere that will circumscribe a Dodecahedron with the side $s = 9$ inches.

$$R = 1.36428 \times 9 = 12.27852 \text{ inches, the answer.}$$

Example 5. Fig. 118. A Frustrum of a Cone having its bottom diameter $D = 13$ inches, the top diameter $d = 5\frac{1}{4}$ inches, and the height $h = 25$ inches. What is the cubic contents $c = ?$

$$c = \frac{1}{12} \pi h (D^2 + D d + d^2) = 0.2618 \times 25 (13^2 + 13 \times 5.25 + 5.25^2) = 20995 \text{ cubic inches.}$$

Example 6. Fig. 125. A Cask having its bung diameter $D = 36$ inches, head diameter $d = 28$ inches, and length $l = 56$ inches, (inside measurement) how many gallons of liquid can be contained in the cask? (The gallon = 231 cub. in.)

$$\text{Gallon} = \frac{56}{2200} (4 \times 36^2 + 2 \times 36 \times 28 + 1.5 \times 28^2) = 214 \text{ gallons.}$$

Example 7. Fig. 50. Require the length of the circle-arc b , when the angle $v = 42^\circ$, and the radius $r = 4$ feet, 3 inches?

$$b = \frac{\pi r v}{180} = \frac{3.14 \times 4.25 \times 42}{180} = 3.113 \text{ feet.}$$

Example 8. Fig. 52. Require the radius of a circle-arc, whose chord is 9 feet, 4 inches, and height, $h = 1$ foot, 8 inches?

$$r = \frac{c^2 + 4h^2}{8h} = \frac{9.33^2 + 4 \times 1.66^2}{8 \times 1.66} = \frac{98.0711}{13.28} = 7.384 \text{ feet.}$$

Example 9. Fig. 68. The three sides in a triangle being, $a = 6.42$, $b = 7.75$, and $c = 8.66$ feet. How high is the triangle over the base b ?

$$d = \frac{a^2 + b^2 - c^2}{2b} = \frac{6.42^2 + 7.75^2 - 8.66^2}{2 \times 8.66} = 1.5175 \text{ feet,}$$








the height $h = \sqrt{a^2 - d^2} = \sqrt{6.42^2 - 1.5175^2} = 6.24$ feet, the answer.

Example 10. Fig. 77. The radius of a walking beam is, $r = 8.36$ feet, the stroke $S = 5.5$ feet. How much is the vibration $V = ?$

$$\text{Vibration, } V = r - \sqrt{r^2 - \frac{S^2}{4}} = 8.36 - \sqrt{8.36^2 - \frac{5.5^2}{4}}$$

$$= 0.471 \text{ feet} = 5.65 \text{ inches} = 5\frac{21}{32}, \text{ the answer.}$$

TABLE OF POLYGONS.

Number of sides in the Polygon.	Centre Angle w .	Polygon Angle v .	Side $= k R$.	Area $= k S^2$.	Apothem $= k R$.	Side $= k r$.	Area $= k r^2$.	
								
Trigon.	3	120°	60°	1.732	0.4330	0.5000	3.4641	5.1961
Tetragon.	4	90°	90°	1.4142	1.0000	0.7071	2.0000	4.0000
Pentagon.	5	72°	108°	1.1755	1.7205	0.8090	1.4536	3.6327
Hexagon.	6	60°	120°	1.0000	2.5980	0.8660	1.1547	3.4640
Heptagon.	7	51°43'	128°17'	0.8677	3.6339	0.9009	0.9631	3.3710
Octagon.	8	45°	135°	0.7653	4.8284	0.9238	0.8284	3.3130
Nonagon.	9	40°	140°	0.6840	6.1820	0.9396	0.7279	3.2750
Decagon.	10	36°	144°	0.6180	7.6942	0.9510	0.6498	3.2490
Undecagon.	11	32°13'	147°47'	0.5634	9.3656	0.9595	0.5872	3.2290
Dodecagon.	12	30°	150°	0.5176	11.196	0.9659	0.5359	3.2152
	14	25°43'	154°17'	0.4450	15.334	0.9762	0.4562	3.1935
	15	24°	156°	0.4158	17.642	0.9781	0.4250	3.1882
	16	22°30'	157°30'	0.3900	20.128	0.9807	0.4068	3.1824
	18	20°	160°	0.3472	25.534	0.9848	0.3526	3.1737
	20	18°	162°	0.3130	40.634	0.9877	0.3166	3.1676
	24	15°	165°	0.2610	45.593	0.9914	0.2632	3.1596

Explanation of the Table for Polygons.

The number of sides in the polygon is noted in the first column.







$k = \text{tabular coefficient}$, to be multiplied as noted on the top of the columns.







Example 1. How long is the side of an inscribed Pentagon, when the radius of the circle is 3 feet, and 4 inches? (4 inches = 0.333 feet.)







$$3.333 \times 1.1755 = 3.9179 \text{ feet, the answer.}$$







Example 2. What is the area of a Heptagon when one of its sides is 13.75 inches







$$13.75^2 \times 3.6339 = 687.02 \text{ square inches.}$$







Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
								
$\frac{1}{32}$	·0981	·00076	5	15·70	19·635	11	34·55	95·033
$\frac{1}{16}$	·1963	·00306		16·10	20·629		34·95	97·205
$\frac{1}{8}$	·3926	·01227	$\frac{1}{4}$	16·49	21·647	$\frac{1}{4}$	35·34	99·402
$\frac{3}{16}$	·5890	·02761		16·88	22·690		35·73	101·62
$\frac{1}{4}$	·7854	·04908	$\frac{1}{2}$	17·27	23·758	$\frac{1}{2}$	36·12	103·86
$\frac{5}{16}$	·9817	·07669		17·67	24·850		36·52	106·13
$\frac{3}{8}$	1·178	·1104	$\frac{3}{4}$	18·06	25·967	$\frac{3}{4}$	36·91	108·43
$\frac{7}{16}$	1·374	·1503		18·45	27·108		37·30	110·75
$\frac{1}{2}$	1·570	·1963	6	18·84	28·274	12	37·69	113·09
$\frac{9}{16}$	1·767	·2485		19·24	29·464		38·09	115·46
$\frac{5}{8}$	1·963	·3067	$\frac{1}{4}$	19·63	30·679	$\frac{1}{4}$	38·48	117·85
$\frac{11}{16}$	2·159	·3712		20·02	31·919		38·87	120·27
$\frac{3}{4}$	2·356	·4417	$\frac{1}{2}$	20·42	33·183	$\frac{1}{2}$	39·27	122·71
$\frac{13}{16}$	2·552	·5184		20·81	34·471		39·66	125·18
$\frac{7}{8}$	2·748	·6013	$\frac{3}{4}$	21·20	35·784	$\frac{3}{4}$	40·05	127·67
$\frac{15}{16}$	2·945	·6902		21·57	37·122		40·44	130·19
1	3·141	·7854	7	21·99	38·484	13	40·84	132·73
	3·534	·9940		22·38	39·871		41·23	135·29
$\frac{1}{2}$	3·927	1·227	$\frac{1}{4}$	22·77	41·282	$\frac{1}{4}$	41·62	137·88
	4·319	1·484		23·16	42·718		42·01	140·50
$\frac{1}{2}$	4·712	1·767	$\frac{1}{2}$	23·56	44·178	$\frac{1}{2}$	42·41	143·13
	5·105	2·073		23·95	45·663		42·80	145·80
$\frac{3}{4}$	5·497	2·405	$\frac{3}{4}$	24·34	47·173	$\frac{3}{4}$	43·19	148·48
	5·890	2·761		24·74	48·707		43·58	151·20
2	6·283	3·141	8	25·13	50·265	14	43·98	153·93
	6·675	3·546		25·52	51·848		44·37	156·69
$\frac{1}{2}$	7·068	3·976	$\frac{1}{4}$	25·91	53·456	$\frac{1}{4}$	44·76	159·48
	7·461	4·430		26·31	55·088		45·16	162·29
$\frac{1}{2}$	7·854	4·908	$\frac{1}{2}$	26·70	56·745	$\frac{1}{2}$	45·55	165·13
	8·246	5·411		27·09	58·426		45·94	167·98
$\frac{3}{4}$	8·639	5·939	$\frac{3}{4}$	27·48	60·132	$\frac{3}{4}$	46·33	170·87
	9·032	6·491		27·88	61·862		46·73	173·78
3	9·424	7·068	9	28·27	63·617	15	47·12	176·71
	9·817	7·669		28·66	65·396		47·51	179·67
$\frac{1}{2}$	10·21	8·295	$\frac{1}{4}$	29·05	67·200	$\frac{1}{4}$	47·90	182·65
	10·60	8·946		29·45	69·029		48·30	185·66
$\frac{1}{2}$	10·99	9·621	$\frac{1}{2}$	29·84	70·882	$\frac{1}{2}$	48·69	188·69
	11·38	10·320		30·23	72·759		49·08	191·74
$\frac{3}{4}$	11·78	11·044	$\frac{3}{4}$	30·63	74·662	$\frac{3}{4}$	49·48	194·82
	12·17	11·793		31·02	76·588		49·87	197·93
4	12·56	12·566	10	31·41	78·539	16	50·26	201·06
	12·95	13·364		31·80	80·515		50·65	204·21
$\frac{1}{2}$	13·35	14·186	$\frac{1}{4}$	32·20	82·516	$\frac{1}{4}$	51·05	207·39
	13·74	15·033		32·59	84·540		51·44	210·59
$\frac{1}{2}$	14·13	15·904	$\frac{1}{2}$	32·98	86·590	$\frac{1}{2}$	51·83	213·82
	14·52	16·800		33·37	88·664		52·22	217·07
$\frac{3}{4}$	14·92	17·720	$\frac{3}{4}$	33·77	90·762	$\frac{3}{4}$	52·62	220·35
	15·31	18·665		34·16	92·885		53·01	223·65

Diameter.	Circ.		Diameter.	Circ.		Diameter.	Circ.	
								
17	53.40	226.98	23	72.25	415.47	29	91.10	660.52
	53.79	230.33		72.64	420.00		91.49	666.22
$\frac{1}{4}$	54.19	233.70	$\frac{1}{4}$	73.04	424.55	$\frac{1}{4}$	91.89	671.95
	54.58	237.10		73.43	429.13		92.28	677.71
$\frac{1}{2}$	54.97	240.52	$\frac{1}{2}$	73.82	433.73	$\frac{1}{2}$	92.67	683.49
	55.37	243.97		74.21	438.30		93.06	689.29
$\frac{3}{4}$	55.76	247.45	$\frac{3}{4}$	74.61	443.01	$\frac{3}{4}$	93.46	695.12
	56.16	250.94		75.	447.69		93.85	700.98
18	56.54	254.46	24	75.39	452.39	30	94.24	706.86
	56.94	258.01		75.79	457.11		94.64	712.76
$\frac{1}{4}$	57.33	261.58	$\frac{1}{4}$	76.18	461.86	$\frac{1}{4}$	95.03	718.69
	57.72	265.18		76.57	466.63		95.42	724.64
$\frac{1}{2}$	58.11	268.80	$\frac{1}{2}$	76.96	471.43	$\frac{1}{2}$	95.81	730.61
	58.51	272.44		77.36	476.25		96.21	736.61
$\frac{3}{4}$	58.90	276.11	$\frac{3}{4}$	77.75	481.10	$\frac{3}{4}$	96.60	742.64
	59.29	279.81		78.14	485.97		96.99	748.69
19	59.69	283.52	25	78.54	490.87	31	97.38	754.76
	60.08	287.27		78.93	495.79		97.78	760.86
$\frac{1}{4}$	60.47	291.03	$\frac{1}{4}$	79.32	500.74	$\frac{1}{4}$	98.17	766.99
	60.86	294.83		79.71	505.71		98.56	773.14
$\frac{1}{2}$	61.26	298.64	$\frac{1}{2}$	80.10	510.70	$\frac{1}{2}$	98.96	779.31
	61.65	302.48		80.50	515.72		99.35	785.51
$\frac{3}{4}$	62.04	306.35	$\frac{3}{4}$	80.89	520.70	$\frac{3}{4}$	99.74	791.73
	62.43	310.24		81.28	525.83		100.1	797.97
20	62.83	314.16	26	81.68	530.93	32	100.5	804.24
	63.22	318.09		82.07	536.04		100.9	810.54
$\frac{1}{4}$	63.61	322.06	$\frac{1}{4}$	82.46	541.18	$\frac{1}{4}$	101.3	816.86
	64.01	326.05		82.85	546.35		101.7	823.21
$\frac{1}{2}$	64.40	330.06	$\frac{1}{2}$	83.25	551.54	$\frac{1}{2}$	102.1	829.57
	64.79	334.10		83.64	556.76		102.4	835.97
$\frac{3}{4}$	65.18	338.16	$\frac{3}{4}$	84.03	562.00	$\frac{3}{4}$	102.8	842.39
	65.58	342.25		84.43	567.26		103.2	848.83
21	65.97	346.36	27	84.82	572.55	33	103.6	855.30
	66.36	350.49		85.21	577.87		104.	861.79
$\frac{1}{4}$	66.75	354.65	$\frac{1}{4}$	85.60	583.20	$\frac{1}{4}$	104.4	868.30
	67.15	358.84		86.	588.57		104.8	874.84
$\frac{1}{2}$	67.54	363.05	$\frac{1}{2}$	86.39	593.95	$\frac{1}{2}$	105.2	881.41
	67.93	367.28		86.78	599.37		105.6	888.00
$\frac{3}{4}$	68.32	371.54	$\frac{3}{4}$	87.17	604.80	$\frac{3}{4}$	106.	894.61
	68.72	375.82		87.57	610.26		106.4	901.25
22	69.11	380.13	28	87.96	615.75	34	106.8	907.92
	69.50	384.46		88.35	621.26		107.2	914.61
$\frac{1}{4}$	69.90	388.82	$\frac{1}{4}$	88.75	626.79	$\frac{1}{4}$	107.5	921.32
	70.29	393.20		89.14	632.35		107.9	928.06
$\frac{1}{2}$	70.68	397.60	$\frac{1}{2}$	89.53	637.94	$\frac{1}{2}$	108.3	934.82
	71.07	402.03		89.92	643.54		108.7	941.60
$\frac{3}{4}$	71.47	406.49	$\frac{3}{4}$	90.32	649.18	$\frac{3}{4}$	109.1	948.41
	71.86	410.97		90.71	654.83		109.5	955.25

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
								
35	109.9	962.11	41	128.8	1320.2	47	147.6	1734.9
$\frac{1}{4}$	110.3	968.99	$\frac{1}{4}$	129.1	1328.3	$\frac{1}{4}$	148.	1744.1
$\frac{1}{2}$	110.7	975.90	$\frac{1}{2}$	129.5	1336.4	$\frac{1}{2}$	148.4	1753.4
$\frac{3}{4}$	111.1	982.84	$\frac{3}{4}$	129.9	1344.5	$\frac{3}{4}$	148.8	1762.7
36	111.5	989.80	42	130.3	1352.6	48	149.2	1772.0
$\frac{1}{4}$	111.9	996.78	$\frac{1}{4}$	130.7	1360.8	$\frac{1}{4}$	149.6	1781.3
$\frac{1}{2}$	112.3	1003.7	$\frac{1}{2}$	131.1	1369.0	$\frac{1}{2}$	150.	1790.7
$\frac{3}{4}$	112.7	1010.8	$\frac{3}{4}$	131.5	1377.2	$\frac{3}{4}$	150.4	1800.1
37	113.	1017.8	43	131.9	1385.4	49	150.7	1809.5
$\frac{1}{4}$	113.4	1024.9	$\frac{1}{4}$	132.3	1393.7	$\frac{1}{4}$	150.1	1818.9
$\frac{1}{2}$	113.8	1032.0	$\frac{1}{2}$	132.7	1401.9	$\frac{1}{2}$	151.5	1828.4
$\frac{3}{4}$	114.2	1039.1	$\frac{3}{4}$	133.1	1410.2	$\frac{3}{4}$	151.9	1837.9
38	114.6	1046.3	44	133.5	1418.6	50	152.3	1847.4
$\frac{1}{4}$	115.	1053.5	$\frac{1}{4}$	133.9	1426.9	$\frac{1}{4}$	152.7	1856.9
$\frac{1}{2}$	115.4	1060.7	$\frac{1}{2}$	134.3	1435.3	$\frac{1}{2}$	153.1	1866.5
$\frac{3}{4}$	115.8	1067.9	$\frac{3}{4}$	134.6	1443.7	$\frac{3}{4}$	153.5	1876.1
39	116.2	1075.2	45	135.	1452.2	51	153.9	1885.7
$\frac{1}{4}$	116.6	1082.4	$\frac{1}{4}$	135.4	1460.6	$\frac{1}{4}$	154.3	1895.3
$\frac{1}{2}$	117.	1089.7	$\frac{1}{2}$	135.8	1469.1	$\frac{1}{2}$	154.7	1905.0
$\frac{3}{4}$	117.4	1097.1	$\frac{3}{4}$	136.2	1477.6	$\frac{3}{4}$	155.1	1914.7
40	117.8	1104.4	46	136.6	1486.1	52	155.5	1924.4
$\frac{1}{4}$	118.2	1111.8	$\frac{1}{4}$	137.	1494.7	$\frac{1}{4}$	155.9	1934.1
$\frac{1}{2}$	118.6	1119.2	$\frac{1}{2}$	137.4	1503.3	$\frac{1}{2}$	156.2	1943.9
$\frac{3}{4}$	118.9	1126.6	$\frac{3}{4}$	137.8	1511.9	$\frac{3}{4}$	156.6	1953.6
41	119.3	1134.1	47	138.2	1520.5	53	157.	1963.5
$\frac{1}{4}$	119.7	1141.5	$\frac{1}{4}$	138.6	1529.1	$\frac{1}{4}$	157.4	1973.3
$\frac{1}{2}$	120.1	1149.0	$\frac{1}{2}$	139.	1537.8	$\frac{1}{2}$	157.8	1983.1
$\frac{3}{4}$	120.5	1156.6	$\frac{3}{4}$	139.4	1546.5	$\frac{3}{4}$	158.2	1993.0
42	120.9	1164.1	48	139.8	1555.2	54	158.6	2002.9
$\frac{1}{4}$	121.3	1171.7	$\frac{1}{4}$	140.1	1564.0	$\frac{1}{4}$	159.	2012.8
$\frac{1}{2}$	121.7	1179.3	$\frac{1}{2}$	140.5	1572.8	$\frac{1}{2}$	159.4	2022.8
$\frac{3}{4}$	122.1	1186.9	$\frac{3}{4}$	140.9	1581.6	$\frac{3}{4}$	159.8	2032.8
43	122.5	1194.5	49	141.3	1590.4	55	160.2	2042.8
$\frac{1}{4}$	122.9	1202.2	$\frac{1}{4}$	141.7	1599.2	$\frac{1}{4}$	160.6	2052.8
$\frac{1}{2}$	123.3	1209.9	$\frac{1}{2}$	142.1	1608.1	$\frac{1}{2}$	161.	2062.9
$\frac{3}{4}$	123.7	1217.6	$\frac{3}{4}$	142.5	1617.0	$\frac{3}{4}$	161.3	2072.9
44	124.	1225.4	50	142.9	1625.9	56	161.7	2083.0
$\frac{1}{4}$	124.4	1233.1	$\frac{1}{4}$	143.3	1634.9	$\frac{1}{4}$	162.1	2093.2
$\frac{1}{2}$	124.8	1240.9	$\frac{1}{2}$	143.7	1643.8	$\frac{1}{2}$	162.5	2103.3
$\frac{3}{4}$	125.2	1248.7	$\frac{3}{4}$	144.1	1652.8	$\frac{3}{4}$	162.9	2113.5
45	125.6	1256.6	51	144.5	1661.9	57	163.3	2123.7
$\frac{1}{4}$	126.	1264.5	$\frac{1}{4}$	144.9	1670.9	$\frac{1}{4}$	163.7	2133.9
$\frac{1}{2}$	126.4	1272.3	$\frac{1}{2}$	145.2	1680.0	$\frac{1}{2}$	164.1	2144.1
$\frac{3}{4}$	126.8	1280.3	$\frac{3}{4}$	145.6	1689.1	$\frac{3}{4}$	164.5	2154.4
46	127.2	1288.2	52	146.	1698.2	58	164.9	2164.7
$\frac{1}{4}$	127.6	1296.2	$\frac{1}{4}$	146.4	1707.3	$\frac{1}{4}$	165.3	2175.0
$\frac{1}{2}$	128.	1304.2	$\frac{1}{2}$	146.8	1716.5	$\frac{1}{2}$	165.7	2185.4
$\frac{3}{4}$	128.4	1312.2	$\frac{3}{4}$	147.2	1725.7	$\frac{3}{4}$	166.1	2195.7

Circ.		Area.	Circ.		Area.	Circ.		Area.
Dia-			meter.			Dia-		
53	166.5	2206.1	59	185.3	2733.9	65	204.2	3318.3
	166.8	2216.6		185.7	2745.5		204.5	3331.0
$\frac{1}{4}$	167.2	2227.0	$\frac{1}{4}$	186.1	2757.1	$\frac{1}{4}$	204.9	3343.8
	167.6	2237.5		186.5	2768.8		205.3	3356.7
$\frac{1}{2}$	168.	2248.0	$\frac{1}{2}$	186.9	2780.5	$\frac{1}{2}$	205.7	3369.5
	168.4	2258.5		187.3	2792.2		206.1	3382.4
$\frac{3}{4}$	168.8	2269.0	$\frac{3}{4}$	187.7	2803.9	$\frac{3}{4}$	206.5	3395.3
	169.2	2279.6		188.1	2815.6		206.9	3408.2
54	169.6	2290.2	60	188.4	2827.4	66	207.3	3421.2
	170.	2300.8		188.8	2839.2		207.7	3434.1
$\frac{1}{4}$	170.4	2311.4	$\frac{1}{4}$	189.2	2851.0	$\frac{1}{4}$	208.1	3447.1
	170.8	2322.1		189.6	2862.8		208.5	3460.1
$\frac{1}{2}$	171.2	2332.8	$\frac{1}{2}$	190	2874.7	$\frac{1}{2}$	208.9	3473.2
	171.6	2343.5		190.4	2886.6		209.3	3486.3
$\frac{3}{4}$	172.	2354.2	$\frac{3}{4}$	190.8	2898.5	$\frac{3}{4}$	209.7	3499.3
	172.3	2365.0		191.2	2910.5		210.	3512.5
55	172.7	2375.8	61	191.6	2922.4	67	210.4	3525.6
	173.1	2386.6		192.	2934.4		210.8	3538.8
$\frac{1}{4}$	173.5	2397.4	$\frac{1}{4}$	192.4	2946.4	$\frac{1}{4}$	211.2	3552.0
	173.9	2408.3		192.8	2958.5		211.6	3565.2
$\frac{1}{2}$	174.3	2419.2	$\frac{1}{2}$	193.2	2970.5	$\frac{1}{2}$	212.	3578.4
	174.7	2430.1		193.6	2982.6		212.4	3591.7
$\frac{3}{4}$	175.1	2441.0	$\frac{3}{4}$	193.9	2994.7	$\frac{3}{4}$	212.8	3605.0
	175.5	2452.0		194.3	3006.9		213.2	3618.3
56	175.9	2463.0	62	194.7	3019.0	68	213.6	3631.6
	176.3	2474.0		195.1	3031.2		214.	3645.0
$\frac{1}{4}$	176.7	2485.0	$\frac{1}{4}$	195.5	3043.4	$\frac{1}{4}$	214.4	3658.4
	177.1	2496.1		195.9	3055.7		214.8	3671.8
$\frac{1}{2}$	177.5	2507.1	$\frac{1}{2}$	196.3	3067.9	$\frac{1}{2}$	215.1	3685.2
	177.8	2518.2		196.7	3080.2		215.5	3698.7
$\frac{3}{4}$	178.2	2529.4	$\frac{3}{4}$	197.1	3092.5	$\frac{3}{4}$	215.9	3712.2
	178.6	2540.5		197.5	3104.8		216.3	3725.7
57	179.	2551.7	63	197.9	3117.2	69	216.7	3739.2
	179.4	2562.9		198.3	3129.6		217.1	3752.8
$\frac{1}{4}$	179.8	2574.1	$\frac{1}{4}$	198.7	3142.0	$\frac{1}{4}$	217.5	3766.4
	180.2	2585.4		199.	3154.4		217.9	3780.0
$\frac{1}{2}$	180.6	2596.7	$\frac{1}{2}$	199.4	3166.9	$\frac{1}{2}$	218.3	3793.6
	181.	2608.0		199.8	3179.4		218.7	3807.3
$\frac{3}{4}$	181.4	2619.3	$\frac{3}{4}$	200.2	3191.9	$\frac{3}{4}$	219.1	3821.0
	181.8	2630.7		200.6	3204.4		219.5	3834.7
58	182.2	2642.0	64	201.	3216.9	70	219.9	3848.4
	182.6	2653.4		201.4	3229.5		220.3	3862.2
$\frac{1}{4}$	182.9	2664.9	$\frac{1}{4}$	201.8	3242.1	$\frac{1}{4}$	220.6	3875.9
	183.3	2676.3		202.2	3254.8		221.	3889.8
$\frac{1}{2}$	183.7	2687.8	$\frac{1}{2}$	202.6	3267.4	$\frac{1}{2}$	221.4	3903.6
	184.1	2699.3		203.	3280.1		221.8	3917.4
$\frac{3}{4}$	184.5	2710.8	$\frac{3}{4}$	203.4	3292.8	$\frac{3}{4}$	222.2	3931.3
	184.9	2722.4		203.8	3305.5		222.6	3945.2

Diameter.	Circ.		Diameter.	Circ.		Diameter.	Circ.	
								
71	223.	3959.2	77	241.9	4656.6	83	260.7	5410.6
	223.4	3973.1		242.2	4671.7		261.1	5426.9
$\frac{1}{4}$	223.8	3987.1	$\frac{1}{4}$	242.6	4686.9	$\frac{1}{4}$	261.5	5443.2
	224.2	4001.1		243.	4702.1		261.9	5459.6
$\frac{1}{2}$	224.6	4015.1	$\frac{1}{2}$	243.4	4717.3	$\frac{1}{2}$	262.3	5476.0
	225.	4029.2		243.8	4732.5		262.7	5492.4
$\frac{3}{4}$	225.4	4043.2	$\frac{3}{4}$	244.2	4747.7	$\frac{3}{4}$	263.1	5508.8
	225.8	4067.3		244.6	4763.0		263.5	5525.3
72	226.1	4071.5	78	245.	4778.3	84	263.8	5541.7
	226.5	4085.6		245.4	4793.7		264.2	5558.2
$\frac{1}{4}$	226.9	4099.8	$\frac{1}{4}$	245.8	4809.0	$\frac{1}{4}$	264.6	5574.8
	227.3	4114.0		246.2	4824.4		265.	5591.3
$\frac{1}{2}$	227.7	4128.2	$\frac{1}{2}$	246.6	4839.8	$\frac{1}{2}$	265.4	5607.9
	228.1	4142.5		247.	4855.2		265.8	5624.5
$\frac{3}{4}$	228.5	4156.7	$\frac{3}{4}$	247.4	4870.7	$\frac{3}{4}$	266.2	5641.1
	228.9	4171.0		247.7	4886.1		266.6	5657.8
73	229.3	4185.3	79	248.1	4901.6	85	267.	5674.5
	229.7	4199.7		248.5	4917.2		267.4	5691.2
$\frac{1}{4}$	230.1	4214.1	$\frac{1}{4}$	248.9	4932.7	$\frac{1}{4}$	267.8	5707.9
	230.5	4228.5		249.3	4948.3		268.2	5724.6
$\frac{1}{2}$	230.9	4242.9	$\frac{1}{2}$	249.7	4963.9	$\frac{1}{2}$	269.6	5741.4
	231.3	4257.3		250.1	4979.5		268.9	5758.2
$\frac{3}{4}$	231.6	4271.8	$\frac{3}{4}$	250.5	4995.1	$\frac{3}{4}$	269.3	5775.0
	232.	4286.3		250.9	5010.8		269.7	5791.9
74	232.4	4300.8	80	251.3	5026.5	86	270.1	5808.8
	232.8	4315.3		251.7	5042.2		270.5	5825.7
$\frac{1}{4}$	233.2	4329.9	$\frac{1}{4}$	252.1	5058.0	$\frac{1}{4}$	270.9	5842.6
	233.6	4344.5		252.5	5073.7		271.3	5859.5
$\frac{1}{2}$	234.	4359.1	$\frac{1}{2}$	252.8	5089.5	$\frac{1}{2}$	271.7	5876.5
	234.4	4373.8		253.2	5105.4		272.1	5893.5
$\frac{3}{4}$	234.8	4388.4	$\frac{3}{4}$	253.6	5121.2	$\frac{3}{4}$	272.5	5910.5
	235.2	4403.1		254.	5137.1		272.9	5927.6
75	235.6	4417.8	81	254.4	5153.0	87	273.3	5944.6
	236.	4432.6		254.8	5168.9		273.7	5961.7
$\frac{1}{4}$	236.4	4447.3	$\frac{1}{4}$	255.2	5184.8	$\frac{1}{4}$	274.1	5978.9
	236.7	4462.1		255.6	5200.8		274.4	5996.0
$\frac{1}{2}$	237.1	4476.9	$\frac{1}{2}$	256.	5216.8	$\frac{1}{2}$	274.8	6013.2
	237.5	4491.8		256.4	5232.8		275.2	6030.4
$\frac{3}{4}$	237.9	4506.6	$\frac{3}{4}$	256.8	5248.8	$\frac{3}{4}$	275.6	6047.6
	238.3	4521.5		257.2	5264.9		276.	6064.8
76	238.7	4536.4	82	257.6	5281.0	88	276.4	6082.1
	239.1	4551.4		258.	5297.1		276.8	6099.4
$\frac{1}{4}$	239.5	4566.3	$\frac{1}{4}$	258.3	5313.2	$\frac{1}{4}$	277.2	6116.7
	239.9	4581.3		258.7	5329.4		277.6	6134.0
$\frac{1}{2}$	240.3	4596.3	$\frac{1}{2}$	259.1	5345.6	$\frac{1}{2}$	278.	6151.4
	240.7	4611.3		259.5	5361.8		278.4	6168.8
$\frac{3}{4}$	241.1	4626.4	$\frac{3}{4}$	259.9	5378.0	$\frac{3}{4}$	278.8	6186.2
	241.5	4641.5		260.3	5394.3		279.2	6203.6

Circ. Area.			Circ. Area.			Circ. Area.		
Dia- meter.			Dia- meter.			Dia- meter.		
89	279.6	6221.1	93	292.1	6792.9	97	304.7	7389.8
	279.9	6238.6		292.5	6811.1		305.1	7408.8
$\frac{1}{4}$	280.3	6256.1	$\frac{1}{4}$	292.9	6829.4	$\frac{1}{4}$	305.5	7427.9
	280.7	6273.6		293.3	6847.8		305.9	7447.0
$\frac{1}{2}$	281.1	6291.2	$\frac{1}{2}$	293.7	6866.1	$\frac{1}{2}$	306.3	7466.2
	281.5	6308.8		294.1	6884.5		306.6	7485.3
$\frac{3}{4}$	281.9	6326.4	$\frac{3}{4}$	294.5	6902.9	$\frac{3}{4}$	307.0	7504.5
	282.3	6344.0		294.9	6921.3		307.4	7523.7
90	282.7	6361.7	94	295.3	6939.7	98	307.8	7542.9
	283.1	6379.4		295.7	6958.2		308.2	7562.2
$\frac{1}{4}$	283.5	6397.1	$\frac{1}{4}$	296.0	6976.7	$\frac{1}{4}$	308.6	7581.5
	283.9	6414.8		296.4	6995.2		309.0	7600.8
$\frac{1}{2}$	284.3	6432.6	$\frac{1}{2}$	296.8	7013.8	$\frac{1}{2}$	309.4	7620.1
	284.7	6450.4		297.2	7032.3		309.8	7639.4
$\frac{3}{4}$	285.1	6468.2	$\frac{3}{4}$	297.6	7050.9	$\frac{3}{4}$	310.2	7658.8
	285.4	6486.0		298.0	7069.5		310.6	7678.2
91	285.8	6503.8	95	298.4	7088.2	99	311.0	7697.7
	286.2	6521.7		298.8	7106.9		311.4	7717.1
$\frac{1}{4}$	286.6	6539.6	$\frac{1}{4}$	299.2	7125.5	$\frac{1}{4}$	311.8	7736.6
	287.0	6557.6		299.6	7144.3		312.1	7756.1
$\frac{1}{2}$	287.4	6575.5	$\frac{1}{2}$	300.0	7163.0	$\frac{1}{2}$	312.5	7775.6
	287.8	6593.5		300.4	7181.8		312.9	7795.2
$\frac{3}{4}$	288.2	6611.5	$\frac{3}{4}$	300.8	7200.5	$\frac{3}{4}$	313.3	7814.7
	288.6	6629.5		301.2	7219.4		313.7	7834.3
92	289.0	6647.6	96	301.5	7238.2	100	314.1	7853.9
	289.4	6665.7		301.9	7257.1		314.5	7853.6
$\frac{1}{4}$	289.8	6683.8	$\frac{1}{4}$	302.3	7275.9	$\frac{1}{4}$	314.9	7893.3
	290.2	6701.9		302.7	7294.9		315.3	7913.1
$\frac{1}{2}$	290.5	6720.0	$\frac{1}{2}$	303.1	7313.8	$\frac{1}{2}$	315.7	7932.7
	290.9	6738.2		303.5	7332.8		316.0	7942.4
$\frac{3}{4}$	291.3	6756.4	$\frac{3}{4}$	303.9	7351.7	$\frac{3}{4}$	316.4	7972.2
	291.7	6776.4		304.3	7370.7		316.8	7991.9

EXPLANATION OF THE TABLE FOR SEGMENTS, &c.

The chord divided by the height is the gauge in the Table, the quotient in the first column.









k = tabular coefficient, always to be multiplied by the chord.







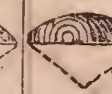

To find the angle of an arc of a circle.





RULE. Divide the base (chord) of the arc by its height, (*sine verse*) and find the quotient in the first column. The corresponding number in the second column is the angle of the arc in degrees of the circle.









To find the radius of an arc of a circle.

RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the third column, multiplied by the chord, is the radius of the arc.

Chord div. by height.	Centre Angle v.	Radius $r = k c.$	Cir. Arc. $b = k c.$	Area Seg. $a = k c^2.$	Surface $a = k c^2.$	Solidity $c = k c^3.$	Chord $c = k r.$
							
458.08	1	57.296	1.0000	.01091	.78539	.00085	.01744
229.18	2	28.649	1.0000	.00218	.78549	.00172	.03490
152.77	3	19.101	1.0000	.00327	.78462	.00255	.05234
114.57	4	14.327	1.0000	.00436	.78574	.00310	.06978
84.747	5	11.462	1.0001	.00647	.78586	.00401	.08722
76.375	6	9.5530	1.0003	.00741	.78599	.00514	.10466
65.943	7	8.1902	1.0004	.00910	.78621	.00592	.12208
57.273	8	7.1678	1.0006	.01089	.78630	.00686	.13950
50.902	9	6.3728	1.0008	.01254	.78665	.00772	.15690
45.807	10	5.7368	1.0011	.01407	.78695	.00857	.17430
41.203	11	5.2167	1.0013	.01552	.78730	.00964	.19168
38.133	12	4.7834	1.0016	.01695	.78725	.01031	.20904
35.221	13	4.4168	1.0019	.01841	.78794	.01114	.22640
32.742	14	4.1027	1.0023	.02000	.78832	.01199	.24372
30.514	15	3.8307	1.0027	.02157	.78889	.01288	.26104
28.601	16	3.5927	1.0029	.02269	.78909	.01375	.27834
26.915	17	3.3827	1.0034	.02434	.78969	.01462	.29560
25.412	18	3.1962	1.0039	.02592	.79028	.01542	.31286
24.068	19	3.0293	1.0044	.02744	.79084	.01635	.33008
22.860	20	2.8793	1.0048	.02878	.79140	.01722	.34728
21.760	21	2.7440	1.0054	.03040	.79234	.01802	.36446
20.777	22	2.6222	1.0059	.03178	.79300	.01897	.38160
19.862	23	2.5080	1.0066	.03343	.79340	.01984	.39872
19.028	24	2.4050	1.0072	.03493	.79416	.02072	.41582
18.261	25	2.3101	1.0078	.03639	.79486	.02159	.43286
17.553	26	2.2233	1.0084	.03784	.79530	.02248	.44990
16.970	27	2.1418	1.0091	.03970	.79639	.02315	.46688
16.288	28	2.0673	1.0101	.04115	.79748	.02424	.48384
15.721	29	1.9969	1.0105	.04230	.79811	.02511	.50076
15.191	30	1.9319	1.0113	.04385	.79907	.02600	.51762
14.970	31	1.8710	1.0121	.04476	.78530	.02692	.53446
14.230	32	1.8140	1.0129	.04710	.80098	.02778	.55126
13.796	33	1.7605	1.0138	.04842	.80181	.02866	.56802
13.382	34	1.7102	1.0146	.04989	.80300	.02956	.58479
12.994	35	1.6628	1.0155	.05137	.80405	.03046	.60140
12.733	36	1.6184	1.0167	.05311	.80531	.03137	.61802
12.473	37	1.5758	1.0174	.05401	.80622	.03226	.63460
11.931	38	1.5358	1.0184	.05628	.80713	.03328	.65112
11.621	39	1.4979	1.0194	.05755	.80850	.03418	.66760
11.342	40	1.4619	1.0204	.05899	.80987	.03506	.68404
11.060	41	1.4266	1.0207	.06001	.81046	.03589	.70040
10.791	42	1.3952	1.0226	.06196	.81240	.03680	.71672
10.534	43	1.3643	1.0237	.06359	.81377	.03773	.73300
10.289	44	1.3347	1.0248	.06574	.81505	.03864	.74920
10.043	45	1.3066	1.0260	.06628	.81756	.03890	.76536
9.8303	46	1.2797	1.0272	.06826	.81795	.04050	.78146
9.6153	47	1.2539	1.0290	.06998	.81939	.04143	.79748
9.4092	48	1.2289	1.0297	.09138	.82064	.04247	.81346

Chord div. by height.	Centre Angle v.	Radius $r = k c.$	Cir. Arc. $b = k c.$	Area Seg. $a = k c^2.$	Surface $a = k c^2.$	Solidity $C = k c^3.$	Chord $c = k r.$
							
9.2113	49	1.2057	1.0309	.07290	.82244	.04330	.82938
9.0214	50	1.1831	1.0323	.07453	.82384	.04424	.84522
8.8387	51	1.1614	1.0336	.07611	.82562	.04519	.86102
8.6629	52	1.1406	1.0349	.07758	.82729	.04614	.87674
8.4462	53	1.1206	1.0364	.07959	.83363	.04685	.89238
8.3306	54	1.1014	1.0378	.08083	.83072	.04805	.90798
8.1733	55	1.0828	1.0393	.08246	.83249	.04901	.92348
8.0215	56	1.0650	1.0407	.08400	.83422	.05002	.93894
7.8750	57	1.0478	1.0422	.08579	.83602	.05098	.95430
7.7334	58	1.0313	1.0431	.08680	.83796	.05191	.96960
7.5895	59	1.0154	1.0454	.08891	.84064	.05299	.98484
7.4565	60	1.0000	1.0470	.09106	.84266	.05400	1.0000
7.3358	61	.98515	1.0486	.09209	.84380	.0546	1.0150
7.2118	62	.97080	1.0503	.09375	.84581	.05583	1.0300
7.0914	63	.95694	1.0520	.09540	.84791	.05684	1.0450
6.9748	64	.94352	1.0537	.09697	.84993	.05784	1.0598
6.8616	65	.93058	1.0555	.09865	.85215	.05885	1.0746
6.7512	66	.91804	1.0573	.10036	.85441	.05987	1.0892
6.6453	67	.90590	1.0591	.10201	.85649	.06088	1.1038
6.5469	68	.89415	1.0610	.10367	.85815	.06181	1.1184
6.4902	69	.88276	1.0629	.10520	.85464	.06201	1.1328
6.3431	70	.87172	1.0648	.10710	.86350	.06396	1.1471
6.2400	71	.86102	1.0668	.10857	.86699	.06515	1.1614
6.1553	72	.85065	1.0687	.11046	.86834	.06604	1.1755
6.0652	73	.84058	1.0708	.11225	.87081	.06709	1.1896
5.9773	74	.83082	1.0728	.11385	.87935	.06815	1.2036
5.8918	75	.82134	1.0749	.11563	.87590	.06921	1.2175
5.8084	76	.81213	1.0770	.11736	.87853	.07037	1.2313
5.7271	77	.80319	1.0792	.11910	.88120	.07136	1.2450
5.6478	78	.79449	1.0814	.12072	.88389	.07244	1.2586
5.5704	79	.78606	1.0836	.12281	.88677	.07352	1.2721
5.4949	80	.77786	1.0859	.12441	.88949	.07462	1.2855
5.4254	81	.76988	1.0882	.12660	.89161	.07512	1.2989
5.3492	82	.76212	1.0905	.12793	.89520	.07683	1.3121
5.2705	83	.75458	1.0920	.12958	.89958	.07819	1.3252
5.2101	84	.74724	1.0953	.13157	.90095	.07907	1.3383
5.1429	85	.74009	1.0977	.13330	.90420	.07960	1.3512
5.0772	86	.73314	1.1012	.13546	.90734	.08102	1.3639
5.0134	87	.72637	1.1027	.13704	.91036	.08440	1.3767
4.9501	88	.71978	1.1054	.13893	.91363	.00836	1.3893
4.8886	89	.71336	1.1079	.14078	.91696	.08450	1.4818
4.8216	90	.70710	1.1105	.14279	.92210	.08621	1.4142
4.7694	91	.70101	1.1132	.14449	.92352	.08716	1.4265
4.7117	92	.69508	1.1159	.14643	.92476	.08798	1.4387
4.6615	93	.68930	1.1186	.14817	.92914	.08932	1.4507
4.5999	94	.68366	1.1211	.15009	.93385	.09076	1.4627
4.5453	95	.67817	1.1242	.15211	.93746	.09197	1.4745
4.4845	96	.67282	1.1271	.15375	.94272	.09348	1.4863

Chord div. by height.	Centre Angle v .	Radius $r = k c$.	Cir. Arc. $b = k c$.	Area Seg. $a = k c^2$.	Surface $a = k c^2$.	Solidity $c = k c^3$.	Chord $c = k r$.
							
4.4398	97	.66760	1.1300	.15600	.94470	.09442	1.4979
4.3859	98	.66250	1.1329	.15801	.94852	.09567	1.5094
4.3383	99	.65754	1.1359	.15995	.95236	.09693	1.5208
4.2862	100	.65270	1.1382	.16180	.95682	.09831	1.5321
4.2406	101	.64798	1.1420	.16393	.96011	.09856	1.5432
4.1930	102	.64338	1.1451	.16610	.96412	.10076	1.5543
4.1570	103	.63889	1.1483	.16925	.96568	.10557	1.5652
4.1006	104	.63450	1.1515	.17001	.97246	.10273	1.5760
4.0555	105	.63023	1.1547	.17204	.97643	.10471	1.5867
4.0113	106	.62607	1.1580	.17414	.98067	.10601	1.5973
3.9679	107	.62200	1.1614	.17619	.98495	.10735	1.6077
3.9252	108	.61803	1.1648	.17832	.98931	.10870	1.6180
3.8832	109	.61416	1.1682	.18041	.99376	.11007	1.6282
3.8419	110	.61039	1.1716	.18257	.98827	.11149	1.6388
3.8013	111	.60670	1.1752	.18472	1.0028	.11284	1.6482
3.7612	112	.60325	1.1790	.18696	1.0077	.11426	1.6581
3.7221	113	.59960	1.1823	.18900	1.0122	.11566	1.6677
3.6837	114	.59618	1.1859	.19117	1.0169	.11709	1.6773
3.6454	115	.59284	1.1897	.19339	1.0218	.11853	1.6867
3.6086	116	.58959	1.1934	.19559	1.0266	.11995	1.6961
3.5712	117	.58641	1.1972	.19787	1.0317	.12145	1.7053
3.5349	118	.58331	1.2011	.20009	1.0368	.12294	1.7143
3.4992	119	.58030	1.2050	.20227	1.0417	.12444	1.7232
3.4641	120	.57735	1.2089	.20453	1.0472	.12596	1.7320
3.4296	121	.57450	1.2130	.20678	1.0525	.12748	1.7407
3.3953	122	.57168	1.2177	.20945	1.0578	.12903	1.7492
3.3616	123	.56895	1.2213	.21175	1.0634	.13060	1.7576
3.3285	124	.56628	1.2253	.21399	1.0690	.13218	1.7659
3.2940	125	.56370	1.2295	.21538	1.0753	.13391	1.7740
3.2637	126	.56116	1.2338	.21859	1.0803	.13558	1.7820
3.2319	127	.55870	1.2381	.22121	1.0862	.13701	1.7898
3.2006	128	.55630	1.2425	.22370	1.0921	.13866	1.7976
3.1716	129	.55396	1.2470	.22617	1.0974	.14028	1.8051
3.1393	130	.55169	1.2515	.22865	1.1040	.14202	1.8126
3.1093	131	.54947	1.2561	.23113	1.1104	.14371	1.8199
3.0805	132	.54732	1.2607	.23372	1.1164	.14537	1.8271
3.0555	133	.54522	1.2654	.23603	1.1212	.14676	1.8341
3.0216	134	.54318	1.2701	.23892	1.1295	.14894	1.8410
2.9777	135	.54120	1.2749	.24198	1.1420	.15209	1.8477
2.9651	136	.53927	1.2798	.24364	1.1428	.15252	1.8543
2.9374	137	.53740	1.2847	.24676	1.1495	.15422	1.8608
2.9115	138	.53557	1.2897	.24938	1.1558	.15605	1.8671
2.8829	139	.53380	1.2948	.25222	1.1634	.15807	1.8733
2.8562	140	.53209	1.2999	.25485	1.1705	.15996	1.8794
2.8299	141	.53042	1.3051	.25759	1.1777	.16201	1.8853
2.8038	142	.52881	1.3065	.25936	1.1851	.16381	1.8910
2.7781	143	.52724	1.3157	.26320	1.1925	.16577	1.8966
2.7527	144	.52573	1.3211	.26604	1.2000	.16776	1.9021

Chord div. by height.	Centre Angle v .	Radius $r = k c$.	Cir. Arc. $b = k c$.	Area Seg. $a = k c^2$.	Surface $\alpha = k c^2$.	Solidity $c = k c^3$.	Chord $c = k r$.
							
2.7276	145	.52426	1.3265	.26889	1.2077	.16965	1.9074
2.7002	146	.52284	1.3320	.27196	1.2166	.17209	1.9126
2.6816	147	.52147	1.3377	.27449	1.2219	.17205	1.9176
2.6533	148	.52015	1.3433	.27772	1.2318	.17605	1.9225
2.6301	149	.51887	1.3491	.28168	1.2396	.17809	1.9272
2.6064	150	.51764	1.3549	.28369	1.2476	.18023	1.9318
2.5830	151	.51645	1.3608	.28674	1.2563	.18666	1.9363
2.5598	152	.51530	1.3668	.28983	1.2648	.18751	1.9406
2.5239	153	.51420	1.3729	.29397	1.2801	.18845	1.9447
2.5143	154	.51315	1.3790	.29607	1.2824	.18913	1.9487
2.4919	155	.51214	1.3852	.29928	1.2914	.19147	1.9526
2.4699	156	.51117	1.3919	.30259	1.3004	.19374	1.9563
2.4478	157	.51014	1.3973	.30560	1.3094	.19607	1.9598
2.4262	158	.50936	1.4043	.30905	1.3191	.20029	1.9632
2.4047	159	.50851	1.4109	.31239	1.3287	.20095	1.9663
2.3835	160	.50771	1.4175	.31575	1.3368	.20342	1.9696
2.3613	161	.50695	1.4243	.31931	1.3490	.20609	1.9725
2.3417	162	.50623	1.4311	.32263	1.3583	.20847	1.9753
2.3211	163	.50555	1.4380	.32618	1.3682	.21105	1.9780
2.3004	164	.50491	1.4450	.32969	1.3791	.21371	1.9805
2.2805	165	.50431	1.4520	.33327	1.3895	.21634	1.9829
2.2605	166	.50374	1.4592	.33684	1.4021	.21904	1.9851
2.2408	167	.50323	1.4665	.34048	1.4111	.22177	1.9871
2.2212	168	.50275	1.4739	.34422	1.4222	.21946	1.9890
2.2013	169	.50231	1.4813	.34802	1.4344	.22766	1.9908
2.1826	170	.50191	1.4889	.35230	1.4476	.23028	1.9924
2.1636	171	.50154	1.4966	.35563	1.4565	.23266	1.9938
2.1447	172	.50122	1.5044	.35953	1.4684	.23650	1.9951
2.1271	173	.50093	1.5123	.36337	1.4797	.23900	1.9962
2.1075	174	.50068	1.5202	.36747	1.4927	.24225	1.9972
2.0892	175	.50047	1.5283	.37152	1.5052	.24537	1.9981
2.0710	176	.50030	1.5365	.37562	1.5179	.24856	1.9988
2.0530	177	.50017	1.5448	.37974	1.5308	.25179	1.9993
2.0352	178	.50007	1.5533	.38401	1.5439	.25531	1.9996
2.0175	179	.50002	1.5618	.38828	1.5573	.25840	1.9999
2.0000	180	.50000	1.5707	.39269	1.5708	.26179	2.0000

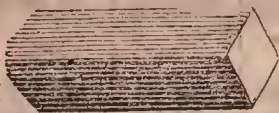
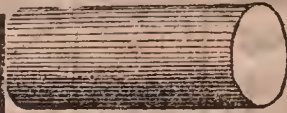

To find the length of an arc of a circle,

RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the fourth column multiplied by the chord is the length of the arc.

To find the area of a segment of a circle,

RULE. Divide the chord of the segment by its height, and find the quotient in the first column. The corresponding number in the fifth column multiplied by the square of the chord, is the area of the segment.

Coefficient for Capacity and Weight,

Names of Substances.								
	<i>FFE.</i>	<i>Fi.</i>	<i>iii.</i>	<i>FF₂</i>	<i>Fi₂</i>	<i>ii₂</i>	<i>F³</i>	<i>i³</i>
Cubic inches, - -	1728	12	1	1356	9.42	0.78	903.7	0.523
Cubic feet, - - -	1	.694	.58	0.785	.549	.44	0.523	.3
Gallons, - - - -	7.476	0.052	.433	5.868	.408	.34	3.91	.226
Water, fresh, - -	62.5	0.433	0.036	49	0.34	.283	32.7	0.019
Water, salt, - - -	64.3	0.445	0.037	50.4	0.35	0.029	33.6	0.02
Oil, - - - - -	57.5	0.4	0.033	45.1	0.313	0.026	30	0.017
Cast-iron, - - -	450	3.12	0.26	353	2.45	0.204	235	0.136
Wrought-iron, - -	487	3.37	0.281	382	2.65	0.221	255	0.147
Steel, - - - - -	490	3.4	0.283	385	2.67	0.222	257	0.149
Brass, - - - - -	532	3.68	0.307	417	2.9	0.241	278	0.161
Tin, - - - - -	456	3.16	0.263	358	2.48	0.207	239	0.138
Lead, - - - - -	710	4.92	0.41	557	3.87	0.322	371	0.215
Zinc, - - - - -	440	3.05	0.254	345	2.4	0.2	230	0.133
Copper, - - - - -	556	3.85	0.321	436	3.03	0.252	291	0.168
Mercury, - - - -	850	5.9	0.491	666	4.63	0.385	445	0.257
Stone, common, -	156	1.08	0.09	122	0.85	0.071	82	0.047
Clay, - - - - -	135	0.936	0.078	106	0.735	0.061	70	0.04
Earth, compact, -	127	0.88	0.0733	99	0.692	0.058	66	0.038
Earth, loose, - -	95	0.66	0.055	74	0.517	0.043	50	0.029
Oak, dry, - - - -	58	0.4	0.033	44	0.316	0.026	30	0.017
Pine, - - - - -	30	0.208	0.017	24	0.163	0.014	16	0.009
Mahogany, - - -	66	0.457	0.038	52	0.36	0.03	34	0.02
Coal, stone, - - -	54	0.375	0.031	42	0.294	0.024	28.2	0.016
Charcoal, - - - -	27.5	0.19	0.016	21	0.15	0.012	14.4	0.008

To Find the Weight and Capacity by this Table

RULE. The product of the dimensions in feet or in inches, as noted in the columns, multiplied by the tabular coefficient, is the capacity of the solid, or weight in pounds avoirdupois.

Example 1. A cistern is 6 feet long, 27 inches wide, and 20 inches deep. How many gallons of liquid can be contained in it?

$$6 \times 27 \times 20 \times 0.052 = 168.48 \text{ gallons.}$$

Example 2. A cast-iron cylinder is 4.5 feet long, and 7.5 inches diameter. Required the weight of it?

$$4.5 \times 7.5^2 \times 2.45 = 620 \text{ pounds.}$$

Angle. W	Ordinates.				Angle. W	Ordinates.			
	1. 7.	2. 6.	3. 5.	4. h.		1. 7.	2. 6.	3. 5.	4. h.
1°	·00084	·00164	·00193	·00218	53°	·05313	·08932	·11063	·11773
2	·00191	·00327	·00409	·00436	54	·05422	·09130	·11318	·12003
3	·00299	·00522	·00561	·00659	55	·05531	·09308	·11510	·12235
4	·00382	·00654	·00818	·00872	56	·05646	·09487	·11731	·12466
5	·00437	·00818	·01023	·01091	57	·05760	·09673	·11950	·12698
6	·00573	·00928	·01228	·01309	58	·05875	·09853	·12170	·12932
7	·00675	·01173	·01432	·01527	59	·05989	·10037	·12393	·13162
8	·00764	·01309	·01639	·01746	60	·06094	·10220	·12612	·13397
9	·00845	·01474	·01842	·01964	61	·06261	·10427	·12840	·13631
10	·00955	·01637	·02047	·02183	62	·06331	·10593	·13054	·13866
11	·01053	·01801	·02250	·02402	63	·06451	·10781	·13281	·14101
12	·01146	·01965	·02456	·02620	64	·06570	·10964	·13505	·14337
13	·01245	·02129	·02662	·02839	65	·06681	·11101	·13765	·14573
14	·01284	·02271	·02861	·03058	66	·06805	·11342	·13956	·14810
15	·01438	·02461	·03081	·03282	67	·06914	·11532	·14181	·15048
16	·01535	·02625	·03277	·03496	68	·07040	·11721	·14409	·15286
17	·01630	·02789	·03484	·03715	69	·07168	·11912	·14637	·15526
18	·01730	·02956	·03693	·03935	70	·07284	·12103	·14864	·15765
19	·01858	·03125	·03996	·04154	71	·07407	·12294	·15087	·16005
20	·01922	·03286	·04103	·04374	72	·07535	·12485	·15323	·16245
21	·02022	·03453	·04309	·04594	73	·07656	·12685	·15555	·16487
22	·02119	·03619	·04522	·04814	74	·07784	·12877	·15785	·16729
23	·02215	·03787	·04720	·05034	75	·07912	·13078	·16016	·16972
24	·02311	·03934	·04930	·05255	76	·08040	·13292	·16247	·17216
25	·02413	·04117	·05138	·05475	77	·08168	·13472	·16482	·17460
26	·02508	·04283	·05346	·05696	78	·08297	·13670	·16716	·17706
27	·02610	·04457	·05552	·05917	79	·08426	·13868	·16951	·17951
28	·02708	·04621	·05761	·06139	80	·08560	·14070	·17187	·18198
29	·02813	·04793	·05970	·06361	81	·08695	·14274	·17423	·18445
30	·02911	·04970	·06188	·06582	82	·08829	·14477	·17660	·18694
31	·03005	·05125	·06386	·06804	83	·08944	·14681	·17901	·18943
32	·03107	·05298	·06596	·07027	84	·09105	·14888	·18140	·19193
33	·03191	·05464	·06806	·07250	85	·09235	·15120	·18379	·19444
34	·03310	·05637	·07016	·07477	86	·09377	·15304	·18622	·19695
35	·03412	·05804	·07424	·07695	87	·09428	·15509	·18865	·19946
36	·03515	·05992	·07452	·07919	88	·09560	·15756	·19108	·20201
37	·03616	·06147	·07646	·08143	89	·09780	·15931	·19350	·20555
38	·03718	·06327	·07858	·08367	90	·09944	·16144	·19597	·20710
39	·03821	·06492	·08069	·08591	91	·10098	·16359	·19842	·20966
40	·03905	·06631	·08243	·08816	92	·10240	·16575	·20092	·21223
41	·04030	·06836	·08494	·09041	93	·10384	·16787	·20338	·21481
42	·04133	·07012	·08707	·09266	94	·10537	·17005	·20589	·21740
43	·04241	·07182	·08920	·09492	95	·10692	·17224	·20837	·22000
44	·04363	·07353	·09130	·09719	96	·10851	·17444	·21091	·22262
45	·04522	·07531	·09346	·09945	97	·10997	·17666	·21342	·22523
46	·04556	·07706	·09562	·10172	98	·11150	·17888	·21596	·22786
47	·04682	·07894	·09790	·10400	99	·11310	·18111	·22800	·23050
48	·04833	·08059	·09991	·10627	100	·11468	·18354	·22107	·23315
49	·04879	·08236	·00207	·10856	101	·11626	·18500	·22364	·23596
50	·04982	·08413	·00422	·11085	102	·11791	·18793	·22623	·23848
51	·05096	·08593	·10639	·11314	103	·11959	·19021	·22876	·24107
52	·05204	·08768	·10855	·11543	104	·12116	·19256	·23147	·24386

RAIL ROAD CURVES.

WHEN Railroads are to be connected by curves, we commonly have given the distance (chord c ,) between the two ends oo of the tracks, and the tangential angle v . By these the curve is to be constructed.

Example 1. Fig. 128. The chord $C = 168$ feet, and the tangential angle $v = 19^\circ 30'$. Required the centre angle $w =$, and the radius $R =$?

$$w = 2(19^\circ 30') = 39^\circ. \quad R = \frac{3}{2}c = 1.4979 \times 168 = 251.647 \text{ feet.}$$

$k =$ See Table for Segments, &c., of a circle.

By Tangential Angles.

The curve to be laid out by the three tangential angles ror , ron , and noo , each angle $= \frac{1}{3}v = 6^\circ 30'$. Required the chord $r =$?

The centre angle for the chord r is

$$2 \times (6^\circ 30') = 13^\circ, \text{ and } r = \frac{1}{2}k R = 0.2264 \times 251.647 = 56.974 \text{ feet.}$$

By Angles of Deflexion.

Divide the centre angle w into an even number of parts $= z$. Set off at o the angle $z = ron$, and bisect it into ror and ron ,—find the chord r , and sub-chord a , and continue as shown by Figure.

Example 2. Fig. 128. The tangential angle $v = 78^\circ$, and the chord $C = 638$ feet. Required the centre-angle $w =$? Radius $R =$? Chord $r =$? and the sub-chord $a =$?

$$w = 2 \times 78^\circ = 156^\circ. \quad R = \frac{1}{2}k c = 0.51117 \times 638 = 326.126 \text{ feet.}$$

Let the curve be laid out by 6 angles of deflexion, and $z = \frac{1}{6} \times 156^\circ = 26^\circ$, and

$$r = \frac{2}{6}k R = 0.4499 \times 326.126 = 146.73 \text{ feet.}$$

$$a = \frac{2}{6}k r = 0.4495 \times 146.73 = 66.012 \text{ feet.}$$

By Ordinates.

Example 3. Fig. 129. The chord $C = 368$ feet, and $v = 33^\circ$. Required the height $h =$?

$$h = \frac{1}{2}C(\operatorname{cosec} v - \cot v).$$

$$\text{From } - - - - - \operatorname{cosec} 36^\circ = 1.70130$$

$$\text{Subtract } - - - - - \cot 36^\circ = 1.37638$$

$$\text{The height } h = 0.32492 \times 184 = 59.785 \text{ feet.}$$

At $x = 92$ feet from h . Required the ordinate y ?

$$\sin z = \frac{2 \times 92 \sin 36^\circ}{368} = 0.2938926 = \sin 17^\circ 6'.$$

$$y = \frac{1}{2} \times 368 \left(\frac{\cos 17^\circ 6'}{\sin 36^\circ} - \cot 36^\circ \right) = 45.9448 \text{ feet.}$$

By Sub-Chords.

Example 4. Fig. 130. The ends o and o of the tracks form different angles w and W to the chord C , and therefore must be connected by two curves of different radii, R and r . The chord $C = 869$ feet, $w = 38^\circ$, and $W = 86^\circ$. Required the distance from o to the height h , $n =$? sub-chord $b =$? sub-chord $a =$? radii R and $r =$?

$$v = \frac{1}{2} \times 38^\circ = 19^\circ, \text{ and } V = \frac{1}{2} \times 86^\circ = 43^\circ.$$

$$n = \frac{869 \tan 19^\circ}{\tan 19^\circ + \tan 43^\circ} = 234.35 \text{ feet.}$$

$$\begin{array}{l} b = 234.35 \sec 43^\circ = 320.42 \text{ feet.} \\ a = \sec 19^\circ (869 - 234.35) = 671.21 \text{ ft.} \end{array} \quad \begin{array}{l} R = \frac{3}{2}ka = 1.5358 \times 671.21 = 1030.2 \text{ ft.} \\ r = \frac{3}{2}kb = 0.73314 \times 320.42 = 234.91 \text{ ft.} \end{array}$$

By Eight Ordinates.

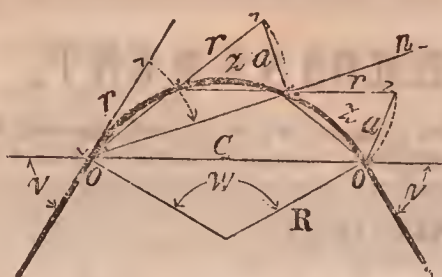
Example 5. Fig. 133. Required 8 ordinates for a curve of chord $C = 710$ feet and the centre angle $w = 69^\circ$? (See Table on the preceding page.)

$$\text{1st and 7th Ordinates } 0.07168 \times 710 = 50.8928 \text{ feet.}$$

$$\text{2nd " 6th " } 0.11912 \times 710 = 84.5752 \text{ "}$$

$$\text{3rd " 5th " } 0.14637 \times 710 = 103.9227 \text{ "}$$

$$\text{4th or height } h \quad 0.15526 \times 710 = 110.2346 \text{ "}$$

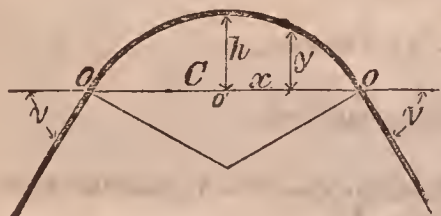


128.

By angles of deflexion.

$$w = 2v, \quad R = {}^w k C = \frac{1}{2} C \operatorname{cosec}.v.$$

$$r = {}^z k R, \quad a = {}^z k r = 2r \sin. \frac{1}{2} z.$$



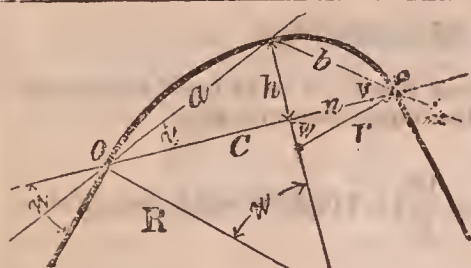
129.

By Ordinates.

$$h = \frac{1}{2} C (\operatorname{cosec}.v - \cot.v).$$

$$y = \frac{1}{2} C \left(\frac{\cos.z}{\sin.v} - \cot.v \right),$$

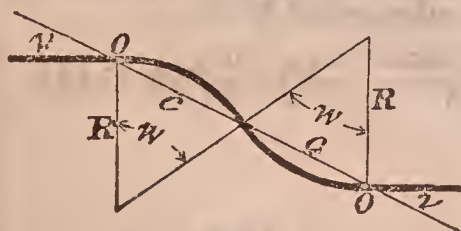
$$\sin.z = \frac{2x \sin.v}{C}.$$

130. *By Sub-chords.*

$$n = \frac{C \tan.v}{\tan.v + \tan.V}, \quad h = n \tan.V,$$

$$b = n \sec.V, \quad w = 2v, \quad W = 2V,$$

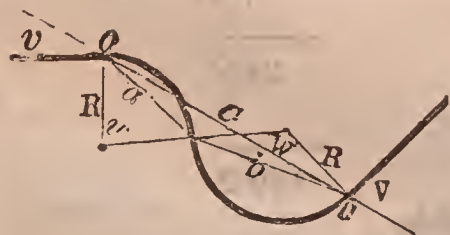
$$a = \sec.v (C - n),$$



131

Parallel tracks by a reverse curve.

Formulas same as above.

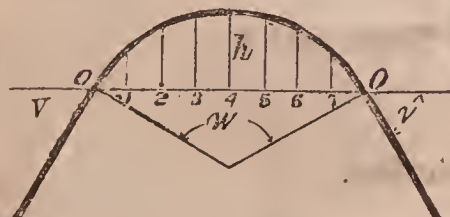
The length $oo = 2c$, length of a circle arc $l = 0.035v R$.

132.

The greatest radius in a reverse curve.

$$w = \frac{1}{2} (V + 3v), \quad W = w + V - v, \\ a = {}^w k R, \quad b = {}^v k R,$$

$$R = C \sec.w (\sin.V - \sqrt{\sin.2V - \cos.2w}).$$



133.

*Curve by 8 Ordinates.*The ordinates are calculated in the accompanying Table, the chord $C = 1$ or the unit.If the angle w is large, or there be some obstacle on the chord C , find the height h and lay out the curve by two or more sets of 8 ordinates.

EXCAVATION AND EMBANKMENT.

Example 1. The Road-way of an excavated channel is $r = 15$ feet, the depth $D = 9$ feet, and the breadth at the top $b = 46\frac{1}{2}$ feet. Require the slope $S = ?$

Formula 6.
$$S = \frac{46.5 - 15}{2 \times 9} = 1.75 \text{ or } 1\frac{3}{4} \text{ to } 1.$$

Example 2. The Road way is to be $r = 15$, $D = 18$, and the slope $S = 1\frac{1}{4}$. Require the breadth $b = ?$ and the cross-section $A = ?$

Formula 4.
$$b = 2 \times 18 \times 1.25 + 15 = 60 \text{ feet.}$$

Formula 7.
$$A = \frac{18}{2} (60 + 15) = 675 \text{ square feet.}$$

Example 3. The Road-way is to be $r = 16$ feet, the slope $S = 1\frac{1}{2}$, and the depth $D = 11$ feet. Required the area of Cross-section $A = ?$

Formula 9.
$$A = 11 (11 \times 1\frac{1}{2} + r) = 357.5 \text{ square feet.}$$

Example 4. The Road-way $r = 18$ feet, slope $S = 1\frac{1}{4}$, $d = 14$ feet 6 inches, and the length from o is $l = 55$ feet. Required the cubic contents $c = ?$

Formula 11.
$$c = 55 \times 14.5 \left(\frac{14.5 \times 1.25}{3} + \frac{18}{2} \right) = 11995.676 \text{ cubic feet, divided by } 27 = 444.28 \text{ cubic yards.}$$

Example 5. The Road-way is $r = 16$ feet, slope $S = 1\frac{1}{2}$ feet, $D = 17.5$, $d = 7.4$ and the length $L = 100$ feet. Required the cubic content $C = ?$

Formula 12.
$$C = 100 \left[1\frac{1}{2} \left(\frac{17.5^2 + 7.4^2 + 17.5 \times 7.4}{3} \right) + \frac{16}{2} (17.5 + 7.4) \right]$$

 $= 44445 \text{ cubic feet, or } 1645.4 \text{ cubic yards.}$

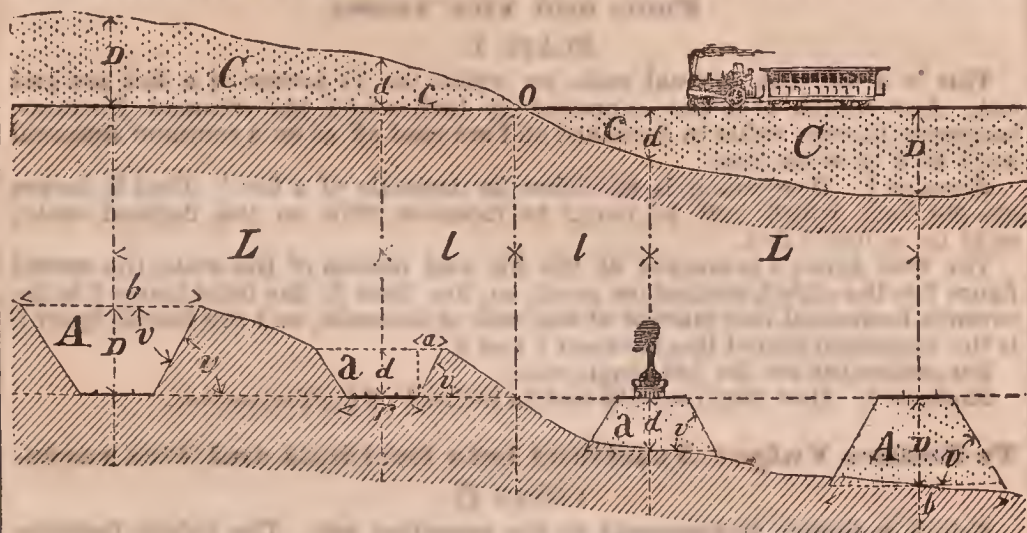
The computation is executed thus.

$\begin{array}{r} 17.5 \\ 7.4 \\ \hline 700 \\ 1225 \\ \hline 129.50 \end{array}$	$\begin{array}{r} 17.5 \\ 7.4 \\ \hline 24.9 \\ 8 \\ \hline 199.2 \end{array}$
$17.5^2 = 306.25$	
$7.4^2 = 54.76$	
$\left. \begin{array}{l} 306.25 \\ 54.76 \end{array} \right\} \text{ From table of Squares.}$	

$$\begin{array}{r} 3) 490.51 (163.5 \\ \quad 81.75 \\ \hline \quad 199.2 \end{array} \left. \vphantom{\begin{array}{r} 3) 490.51 \\ \quad 81.75 \\ \hline \quad 199.2 \end{array}} \right\} \text{ Slope, add } \frac{1}{2}$$

$$\times 100 = 44445. \text{ cubic feet.}$$

134



Letters in the Formulas correspond with the Figure.

$$S = \cot. v, \quad - \quad - \quad 1.$$

$$a = D S, \quad - \quad - \quad 2.$$

$$a = D \cot. v, \quad - \quad - \quad 3.$$

$$b = 2 D S + r, \quad - \quad - \quad 4.$$

$$S = \frac{a}{D}, \quad - \quad - \quad 5.$$

$$S = \frac{b - r}{2 D}, \quad - \quad - \quad 6.$$

$$A = \frac{D}{2} (b + r), \quad - \quad 7.$$

$$a = \frac{d}{2} (b + r), \quad - \quad 8.$$

$$A = D (D S + r), \quad 9.$$

$$a = d (d S + r), \quad - \quad 10.$$

$$c = l d \left(\frac{d S}{3} + \frac{r}{2} \right), \quad 11.$$

$$C = L \left[S \left(\frac{D^2 + d^2 + Dd}{3} \right) + \frac{r}{2} (D + d) \right]. \quad - \quad 12.$$

Letters Denote,

A and **a** = Cross-Sections in square feet, of the excavated channel or embankment.

D and **d** = depth in feet, of the Sections.

r = width in feet of the Road-Way.

b = Base in feet of the embankment, or top breadth of the channel.

L = length in feet, between the two Sections **A** and **a**.

l = length in feet, from the Section **a** to the point **o** where the ground is level with the road.

C = cubic contents in feet, between **A** and **a**.

c = cubic contents in feet, between **a** and **o**.

S = slope of the sides. The slope is commonly given in proportions, thus: "Slope = $1\frac{1}{2}$ to 1," which means, that the side slopes $1\frac{1}{2}$ feet horizontally for 1 foot vertical.

v = angle of the slope.

To Reduce Inches and Fractions thereof to Decimals of a Foot, and vice versa.

PLATE I.

This is a common decimal scale on which the 12 inches of a foot are laid out. Any length of a foot expressed by inches and the common fractions, intersects its own value in decimals, and are read off as on a common diagonal scale with 10 to the base.

Example 1. How much is $8\frac{1}{4}$ inches in decimals of a foot? Find $8\frac{1}{4}$ inches on the rule, which will be found to intersect 6875 on the decimal scale; or $8\frac{1}{4}$ in. = 0.6875 feet.

The first figure 6 is marked at the top and bottom of the scale, the second figure 8 is the eighth vertical (or nearly so) line from 6; the third figure 7 is the seventh horizontal line marked at the ends of the scale, and the fourth figure 5 is the horizontal dotted line between 7 and 8.

For sixteenths see the fine single rule.

Example 2. How much is 0.526 feet in inches? $6\frac{5}{16}$ the answer.

To Reduce Vulgar Fractions into Decimals and vice versa.

PLATE II.

This is a similar arrangement to the preceding one. The vulgar fractions are laid out so that the denominators are marked at the ends of the scale and the nominators on the line that joins the given denominators; by this arrangement the nominator of the vulgar fraction intersects its own value in decimals on the scale. To facilitate the operations it is best to imagine in which quarter the vulgar fraction is; as $\frac{3}{16}$ is in the first, $\frac{3}{8}$ in the second; $\frac{5}{8}$ in the third, and $\frac{11}{16}$ in the fourth quarter, &c., &c.; each rule occupies a quarter of the scale, on which the vulgar fraction is to be found accordingly.

Example 1. How much is $\frac{9}{16}$ in decimals?

On the third rule it will be found at 5625 of decimals, or $\frac{9}{16} = 0.5625$. The first and second figures 56 are marked as described for Plate I, and the third, fourth, &c., &c., figures are written down on the horizontal line of the given fraction.

Example 2. What nearest vulgar fraction answers to the decimals 0.39583?

$\frac{19}{48}$, the answer.

These two diagrams are exceedingly useful in practice. By Plate II, Vulgar Fractions can be added and subtracted.

TRIGONOMETRY.

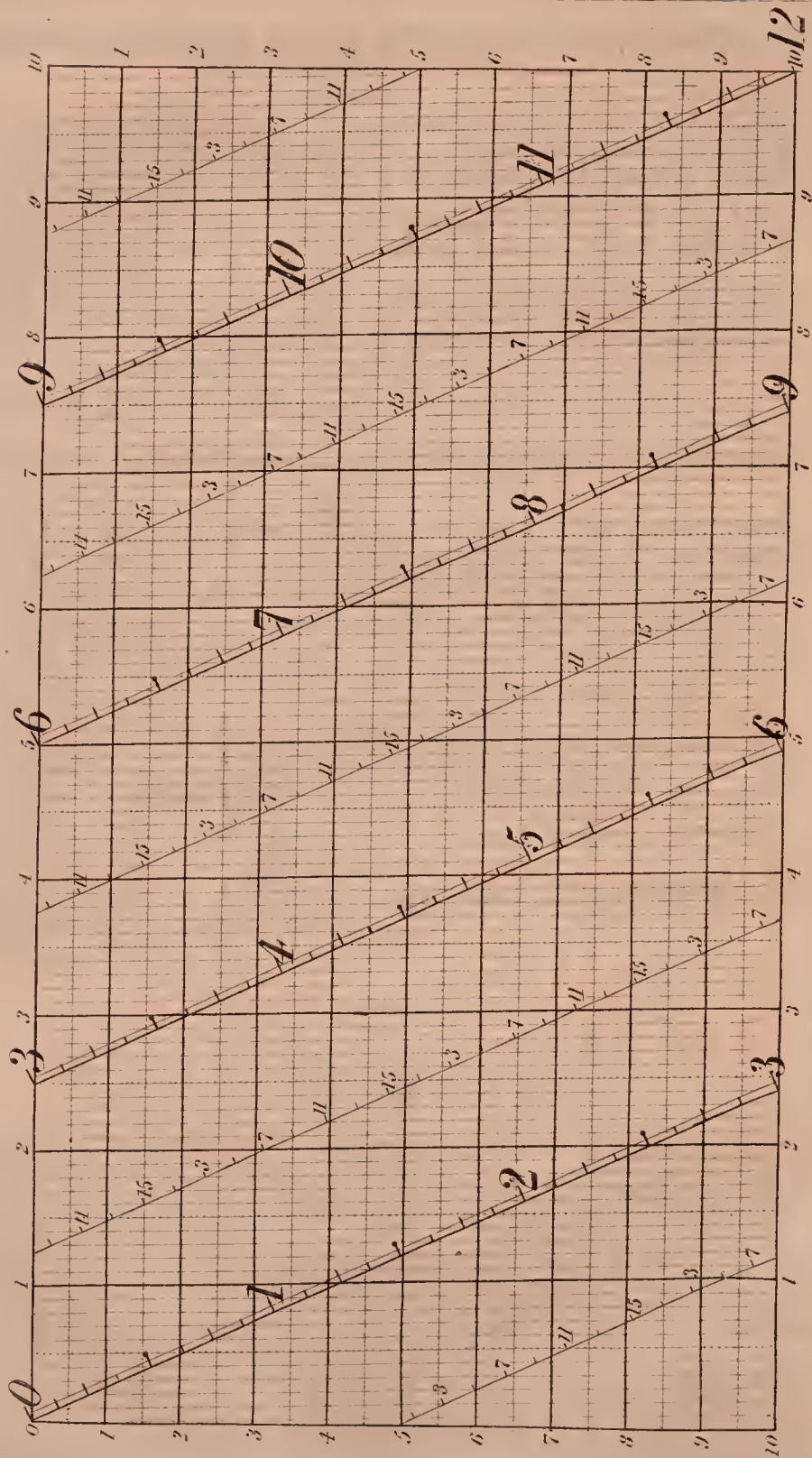
TRIGONOMETRY is that part of Geometry which treats of Triangles. It is divided into two parts, viz.: *plane* and *spherical*.

Plane Trigonometry treats of triangles which are drawn (or imagined to be) on a plane. Spherical Trigonometry treats of the triangles which are drawn (or imagined to be) on a sphere.

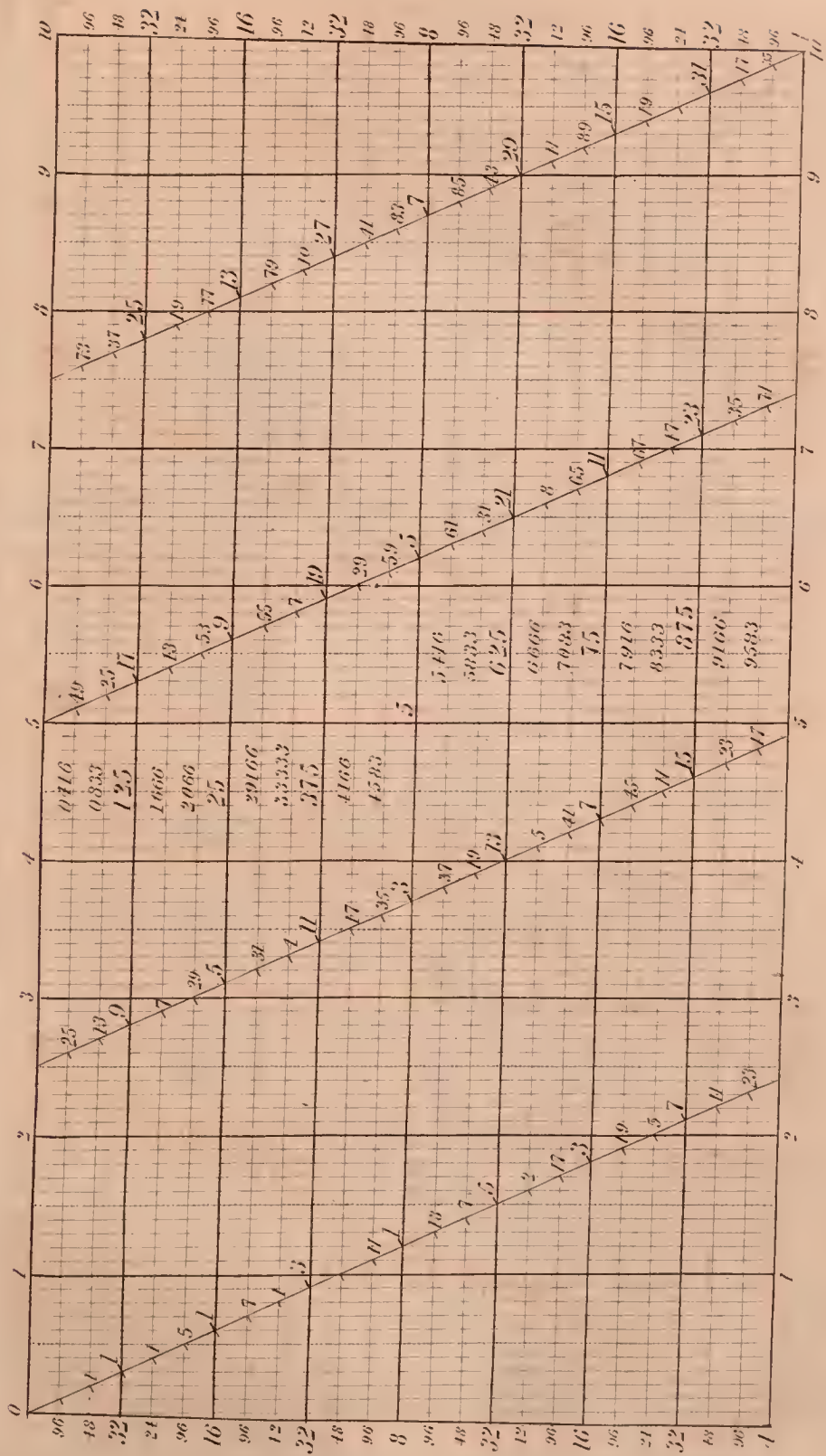
A triangle contains seven quantities, namely, three sides, three angles, and the surface; when any three of these quantities are given, the four remaining ones can by them be ascertained, (one side or the area must be one of the given quantities) and the operation is called *solving the triangle*, which is only an application of arithmetic on Geometrical objects.

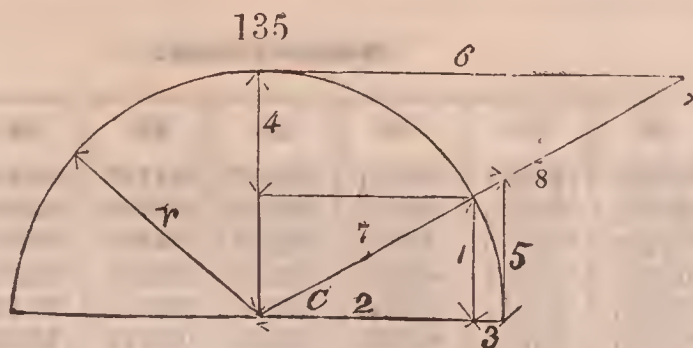
For the foundation of the above mentioned solution, there are assumed eight help quantities which are called *Trigonometrical functions*, and are here denoted with their names and number, corresponding with Figure 1? In the accompanying Tables, the functions are calculated at every 10 minutes per degree in the quadrant of the circle represented by Fig. 1? The angle for which the functions are mentioned, is the opening between the two lines 7 and 2, 3, this angle is denoted by the letter *C*, and the expression *sin.C* means the line 1 compared with the radius *r* as a unit.

To reduce inches to feet.



To reduce vulgar fractions to decimals.





1 Sinus	abbreviated	$\sin.C.$
2 Cosinus	"	$\cos.C.$
3 Sinus-versus	"	$\sin v.C.$
4 Cosinus-versus	"	$\cos v.C.$
5 Tangent	"	$\tan.C.$
6 Cotangent	"	$\cot.C.$
7 Secant	"	$\sec.C.$
8 Cosecant	"	$\csc.C.$

r = Radius of the circle, which is the unit by which the functions are measured.

$$r^2 = \sin.^2 C + \cos.^2 C.$$

$$\tan.C = \frac{\sin.C}{\cos.C},$$

$$\tan.C = \frac{1}{\cot.C},$$

$$\cot.C = \frac{\cos.C}{\sin.C},$$

$$\cot.C = \frac{1}{\tan.C},$$

$$\sec.C = \frac{1}{\cos.C},$$

$$\csc.C = \frac{1}{\sin.C},$$

$$\sin v.C = 1 - \cos.C,$$

$$\cos v.C = 1 - \sin.C,$$

$$\sin.2C = 2 \sin.C \cos.C,$$

$$\sin.\frac{1}{2}C = \frac{1}{2}\sqrt{(\sin.^2 C + \sin v.^2 C)},$$

$$\sin.(C \pm B) = \sin.C \cos.B \pm \sin.B \cos.C.$$

Positive and Negative Signs.

Angles.	sin.	cos.	sin v.	co	tan.	cot.	sec.	cosec.
+0°	+0	+1	+0	+1	+0	+∞	+1	+∞
+90°	+1	+0	+1	+0	+∞	+0	+∞	+1
+180°	+0	-1	+2	+1	+0	+∞	-1	+∞
+270°	-1	+0	+1	+2	+∞	+0	+∞	-1
+360°	+0	+1	+0	+1	+0	-∞	+1	-∞

When a quantity has reached 0 or ∞, it has ceased to exist, because it can not be increased or diminished.

Example. What is the length of the secant for an angle of 74° 18'?

$$\text{Secant } C = \frac{1}{\cos. 74^\circ 18'} = 3.695.$$

Natural Sine.

Deg.	0'	10'	20'	30'	40'	50'	60'	Deg.
0	·00000	·00291	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02326	·02617	·02908	·03199	·03489	88
2	·03489	·03780	·04071	·04361	·04652	·04943	·05233	87
3	·05233	·05524	·05814	·06104	·06395	·06685	·06975	86
4	·06975	·07265	·07555	·07845	·08135	·08425	·08715	85
5	·08715	·09005	·09294	·09584	·09874	·10163	·10452	84
6	·10452	·10742	·11031	·11320	·11609	·11898	·12186	83
7	·12186	·12475	·12764	·13052	·13340	·13629	·13917	82
8	·13917	·14205	·14493	·14780	·15068	·15356	·15643	81
9	·15643	·15930	·16217	·16504	·16791	·17078	·17364	80
10	·17364	·17651	·17937	·18223	·18509	·18795	·19080	79
11	·19080	·19366	·19651	·19936	·20221	·20506	·20791	78
12	·20791	·21075	·21359	·21643	·21927	·22211	·22495	77
13	·22495	·22778	·23061	·23344	·23627	·23909	·24192	76
14	·24192	·24474	·24756	·25038	·25319	·25600	·25881	75
15	·25881	·26162	·26443	·26723	·27004	·27284	·27563	74
16	·27563	·27843	·28122	·28401	·28680	·28958	·29237	73
17	·29237	·29515	·29793	·30070	·30347	·30624	·30901	72
18	·30901	·31178	·31454	·31730	·32006	·32281	·32556	71
19	·32556	·32831	·33106	·33380	·33654	·33928	·34202	70
20	·34202	·34475	·34748	·35020	·35293	·35565	·35836	69
21	·35836	·36108	·36379	·36650	·36920	·37190	·37460	68
22	·37460	·37730	·37999	·38268	·38536	·38805	·39073	67
23	·39073	·39340	·39607	·39874	·40141	·40407	·40673	66
24	·40673	·40939	·41204	·41469	·41733	·41998	·42261	65
25	·42261	·42525	·42788	·43051	·43313	·43575	·43837	64
26	·43837	·44098	·44359	·44619	·44879	·45139	·45399	63
27	·45399	·45658	·45916	·46174	·46432	·46690	·46947	62
28	·46947	·47203	·47460	·47715	·47971	·48226	·48480	61
29	·48480	·48735	·48988	·49242	·49495	·49747	·50000	60
30	·50000	·50251	·50502	·50753	·51004	·51254	·51503	59
31	·51503	·51752	·52001	·52249	·52497	·52745	·52991	58
32	·52991	·53238	·53484	·53729	·53975	·54219	·54463	57
33	·54463	·54707	·54950	·55193	·55436	·55677	·55919	56
34	·55919	·56160	·56400	·56640	·56880	·57119	·57357	55
35	·57357	·57595	·57833	·58070	·58306	·58542	·58778	54
36	·58778	·59013	·59248	·59482	·59715	·59948	·60181	53
37	·60181	·60413	·60645	·60876	·61106	·61336	·61566	52
38	·61566	·61795	·62023	·62251	·62478	·62705	·62932	51
39	·62932	·63157	·63383	·63607	·63832	·64055	·64278	50
40	·64278	·64501	·64723	·64944	·65165	·65386	·65605	49
41	·65605	·65825	·66043	·66262	·66479	·66696	·66913	48
42	·66913	·67128	·67344	·67559	·67773	·67986	·68199	47
43	·68199	·68412	·68624	·68835	·69046	·69256	·69465	46
44	·69465	·69674	·69883	·70090	·70298	·70504	·70710	45
	60'	50'	40'	30'	20'	10'	0'	Deg.

Natural Cosine.

Natural Sine.

De.	0'	10'	20'	30'	40'	50'	60'	
45	·70710	·70916	·71120	·71325	·71528	·71731	·71933	44
46	·71933	·72135	·72336	·72537	·72737	·72936	·73135	43
47	·73135	·73333	·73530	·73727	·73923	·74119	·74314	42
48	·74314	·74508	·74702	·74895	·75088	·75279	·75470	41
49	·75470	·75661	·75851	·76040	·76229	·76417	·76604	40
50	·76604	·76791	·76977	·77162	·77347	·77531	·77714	39
51	·77714	·77897	·78079	·78260	·78441	·78621	·78801	38
52	·78801	·78979	·79157	·79335	·79512	·79688	·79863	37
53	·79863	·80038	·80212	·80385	·80558	·80730	·80901	36
54	·80901	·81072	·81242	·81411	·81580	·81748	·81915	35
55	·81915	·82081	·82247	·82412	·82577	·82740	·82903	34
56	·82903	·83066	·83227	·83388	·83548	·83708	·83867	33
57	·83867	·84025	·84182	·84339	·84495	·84650	·84804	32
58	·84804	·84958	·85111	·85261	·85415	·85566	·85716	31
59	·85716	·85866	·86014	·86162	·86310	·86456	·86602	30
60	·86602	·86747	·86891	·87035	·87178	·87320	·87461	29
61	·87461	·87602	·87742	·87881	·88020	·88157	·88294	28
62	·88294	·88430	·88566	·88701	·88835	·88968	·89100	27
63	·89100	·89232	·89363	·89493	·89622	·89751	·89879	26
64	·89879	·90006	·90132	·90258	·90383	·90507	·90630	25
65	·90630	·90753	·90875	·90996	·91116	·91235	·91354	24
66	·91354	·91472	·91589	·91706	·91811	·91936	·92050	23
67	·92050	·92163	·92276	·92387	·92498	·92609	·92718	22
68	·92718	·92826	·92934	·93041	·93147	·93253	·93358	21
69	·93358	·93461	·93564	·93667	·93768	·93869	·93969	20
70	·93969	·94068	·94166	·94264	·94360	·94456	·94551	19
71	·94551	·94646	·94739	·94832	·94924	·95015	·95105	18
72	·95105	·95195	·95283	·95371	·95458	·95545	·95630	17
73	·95630	·95715	·95798	·95881	·95964	·96045	·96126	16
74	·96126	·96205	·96284	·96363	·96440	·96516	·96592	15
75	·96592	·96667	·96741	·96814	·96887	·96958	·97029	14
76	·97029	·97099	·97168	·97236	·97304	·97371	·97437	13
77	·97437	·97402	·97566	·97629	·97692	·97753	·97814	12
78	·97814	·97874	·97934	·97992	·98050	·98106	·98162	11
79	·98162	·98217	·98272	·98325	·98378	·98429	·98480	10
80	·98480	·98530	·98580	·98628	·98676	·98722	·98768	9
81	·98768	·98813	·98858	·98901	·98944	·98985	·99026	8
82	·99026	·99066	·99106	·99144	·99182	·99218	·99254	7
83	·99254	·99289	·99323	·99357	·99389	·99421	·99452	6
84	·99452	·99482	·99511	·99539	·99567	·99593	·99619	5
85	·99619	·99644	·99668	·99691	·99714	·99735	·99756	4
86	·99756	·99776	·99795	·99813	·99830	·99847	·99862	3
87	·99862	·99877	·99891	·99904	·99917	·99928	·99939	2
88	·99939	·99948	·99957	·99965	·99972	·99979	·99984	1
89	·99984	·99989	·99993	·99996	·99998	·99999	1·0000	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

Natural Cosine.

Natural Tangent.

Deg.	0'	10'	20'	30'	40'	50'	60'	
0	·00000	·00290	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02327	·02618	·02909	·03200	·03492	88
2	·03492	·03783	·04074	·04366	·04657	·04949	·05240	87
3	·05240	·05532	·05824	·06116	·06408	·06700	·06992	86
4	·06992	·07285	·07577	·07870	·08162	·08455	·08748	85
5	·08748	·09042	·09335	·09628	·09922	·10216	·10510	84
6	·10510	·10804	·11098	·11393	·11688	·11983	·12278	83
7	·12278	·12573	·12869	·13165	·13461	·13757	·14054	82
8	·14054	·14350	·14647	·14945	·15242	·15540	·15838	81
9	·15838	·16136	·16435	·16734	·17033	·17332	·17632	80
10	·17632	·17932	·18233	·18533	·18834	·19136	·19438	79
11	·19438	·19740	·20042	·20345	·20648	·20951	·21255	78
12	·21255	·21559	·21864	·22169	·22474	·22780	·23086	77
13	·23086	·23393	·23700	·24207	·24315	·24624	·24932	76
14	·24932	·25242	·25551	·25861	·26172	·26483	·26794	75
15	·26794	·27106	·27419	·27732	·28045	·28359	·28674	74
16	·28674	·28989	·29305	·29621	·29938	·30255	·30573	73
17	·30573	·30891	·31210	·31529	·31849	·32170	·32491	72
18	·32491	·32813	·33136	·33459	·33783	·34107	·34432	71
19	·34432	·34758	·35084	·35411	·35739	·36067	·36397	70
20	·36397	·36726	·37057	·37388	·37720	·38053	·38386	69
21	·38386	·38720	·39055	·39391	·39727	·40064	·40402	68
22	·40402	·40741	·41080	·41421	·41762	·42104	·42447	67
23	·42447	·42791	·43135	·43481	·43827	·44174	·44522	66
24	·44522	·44871	·45221	·45572	·45924	·46277	·46630	65
25	·46630	·46985	·47340	·47697	·48055	·48413	·48773	64
26	·48773	·49133	·49495	·49858	·50221	·50586	·50952	63
27	·50952	·51319	·51687	·52056	·52426	·52798	·53170	62
28	·53170	·53544	·53919	·54295	·54672	·55051	·55430	61
29	·55430	·55811	·56193	·56577	·56961	·57347	·57735	60
30	·57735	·58123	·58513	·58904	·59296	·59690	·60086	59
31	·60086	·60482	·60880	·61280	·61680	·62083	·62486	58
32	·62486	·62892	·63298	·63707	·64116	·64527	·64940	57
33	·64940	·65355	·65771	·66188	·66607	·67028	·67450	56
34	·67450	·67874	·68300	·68728	·69157	·69588	·70020	55
35	·70020	·70455	·70891	·71329	·71769	·72210	·72654	54
36	·72654	·73099	·73546	·73996	·74447	·74900	·75355	53
37	·75355	·75812	·76271	·76732	·77195	·77661	·78128	52
38	·78128	·78598	·79069	·79543	·80019	·80497	·80978	51
39	·80978	·81461	·81946	·82433	·82923	·83415	·83909	50
40	·83909	·84406	·84906	·85408	·85912	·86414	·86928	49
41	·86928	·87440	·87955	·88472	·88992	·89515	·90040	48
42	·90040	·90568	·91099	·91633	·92169	·92704	·93251	47
43	·93251	·93796	·94345	·94896	·95450	·96008	·96568	46
44	·96568	·97532	·97699	·98269	·98843	·99419	1·0000	45
	60'	50'	40'	30'	20'	10'	0'	Deg.

Natural Cotangent.

Natural Tangent.

Deg.	0'	10'	20'	30'	40'	50'	60'	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2130	1.2203	1.2275	1.2348	39
51	1.2348	1.2422	1.2496	1.2571	1.2647	1.2722	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3679	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4732	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6752	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8939	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1608	2.1774	2.1942	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8769	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1652	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6011	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.760	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.074	19.081	3
87	19.081	20.205	21.470	22.003	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.289	1
89	57.289	68.750	85.939	114.58	171.88	343.77	∞	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

Natural Cotangent.

Natural Secant.

Deg.	0'	10'	20'	30'	40'	50'	60'	
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0001	89
1	1.0001	1.0002	1.0002	1.0003	1.0004	1.0005	1.0006	88
2	1.0006	1.0007	1.0008	1.0009	1.0010	1.0012	1.0013	87
3	1.0013	1.0015	1.0016	1.0018	1.0020	1.0022	1.0024	86
4	1.0024	1.0026	1.0028	1.0031	1.0033	1.0035	1.0038	85
5	1.0038	1.0040	1.0043	1.0046	1.0049	1.0052	1.0055	84
6	1.0055	1.0058	1.0061	1.0064	1.0068	1.0071	1.0075	83
7	1.0075	1.0078	1.0082	1.0086	1.0090	1.0094	1.0098	82
8	1.0098	1.0102	1.0106	1.0111	1.0115	1.0120	1.0124	81
9	1.0124	1.0129	1.0134	1.0139	1.0144	1.0149	1.0154	80
10	1.0154	1.0159	1.0164	1.0170	1.0175	1.0181	1.0187	79
11	1.0187	1.0192	1.0198	1.0204	1.0210	1.0217	1.0223	78
12	1.0223	1.0229	1.0236	1.0242	1.0249	1.0256	1.0263	77
13	1.0263	1.0269	1.0277	1.0284	1.0291	1.0298	1.0306	76
14	1.0306	1.0313	1.0321	1.0329	1.0336	1.0344	1.0352	75
15	1.0352	1.0360	1.0369	1.0377	1.0385	1.0394	1.0403	74
16	1.0403	1.0411	1.0420	1.0429	1.0438	1.0447	1.0456	73
17	1.0456	1.0466	1.0475	1.0485	1.0494	1.0504	1.0514	72
18	1.0514	1.0524	1.0534	1.0544	1.0555	1.0565	1.0576	71
19	1.0576	1.0586	1.0597	1.0608	1.0619	1.0630	1.0641	70
20	1.0641	1.0653	1.0664	1.0676	1.0687	1.0699	1.0711	69
21	1.0711	1.0723	1.0735	1.0747	1.0760	1.0772	1.0785	68
22	1.0785	1.0798	1.0810	1.0823	1.0837	1.0850	1.0863	67
23	1.0863	1.0877	1.0890	1.0904	1.0918	1.0932	1.0946	66
24	1.0946	1.0960	1.0974	1.0989	1.1004	1.1018	1.1033	65
25	1.1033	1.1048	1.1063	1.1079	1.1094	1.1110	1.1126	64
26	1.1126	1.1141	1.1157	1.1174	1.1190	1.1206	1.1223	63
27	1.1223	1.1239	1.1256	1.1273	1.1290	1.1308	1.1325	62
28	1.1325	1.1343	1.1361	1.1378	1.1396	1.1415	1.1433	61
29	1.1433	1.1452	1.1470	1.1489	1.1508	1.1527	1.1547	60
30	1.1547	1.1566	1.1586	1.1605	1.1625	1.1646	1.1666	59
31	1.1666	1.1686	1.1707	1.1728	1.1749	1.1770	1.1791	58
32	1.1791	1.1833	1.1835	1.1856	1.1878	1.1901	1.1923	57
33	1.1923	1.1946	1.1969	1.1992	1.2015	1.2038	1.2062	56
34	1.2062	1.2085	1.2109	1.2134	1.2158	1.2182	1.2207	55
35	1.2207	1.2232	1.2257	1.2283	1.2308	1.2334	1.2360	54
36	1.2360	1.2386	1.2413	1.2440	1.2466	1.2494	1.2521	53
37	1.2521	1.2548	1.2576	1.2604	1.2632	1.2661	1.2690	52
38	1.2690	1.2719	1.2748	1.2777	1.2807	1.2837	1.2867	51
39	1.2867	1.2898	1.2928	1.2959	1.2990	1.3022	1.3054	50
40	1.3054	1.3086	1.3118	1.3150	1.3183	1.3216	1.3250	49
41	1.3250	1.3283	1.3317	1.3351	1.3386	1.3421	1.3456	48
42	1.3456	1.3491	1.3527	1.3563	1.3599	1.3636	1.3673	47
43	1.3673	1.3710	1.3748	1.3785	1.3824	1.3862	1.3901	46
44	1.3901	1.3940	1.3980	1.4020	1.4060	1.4101	1.4142	45
	60'	50'	40'	30'	20'	10'	0'	Deg.

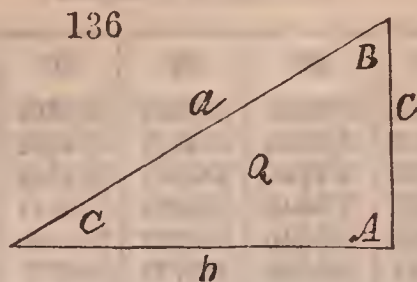
Natural Cosecant.

Natural Secant.

Deg.	0'	10'	20'	30'	40'	50'	60'	
45	1.4142	1.4183	1.4225	1.4267	1.4309	1.4352	1.4395	44
46	1.4395	1.4439	1.4483	1.4527	1.4572	1.4617	1.4662	43
47	1.4662	1.4708	1.4755	1.4801	1.4849	1.4896	1.4944	42
48	1.4944	1.4993	1.5042	1.5091	1.5141	1.5191	1.5242	41
49	1.5242	1.5293	1.5345	1.5397	1.5450	1.5503	1.5557	40
50	1.5557	1.5611	1.5666	1.5721	1.5777	1.5833	1.5890	39
51	1.5890	1.5947	1.6005	1.6063	1.6122	1.6182	1.6242	38
52	1.6242	1.6303	1.6364	1.6426	1.6489	1.6552	1.6616	37
53	1.6616	1.6680	1.6745	1.6811	1.6878	1.6945	1.7013	36
54	1.7013	1.7081	1.7150	1.7220	1.7291	1.7362	1.7434	35
55	1.7434	1.7507	1.7580	1.7655	1.7730	1.7806	1.7882	34
56	1.7882	1.7960	1.8038	1.8118	1.8198	1.8278	1.8360	33
57	1.8360	1.8443	1.8527	1.8611	1.8697	1.8783	1.8870	32
58	1.8870	1.8959	1.9048	1.9138	1.9230	1.9322	1.9416	31
59	1.9416	1.9510	1.9606	1.9702	1.9800	1.9899	2.0000	30
60	2.0000	2.0101	2.0203	2.0307	2.0412	2.0519	2.0626	29
61	2.0626	2.0735	2.0845	2.0957	2.1070	2.1184	2.1300	28
62	2.1300	2.1417	2.1536	2.1656	2.1778	2.1901	2.2026	27
63	2.2026	2.2153	2.2281	2.2411	2.2543	2.2676	2.2811	26
64	2.2811	2.2948	2.3087	2.3228	2.3370	2.3515	2.3662	25
65	2.3662	2.3810	2.3961	2.4114	2.4269	2.4426	2.4585	24
66	2.4585	2.4747	2.4911	2.5078	2.5247	2.5418	2.5593	23
67	2.5593	2.5769	2.5949	2.6131	2.6316	2.6503	2.6694	22
68	2.6694	2.6888	2.7085	2.7285	2.7488	2.7694	2.7904	21
69	2.7904	2.8117	2.8334	2.8554	2.8778	2.9006	2.9238	20
70	2.9238	2.9473	2.9713	2.9957	3.0205	3.0458	3.0715	19
71	3.0715	3.0977	3.1243	3.1515	3.1791	3.2073	3.2360	18
72	3.2360	3.2653	3.2951	3.3255	3.3564	3.3880	3.4203	17
73	3.4203	3.4531	3.4867	3.5209	3.5558	3.5915	3.6279	16
74	3.6279	3.6651	3.7031	3.7419	3.7816	3.8222	3.8637	15
75	3.8637	3.9061	3.9495	3.9939	4.0393	4.0859	4.1335	14
76	4.1335	4.1823	4.2323	4.2836	4.3362	4.3900	4.4454	13
77	4.4454	4.5021	4.5604	4.6202	4.6816	4.7448	4.8097	12
78	4.8097	4.8764	4.9451	5.0158	5.0886	5.1635	5.2408	11
79	5.2408	5.3204	5.4026	5.4874	5.5749	5.6653	5.7587	10
80	5.7587	5.8553	5.9553	6.0588	6.1660	6.2771	6.3924	9
81	6.3924	6.5120	6.6363	6.7654	6.8997	7.0396	7.1852	8
82	7.1852	7.3371	7.4957	7.6612	7.8344	8.0156	8.2055	7
83	8.2055	8.4046	8.6137	8.8336	9.0651	9.3091	9.5667	6
84	9.5667	9.8391	10.127	10.437	10.758	11.104	11.473	5
85	11.473	11.868	12.291	12.745	13.234	13.763	14.335	4
86	14.335	14.957	15.636	16.380	17.198	18.102	19.107	3
87	19.107	20.230	21.493	22.925	24.562	26.450	28.653	2
88	28.653	31.257	34.382	38.201	42.975	49.114	57.298	1
89	57.298	68.757	85.945	114.59	171.88	343.77	∞	0
	60'	50'	40'	30'	20'	10'	0'	Deg.

Natural Cosecant.

FORMULA FOR RIGHT-ANGLED TRIANGLES.



$a = \sqrt{b^2 + c^2},$	1,	$Q = \frac{a^2 \sin. 2C}{4},$	10,
$a = \frac{c}{\sin. C},$	2,	$Q = \frac{1}{2} b^2 \tan. C,$	11,
$a = \frac{b}{\cos. C},$	3,	$Q = \frac{1}{2} c^2 \cot. C,$	12,
$a = 2\sqrt{\frac{Q}{\sin. 2C}},$	4,	$Q = \frac{1}{2} c \sqrt{(a+c)(a-c)}$	13,
$b = a \cos. C,$	5,	$\sin C = \frac{c}{a},$	14,
$b = c \cot. C,$	6,	$\cos. C = \frac{b}{a},$	15,
$b = a \sin. B,$	7,	$\tan. C = \frac{c}{b},$	16,
$b = c \tan. B,$	8,	$\sin. 2C = \frac{4Q}{a^2},$	17,
$b = \sqrt{\frac{2Q}{\tan. C}},$	9,	$\tan. C = \frac{2Q}{b^2},$	18,

Say the angle to be $C = 60^\circ$. In the first column of the table of *sines*, 60° corresponds with 0.86602 in the next column, which is the length of $\sin. 60^\circ$, when the radius of the circle is one, or the unit, and the expression $\sin. 60^\circ \times 36$ means $0.86602 \times 36 = 31.17672$, and likewise with all the other Trigonometrical expressions.

In a triangle the functions for an angle have a certain relation to the opposite side; it is this relationship which enables us to solve the triangle by the application of Simple Arithmetic.

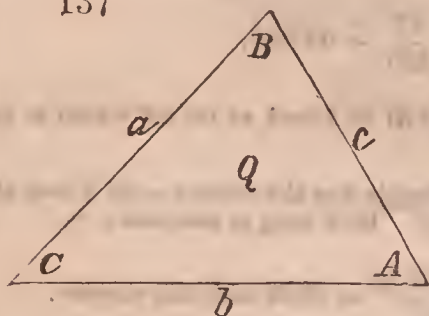
In triangles the sides are denoted by the letters a , b , and c ; their respective opposite angles are denoted by A , B , and C , and the area by Q .

Example 1. Fig. 136 The side c in a right angled Triangle being 365 feet, and the angle $C = 39^\circ 20'$. How long is the side $a = ?$

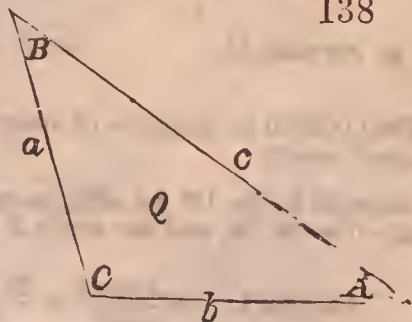
Formula 2. $a = \frac{c}{\sin. C} = \frac{365}{\sin. 39^\circ 20'} = \frac{365}{0.63383} = 575.86 \text{ feet, the answer.}$

FORMULA FOR OBLIQUE-ANGLED TRIANGLES.

137



138



$a : b = \sin. A : \sin. B$, and $b : c = \sin. B : \sin. C$.
 $a : c = \sin. A : \sin. C$, and $Q : ab = \sin. C : 2$.

$$a = \frac{c \sin. A}{\sin. C},$$

1,

$$a = \frac{c \sin. A}{\sin. (A+B)},$$

2,

$$a = \frac{2Q}{b \sin. C},$$

3,

$$b = \frac{c \sin. B}{\sin. C},$$

4,

$$b = \frac{2Q}{c \sin. A},$$

5,

$$\sin. C = \frac{c \sin. B}{b},$$

6,

$$\sin. C = \frac{c \sin. A}{a},$$

7,

$$\sin. A = \frac{2Q}{bc},$$

8,

$$\sin. A = \frac{a \sin. C}{c},$$

9,

$$a = \sqrt{b^2 + c^2 - 2bc \cos. A},$$

10,

$$a = \sqrt{\frac{2Q \sin. A}{\sin. C \sin. (A+B)}}$$

11,

$$S = \frac{1}{2}(a+b+c),$$

12,

$$\sin. \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

13,

$$\sin. \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}},$$

14,

$$\cos. \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}},$$

15,

$$\cos. \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}},$$

16,

$$Q = \frac{bc \sin. A}{2},$$

17,

$$Q = \frac{ab \sin. C}{2},$$

18,

$$Q = \frac{c^2 \sin. A \sin. B}{2 \sin. (A+B)},$$

19,

$$Q = \sqrt{S-a)(S-b)(S-c)}$$

20,

$$b = \sqrt{\frac{2Q \sin. (A+C)}{\sin. A \sin. C}},$$

21,

$$c = \sqrt{\frac{2Q \sin. C}{\sin. A \sin. (A+C)}},$$

22,

Example 2. Fig. 136. An inclined plane $a = 150$ feet long, and $c = 27$ feet, the height over its base. What is the angle of inclination $C = ?$

$$\text{Formula 14.} \quad \sin.C = \frac{c}{a} = \frac{27}{150} = 0.18000.$$

Find 0.18000 in the table of *sines*, which will be found at $10^\circ 30'$ which is the angle C nearly.

Example 3. Fig. 137 An oblique angled triangle has the sides $c = 27.6$ feet, the angle $C = 34^\circ 10'$, and the angle $A = 47^\circ 40'$. How long is the side $a = ?$

$$\text{Formula 1.} \quad a = \frac{c \sin.A}{\sin.C} = \frac{27.6 \times \sin.47^\circ 40'}{\sin.34^\circ 10'} = 36.33 \text{ feet, the answer.}$$

By Logarithms.

$$\log.a = \log.c + \log.\sin.A - \log.\sin.C.$$

$$c \quad + \quad \log.27.6 = 1.44090$$

$$A \quad + \quad \log.\sin.47^\circ 40' = 1.86878$$

$$\hline 1.30968$$

$$C \quad - \quad \log.\sin.34^\circ 10' = 1.74942$$

$$\log.36.4 = 1.56026, \text{ or } a = 36.4 \text{ feet.}$$

Example 1. Two ships of war notice a strong firing from a castle; in order to be safe, they keep themselves at a distance beyond the reach of the balls from the castle. To measure the distance from the castle, they place the vessels 800 yards from each other, and observe the angles between the castle and the vessels to be $A = 63^\circ 45'$, $B = 75^\circ 50'$. What will be the two distances from the castle?

$$C = 180 - 63^\circ 45' - 75^\circ 50' = 40^\circ 25'.$$

To A the distance will be,

$$b = \frac{c \sin.B}{\sin.C} = \frac{800 \times \sin.75^\circ 50'}{\sin.40^\circ 25'} = 1195.75 \text{ yards.}$$

To B the distance will be,

$$a = \frac{c \sin.A}{\sin.C} = \frac{800 \times \sin.63^\circ 45'}{\sin.40^\circ 25'} = 1106.6 \text{ yards.}$$

Example 2. From a window in the lower floor of a house which lays level with the foot of a tower, is observed an angle $= 40^\circ$ to the top of the tower. From another window in the upper story, in the same perpendicular as the lower window, the altitude of the tower is observed to be $= 37^\circ 30'$, which is 18 feet above the lower window.

$$\text{Then we have } A = 90 - 40^\circ = 50^\circ. \quad C = 90 + 37^\circ 30' = 127^\circ 30'.$$

$$B = 180 - 50 - 127^\circ 39' = 2^\circ 30'. \quad b = 18 \text{ feet.}$$

What will be the height of the tower and the distance from it?

The distance from the lower window to the top of the tower $= c$.

$$c = \frac{b \sin.C}{\sin.B} = \frac{18 \times \sin.127^\circ 35'}{\sin.2^\circ 30'} = 327.3 \text{ feet.}$$

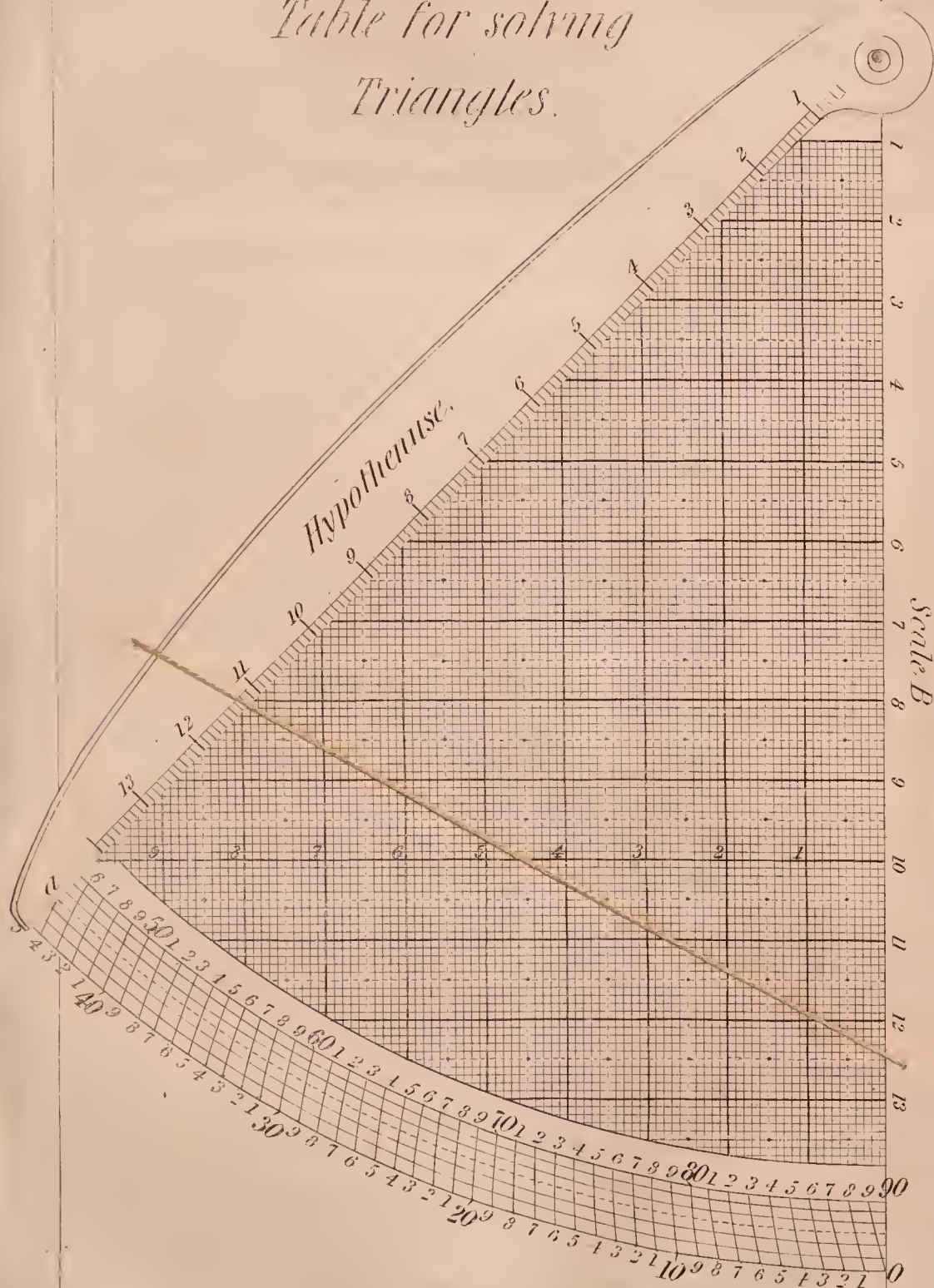
The height of the tower $= h$.

$$h = c \sin.A = 327.3 \times \sin.40^\circ = 210 \text{ feet.}$$

The distance to the tower $= d$.

$$d = c \cos.A = 327.3 \times \cos.40^\circ = 250.8 \text{ feet.}$$

Table for solving Triangles.





To Solve Triangles Mechanically.

PLATE III.

THE accompanying diagram is so constructed that the moveable arm represents the hypotenuse, the square lines the two sides, and the circular scale the angles in a right-angled triangle. The scale numbered from the centre towards 0 will be called b , and the one at right angle to be called c .

Example 1. Let the lines that form the right-angle be given as $b = 12$ and $c = 4$ inches. Required the hypotenuse a , and the angles B and C ?

Find where the two lines 12 and 4 crosses each other, move the arm to this crossing-point, which then indicates the length of the hypotenuse $= 12.65$ on the arm a ; the two angles will be found at a on the scale. $B = 71^\circ 40'$ and $C = 18^\circ 20'$.

If one angle and a side is given, set the arm on the given angle, and the intersection of the given side with the arm shows the length of the hypotenuse and the other side.

An oblique angled triangle can be two right angled triangles by drawing a line from the largest angle perpendicular to the opposite side, and can be solved by this diagram.

Example 2. An oblique angled triangle being $a = 65$ feet, $C = 34^\circ 30'$ and $B = 68^\circ 20'$.

Required the two sides b and c ?

Set the arm on the given angle $34^\circ 30'$, and at 65 feet on the arm will be found the height of the triangle $= 37$ feet on scale c , and one part of the side b is 54 feet on the scale b .

From the given angle

Subtract the complement of $34^\circ 30'$

$B = 68^\circ 20'$

$= 55^\circ 30'$

Set the arm on the angle,

$12^\circ 50'$

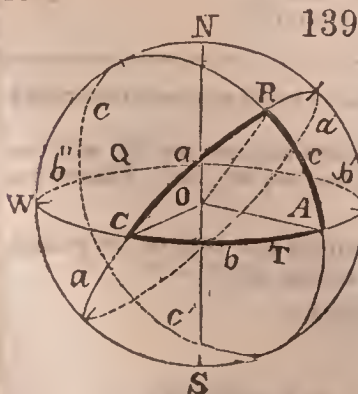
Now at the height 37 on the scale b will be found 38 feet on the arm, which is the length of the side c , and the other part of the side b is 9 feet on the scale c , then $b = 54 + 9 = 63$ feet.

In a similar manner any plane triangle can be so solved. By a little practice, this Table is very useful for approximating triangles.

SPHERICAL TRIGONOMETRY.

Spherical Trigonometry treats of triangles which are drawn (or imagined to be) on the surface of a sphere; their sides are arcs of the great circle of the sphere, and measures by the angle of the arc. Therefore the trigonometrical functions bear quite a different relation to the sides.

Every section of a sphere cut by a plane is a *circle*. A line drawn through the centre and at right angles to the sectional circle is called an *axis*, and the two points where the axis meets the surface of the sphere are called the *poles* of the sectional circle.

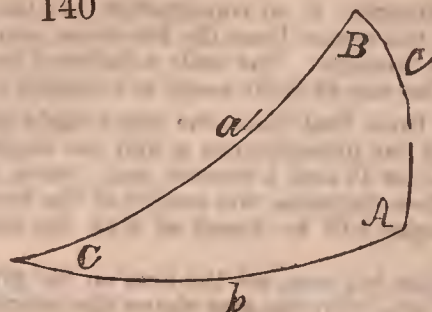


When the cutting plane goes through the centre of the sphere, it will pass through the great circle, and is then called the *Equator* for the poles. Axis = N.S. Equator = G.E.T.W.

Three great circle-planes, $aa'a''a'''$, $bb'b''$, and $cc'c''$, cutting a sphere, $NESW$, will form a solid angle at the centre O , and a triangle ABC on the surface of the sphere, in which the arcs a, b, c , are the sides. The angles formed by each two planes are congruent to each of the appertinent angles A, B , and C .

RIGHT-ANGLED SPHERICAL TRIANGLE.

140



$\sin.b = \sin.a \sin.B,$	1,	$\sin.B = \frac{\sin.b}{\sin.a},$	12,
$\tan.c = \tan.a \cos.B,$	2,		
$\cot.C = \cos.a \tan.B,$	3,	$\sin.C = \frac{\tan.b}{\tan.a},$	13,
$\tan.c = \sin.b \tan.C,$	4,		
$\cos.a = \cos.b \cos.c,$	5,	$\tan.C = \frac{\tan.c}{\sin.b},$	14,
$\cos.B = \cos.b \sin.C,$	6,		
$\tan.a = \frac{\tan.b}{\tan.C},$	7,	$\tan.B = \frac{\tan.b}{\sin.c},$	15,
$\sin.c = \frac{\tan.b}{\tan.B},$	8,		
$\sin.a = \frac{\sin.b}{\tan.B},$	9,	$\cos.c = \frac{\cos.C}{\sin.B},$	16,
$\sin.C = \frac{\cos.B}{\cos.b},$	10,	$\cos.b = \frac{\cos.B}{\sin.C},$	17,
$\cos.c = \frac{\cos.a}{\cos.b},$	11,	$\cos.a = \frac{\cot.C}{\tan.B},$	18.

The sum of the three angles in a spherical triangle is greater than two right angles, and less than six right angles.

By Spherical Trigonometry we ascertain distances and courses on the surface of the earth; positions and motions of the heavenly bodies, &c., &c. Examples will be furnished in Geography and Astronomy.

Example 1. Fig. 140 In a right-angled spherical triangle the side or hypotenuse $a = 36^{\circ} 20'$, the angle $B = 68^{\circ} 50'$. How long is the side $b = ?$

Formula 1. $\sin.b = \sin.a \sin.B = \sin.36^{\circ} 20' \times \sin.68^{\circ} 50'.$

a $\log.\sin. 36^{\circ} 20' = 1.77267$

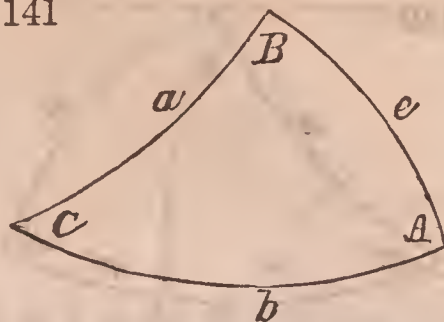
B $\log.\sin. 68^{\circ} 50' = 1.96966$

The answer, $\log.\sin. 33^{\circ} 32' = 1.74233$

or $b = 33^{\circ} 32'.$

OBLIQUE-ANGLED SPHERICAL TRIANGLE.

141



$$\sin.a : \sin.b = \sin.A : \sin.B,$$

$$\sin.a = \frac{\sin.b \sin.A}{\sin.B}, \quad 19,$$

$$\sin.b : \sin.c = \sin.B : \sin.C,$$

$$\sin.b = \frac{\sin.c \sin.B}{\sin.C}, \quad 20,$$

$$\tan.(a+b) = \tan.\frac{1}{2}c \frac{\cos.\frac{1}{2}(A-B)}{\cos.\frac{1}{2}(A+B)}, \quad 21,$$

$$\tan.(a-b) = \tan.c \frac{\sin.\frac{1}{2}(A-B)}{\sin.\frac{1}{2}(A+B)}, \quad 22,$$

$$\tan.\frac{1}{2}(A+B) = \cot.\frac{1}{2}A \frac{\cos.\frac{1}{2}(b-c)}{\cos.\frac{1}{2}(b+c)}, \quad 23,$$

$$\tan.\frac{1}{2}(A-B) = \cot.\frac{1}{2}A \frac{\sin.\frac{1}{2}(b-c)}{\sin.\frac{1}{2}(b+c)}, \quad 24,$$

$$\cot.\frac{1}{2}A = \tan.\frac{1}{2}(B-C) \frac{\sin.\frac{1}{2}(b+c)}{\sin.\frac{1}{2}(b-c)}, \quad 25,$$

$$\tan.\frac{1}{2}c = \tan.\frac{1}{2}(a-b) \frac{\sin.\frac{1}{2}(A+B)}{\sin.\frac{1}{2}(A-B)}, \quad 26,$$

Example 2. Fig. 141 Oblique angled spherical triangle. $c = 72^\circ 30'$. $B = 17^\circ 30'$. $C = 79^\circ 50'$.

How long is the side $b = ?$

Formula 20. $\sin.b = \frac{\sin.c \sin.B}{\sin.C} = \frac{\sin.72^\circ 30' \times \sin.17^\circ 30'}{\sin.79^\circ 50'}$

$$c \quad + \log.\sin. 72^\circ 30' = 1.97942$$

$$B \quad + \log.\sin. 17^\circ 30' = 1.47812$$

$$+ \quad = 11.45754$$

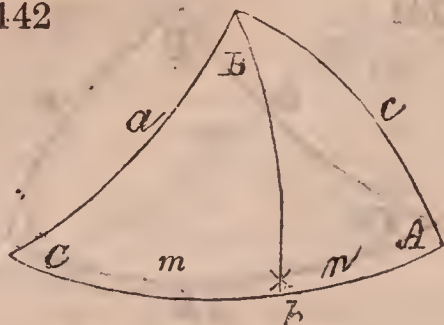
$$C \quad + \log. \sin. 79^\circ 50' = 1.99312$$

The answer

$$\log.\sin. 16^\circ 56' = 1.46442$$

$$\text{or } b = 16^\circ 56'.$$

142



$$\sin. \frac{1}{2}a = \sqrt{\frac{\cos. S \cos. (S - A)}{\sin. B \sin. C}}, \quad 33,$$

$$\sin.\frac{1}{2}Q = \frac{\sin.\frac{1}{2}c \sin.\frac{1}{2}a \sin.B}{\cos.\frac{1}{2}b}, \quad - \quad - \quad - \quad - \quad 2,$$

CONIC SECTIONS.

A Conic Section is the section obtained when a plane cuts a cone. The conic sections are of five different kinds, namely.

1st. *Triangle*. When the plane cuts the cone through its axis.

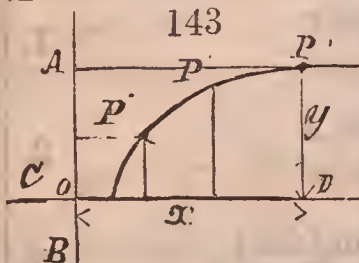
2d. *Circle*. When the plane cuts the cone at right angles to its axis.

3d. *Ellipse*. When the plane cuts the cone obliquely passing through the two sides.

4th. *Parabola*. When the plane cuts the cone parallel to one side.

5th. *Hyperbola*. When the plane cuts the cone at an angle to the axis less than the angle of the axis and the side of the cone.

The position of a point in a plane surface is determined by its course and distance from a given point on a straight line, or by its distances from two lines inclined to one another



Let AB and CD be two infinite lines inclined to one another, and P being a point in the same plane as the lines. It is evident that the position of this point P is determined by the distances x and y from the lines AB and CD . Those lines and distances are called, AB the axis of ordinate, and CD the axis of abscissa, y the ordinate, x the abscissa, and o the origin.

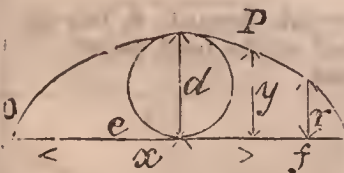
The abscissa x is commonly taken on the absciss' axis. Now take different values of the abscissa x , and by some formula or rule calculate the ordinate y ; then a number of points $P, P', P'',$ &c., may be obtained; join those points by a line, then the rule or formula is called an equation for that line. Equations of this kind will here be furnished for the curve in the conic sections.

Transverse-axis is the longest line that can be drawn in an Ellipse.

Conjugate-axis is a line drawn through the centre, at right angles to the transverse axis.

Parameter of any diameter is a third proportional to that diameter, and its conjugate.

Focus is the point in the axis where the ordinate is equal to half the diameter.



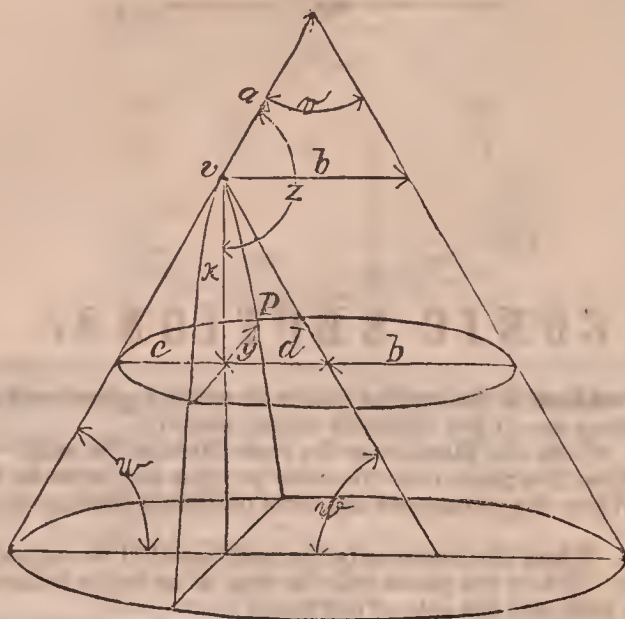
144.

Cycloid.

$$y = 0.637 \sqrt{x(\pi d - x)},$$

$$e = 1.211d, \quad p = 0.637d.$$

CONIC SECTIONS.



145.

$$a : b = \sin.w : \sin.v,$$

$$b = \frac{a \sin.v}{\sin.w}, \quad 1,$$

$$x : c = \sin.w : \sin.z,$$

$$c = \frac{x \sin.z}{\sin.w}, \quad 2,$$

$$x : d = \sin.w : \sin.(z+v),$$

$$d = \frac{x \sin.(z+v)}{\sin.w}, \quad 3,$$

$$y^2 = c(d+b). \quad \text{See Fig. 64, page. 91,} \quad 4,$$

$$y^2 = \frac{x \sin.z}{\sin.w} \left(\frac{a \sin.(z+v)}{\sin.w} + \frac{a \sin.v}{\sin.w} \right), \quad 5,$$

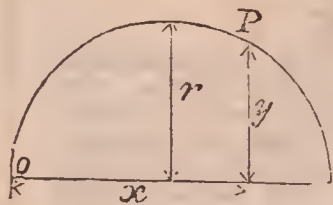
$$y^2 = \frac{x \sin.z}{\sin.^2 w} (a \sin.[z+v] + a \sin.v). \quad 6,$$

This is the general formula for all conic sections.

In any conic section, a point P can be calculated by this formula 6, but for the different conic sections, it will be found greatly simplified on the next pages.

For a Parabola $z+v = 180$, therefore $\sin.z = \sin.v$, and

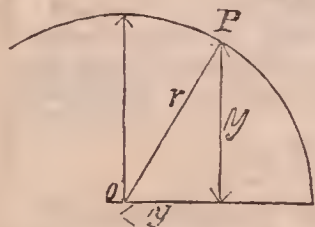
$$y^2 = \frac{x \sin.^2 v}{\sin.^2 w}.$$

146. *Circle.*

$$y = \sqrt{2rx - x^2},$$

$$r = \frac{y^2 + x^2}{2x},$$

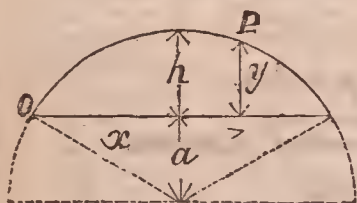
$$x = r + \sqrt{r^2 - y^2}.$$

147. *Circle.*

$$y = \sqrt{r^2 - x^2},$$

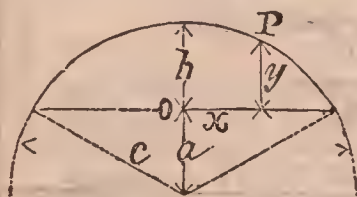
$$r = \sqrt{y^2 + x^2},$$

$$x = \sqrt{r^2 - y^2}.$$

148. *Circle Arc.*

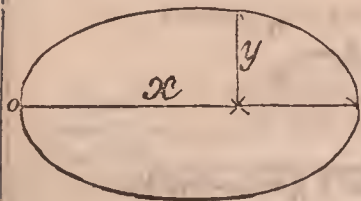
$$y = \sqrt{a^2 + cx - x^2} - a,$$

$$a = \frac{c^2 - 4h^2}{8h}.$$

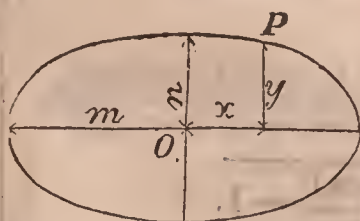
149. *Circle Arc.*

$$y = \sqrt{\left(\frac{c^2 + h^2}{8h}\right)^2 - x^2} - a.$$

$$a = \frac{c^2 - 4h^2}{8h}.$$

150. *Ellipse.*

$$y = \frac{n}{m} \sqrt{2mx - x^2}.$$

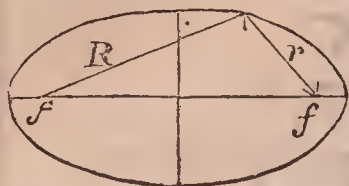
151. *Ellipse.*

$$y = \sqrt{n^2 - \left(\frac{nx}{m}\right)^2}.$$

152. *Ellipse.*

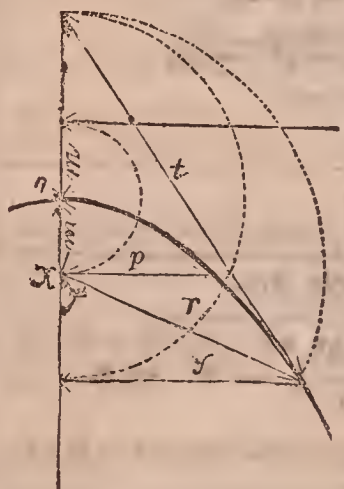
$$e = \sqrt{m^2 - n^2},$$

$$p = \frac{2n^2}{m}.$$

153. *Ellipse.*

$$R = 2m - r,$$

$$r = 2m - R.$$

154. *Parabola.*

$$y = \sqrt{px}, \quad p = 4m,$$

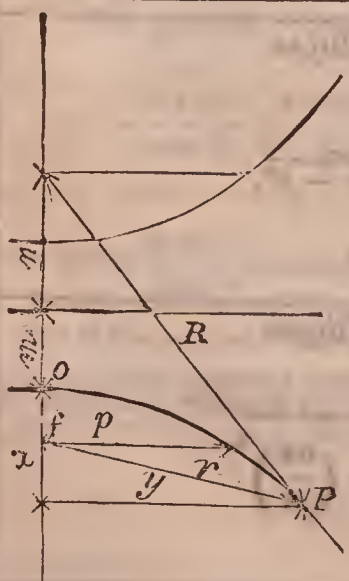
$$r = \sqrt{y^2 + (x - m)^2} = x + m,$$

$$y = \sqrt{t^2 - 4x^2}.$$

$$y = 2\sqrt{x(x+m) - x^2},$$

$$t = \sqrt{4x^2 + y^2},$$

$$t = 2\sqrt{x(x+m)}.$$

55. *Hyperbola.*

$$R - r = 2m, \quad e = \frac{n}{m}.$$

$$y = \frac{1}{m} \sqrt{(e^2 - 1)(x^2 - m^2)},$$

$$b^2 = \frac{e^2 - 1}{m^2},$$

$$y = b \sqrt{\frac{(x+m)^2}{m^2} - 1}.$$

MECHANICS.

Mechanics is that branch of Natural Philosophy which treats of the action of force, motion, and power. Mechanics is divided into four parts, namely, *Statics* the science of forces in equilibrium.

Dynamics, the science of forces in motion, it produces power or effect.

Hydrostatics, the science of fluids in equilibrium

Hydrodynamics the science of fluids in motion, its causes, power or effects.

Statics. Lever. Momentum.

Lever is an inflexible bar, supported in one point called the *Fulcrum*, or centre of motion. The length of a *lever* is measured from the *fulcrum* to where the *force* or *resistance* acts, (when the force acts at right angles to the lever) or, the length of a lever is measured from the fulcrum at right angles to the direction of the *force*.

$$\left. \begin{array}{l} W = \text{Weight,} \quad \text{and } l = \text{lever for } W \\ F = \text{Force,} \quad \text{and } L = \text{lever for } F \end{array} \right\} \text{ See Fig. 156.}$$

Momentum is the product of *force* or *weight*, multiplied by the length of the *lever* it acts upon.

The products Wl and FL are called *Statics Momentums*; when these momentums are equal there will be no motion, and the weight W will balance the force F . When one momentum is greater than the other, there will be a motion, and the velocity of that motion is measured by the difference of the momentums.

Levers are of three distinct kinds, with reference to the relative positions of the *Force F*, *Weight W*, and *Fulcrum C*.

1st. *Fulcrum C*, is between the *force F*, and the *weight W*.

2d. *Weight W*, is between the *fulcrum C*, and the *force F*.

3d. *Force F*, is between the *fulcrum C*, and the *weight W*.

Example 1. Figure 156. The weight $W = 68$ pounds, the lever $l = 3.86$ feet, and $L = 10$ feet 6 inches.

Required the force $F = ?$

$$\text{Formula 1. } F = \frac{Wl}{L} = \frac{68 \times 3.86}{10.5} = 25 \text{ pounds nearly.}$$

a = distance between the force F and the weight W .

The formula 3, 4, 7, 8, 11, 12, are for finding the fulcrum C , when the *force F*, *weight W*, and the distance a , are given.

Example 2. Fig. 157. The force $F = 360$ pounds, $W = 1870$, and $a = 8$ feet, 4 inches.

Required the position of the fulcrum c ?

$$\text{Formula 7. } l = \frac{Fa}{W - F} = \frac{360 \times 8.333}{1870 - 360} = \frac{2999.988}{1510} = 19.86 \text{ feet.}$$

$$L = 8.333 + 19.86 = 28.193 \text{ feet, the answer.}$$

Example 3. Fig. 161. The weight of the lever is $Q = 18$ pounds. The centre of gravity is $x = 2.25$ feet from the fulcrum. $W = 299$ pounds, $l = 5.5$ feet, and $L = 11.95$.

Required the force $F = ?$ in pounds.

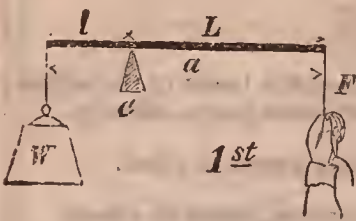
$$F = \frac{Wl - Qx}{L} = \frac{299 \times 5.5 - 18 \times 2.25}{11.95} = 134.25 \text{ pounds.}$$

Inclined Plane.

Example 4. Fig. 180. A load $W = 3466$ pounds, is to be drawn up an inclined plane, $l = 638$ feet long, and $h = 86$ feet high.

What force is required to keep the load on the inclined plane?

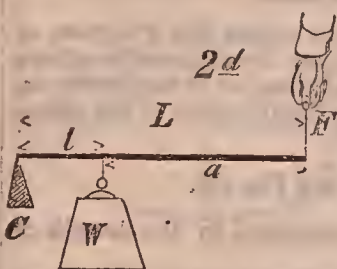
$$F = \frac{hW}{l} = \frac{86 \times 3466}{638} = 467.2 \text{ pounds.}$$



$$156. F : W = l : L, \quad FL = Wl,$$

$$F = \frac{Wl}{L}, \quad W = \frac{FL}{l}, \quad 1, 2,$$

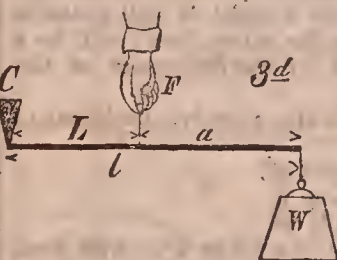
$$l = \frac{Fa}{W+F}, \quad L = \frac{Wa}{W+F}, \quad 3, 4,$$



$$157. F : W = l : L, \quad FL = Wl,$$

$$F = \frac{Wl}{L}, \quad W = \frac{FL}{l}, \quad 5, 6,$$

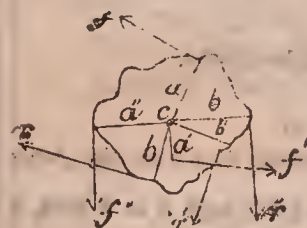
$$L = \frac{Wa}{W-F}, \quad l = \frac{Fa}{W-F}, \quad 7, 8,$$



$$158. F : W = l : L, \quad FL = Wl,$$

$$F = \frac{Wl}{L}, \quad W = \frac{FL}{l}, \quad 9, 10,$$

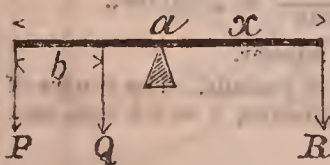
$$L = \frac{Wa}{F-W}, \quad L = \frac{Fa}{F-W}, \quad 11, 12.$$



159.

$$af + a'f' + a''f'' = br + b'r' + b''r''.$$

If the sum of the momentums that act to move the body in one direction are equal to the sum of the momentums that act opposite, the acting forces will be in equilibrium; c being the centre or fulcrum.

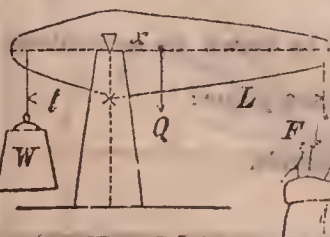


160.

To find the fulcrum c when three forces act on one lever.

$$R x = Q(a - b - x) + P(a - x),$$

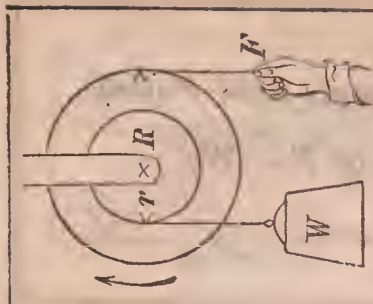
$$x = \frac{Qa + Pa - Qb}{R + Q + P}.$$



161.

Q = weight of the lever. x = distance from the centre of gravity of the lever to the fulcrum. Balance the lever over a sharp edge, and the centre of gravity is found.

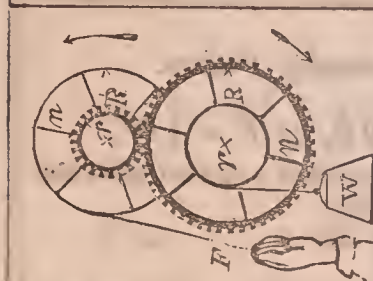
$$F = \frac{Wl - Qx}{L}, \quad W = \frac{FL + Qx}{l}.$$



$$162. \quad F : W = r : R. \quad F R = W r,$$

$$F = \frac{W r}{R}, \quad R = \frac{W r}{F},$$

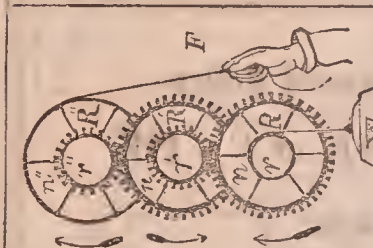
$$W = \frac{R F}{r}, \quad r = \frac{R F}{W}.$$



163.

$$F = \frac{W r r'}{R R'}, \quad W = \frac{F R R'}{r r'},$$

n = number of revolutions of the wheels,
 $n : n' = r' : R$, $v : v' = r r' : R R'$,
 v = velocity of W , v' = velocity of F .

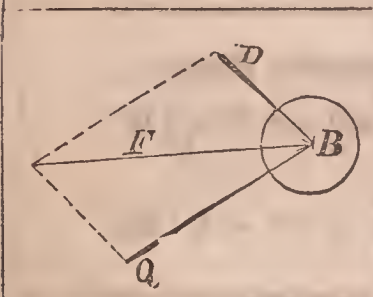


$$164. \quad F = \frac{W r r' r''}{R R' R''}, \quad W = \frac{F R R' R''}{r r' r''},$$

$n : n'' = r' r'' : R R'$, $v : v' = r r' r'' : R R' R''$.

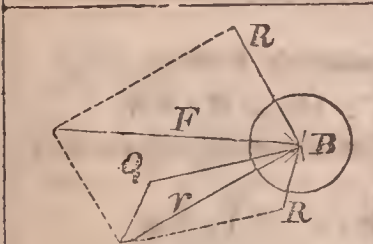
$r r' r''$ &c. = radii of the pinions.

$R R' R''$ &c. = radii of the wheels.



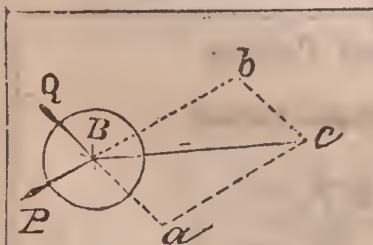
165.

Let P and Q represent the magnitudes and directions of two forces which act to move the body B . By completing the parallelogram, there will be obtained a diagonal force F , whose magnitude and direction is equal to the sum P and Q . F is called the resultant of P and Q .



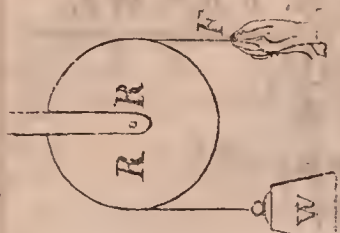
166.

If three or more forces act in different directions to move a body B , find the resultant of any two of them, and consider it as a single force. Between this and the next force find a second resultant. thus: P , Q , and R are magnitudes and directions of the forces. $P + Q = r$, $r + R = F = P + Q + R$, or F is the magnitude and direction of the three forces, P , Q , and R .



167.

A force Q acting (alone) on the body B , can move it to a in a unit of time, another force P is able to move it to b in the same time; now if the two forces act at the same time, they will move the body to c . c is the resultant of a and b .

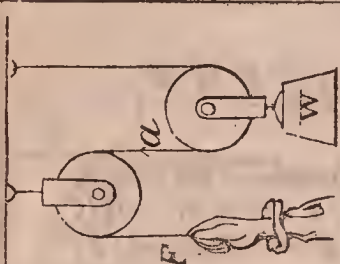


168.

A single fixed Pulley.

$$F : W = R : R, \text{ or } F = W,$$

$$v = v'.$$



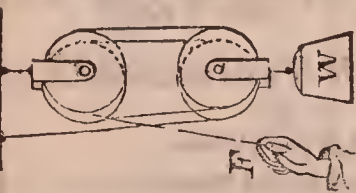
169.

A single moveable Pulley.

$$F : W = R : 2R, \text{ or } F = \frac{1}{2} W,$$

If the force F being applied at a and act upwards,
the result will be the same.

$$v' = 2v.$$

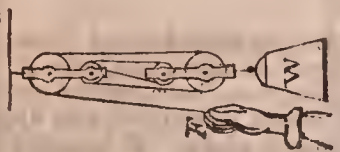


170.

A double moveable Pulley.

$$F : W = R : 4R, \text{ or } F = \frac{1}{4} W,$$

$$v' = 4v.$$



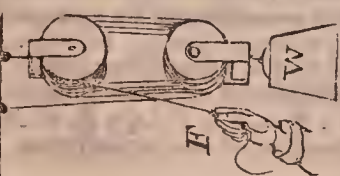
171.

A triple moveable Pulley.

$$F : W = R : 8R, \text{ or } F = \frac{1}{8} W,$$

$$F = \frac{W}{2^u},$$

$$v : v' = 1 : 2u.$$



172.

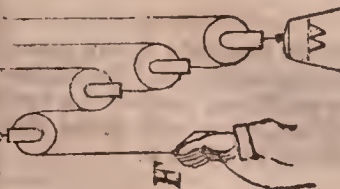
Quadruple moveable Pulley.

$$F = \frac{1}{8} W. \quad F : W = R : 8R.$$

Let u = any number of moveable pulleys, then

$$F = \frac{W}{2^u},$$

$$v : v' = 1 : 2u.$$



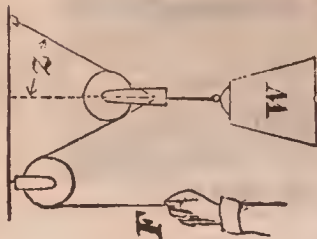
173.

Compound Pulleys.

u = number of moveable Pulleys.

$$F = \frac{W}{2^u} \quad W = 2^u F,$$

$$v : v' = 1 : 2^u.$$

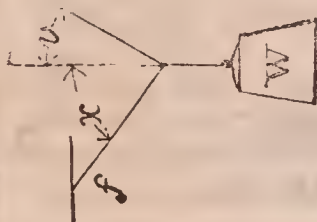


174.

An oblique fixed Pulley.

$$F : W = 2 \cos. z : 1,$$

$$W = \frac{F}{2 \cos. z}, \quad F = 2 W \cos. z.$$

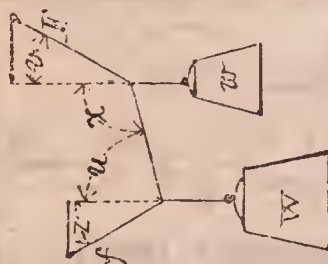


175.

$$W : F = \sin. (x+v) : \sin. v,$$

$$F = \frac{W \sin. x}{\sin. (x+v)}, \quad f = \frac{W \sin. v}{\sin. (x+v)},$$

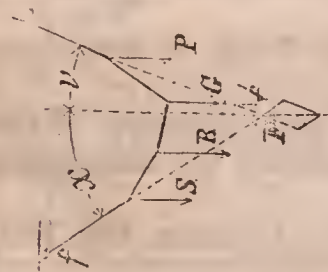
$$W : f = \sin. (x+v) : \sin. v.$$



176.

$$F = \frac{w \sin. x}{\sin. (x+v)},$$

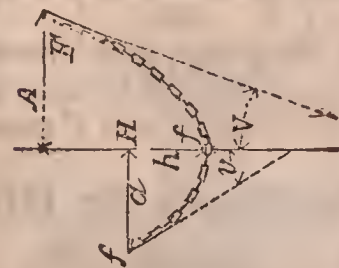
$$f = \frac{W \sin. u}{\sin. (u+z)}.$$



177.

$$W = P + Q + R + S, \quad F = F', \quad f = f',$$

$$F = \frac{W \sin. x}{\sin. (x+v)}, \quad f = \frac{W \sin. v}{\sin. (x+v)}.$$

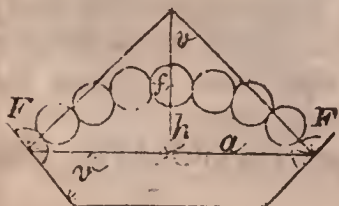


178.

$$l = \sqrt{\frac{4h^2}{3} + a^2}, \quad h = 0.5625 \sqrt{l^2 - a^2},$$

l = length, and W = weight of half the chain, ff' ,

$$\cot. v = \frac{2h}{a}, \quad F = \frac{W \sin. V}{\sin. 2V}, \quad f' = W \tan. v.$$

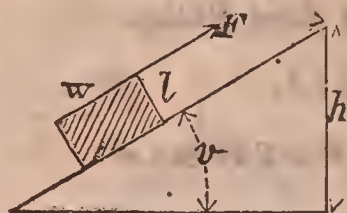


179.

$$F = \frac{W \sin. v}{\sin. 2v}, \quad f = W \tan. v,$$

$$\cot. v = \frac{2h}{a}.$$

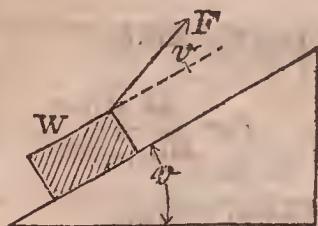
W = weight of half the number of balls.



$$180. \quad F = \frac{W h}{l} = W \sin.v,$$

$$W = \frac{F l}{h} = \frac{F}{\sin.v},$$

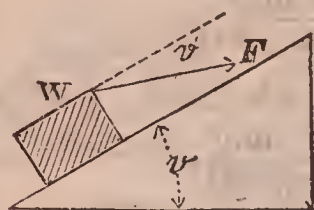
$$W = \frac{W l}{b} = W \cos.v.$$



$$181. \quad F = W \sin.(v+v'),$$

$$W = \frac{F}{\sin.(v+v')},$$

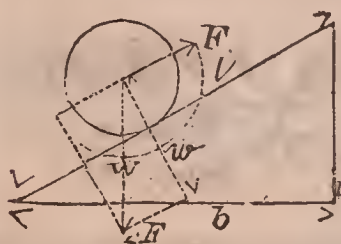
$$W = W \cos.(v+v').$$



$$182. \quad F = \frac{W \sin.v}{\cos.v},$$

$$W = \frac{F \cos.v'}{\sin.v},$$

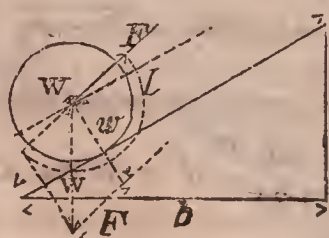
$$W = W (\cos.v + \sin.v. \tan v').$$



183.
To solve an Inclined Plane by diagrams.

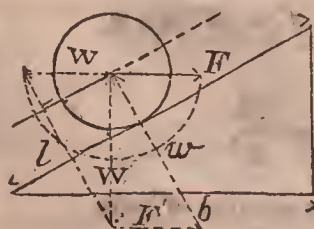
F = magnitude and direction of the force, which is obtained by completing the parallelogram.

By calculation see Formula, Fig. 180.



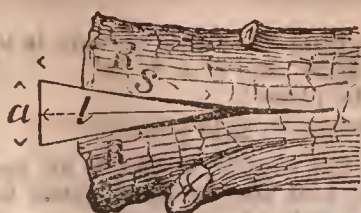
184.
 W = weight of the body, and direction of the force of gravity; to be drawn at right-angles to the base b , and F parallel to F .

By calculation see Formula, Fig. 181.



185.
 w = the force with which the body presses against the plane, to be drawn at right-angles to the plane l ; then the parallelogram is completed.

By calculation see Formula, Fig. 182.

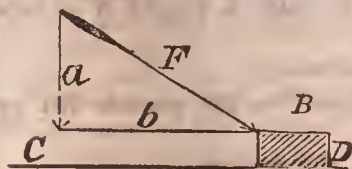


186

Wedge.

$$F = \frac{R a}{l}, \quad R = \frac{F l}{a}.$$

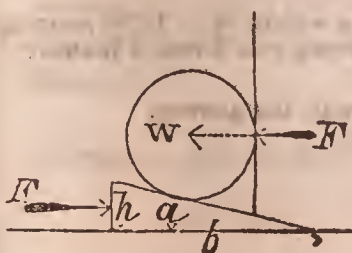
F = force acquired to drive the wedge.



187.

Let the line F represent the magnitude and direction of a force acting to move the body B on the line CD ; then the line a represents a part of F which presses the body B against CD , and the line b represents the magnitude of the force which actually moves the body B .

$$b = \sqrt{F^2 - a^2}, \quad b = F \cos v.$$

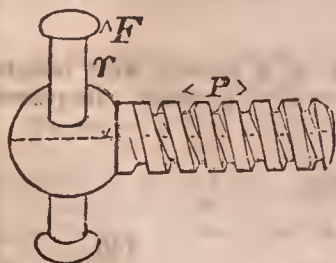


188.

$$F' : W = h : b = \sin v : \cos v = \tan v.$$

$$F = \frac{W h}{b} = W \tan v. \quad F' = F.$$

$$W = \frac{F b}{h} = \frac{F}{\tan v} = F \cot v.$$

189. *Force by a Screw.*

P = Pitch of the screw,

r = radius on which the force F acts.

$$F : W = P : 2\pi r.$$

$$F = \frac{W P}{2\pi r}, \quad W = \frac{F 2\pi r}{P}.$$

190. *Force by Compound Screws.*

P = Pitch of the large screw,

p = Pitch of the endless screw.

R = radius of spur-wheel for the endless screw.

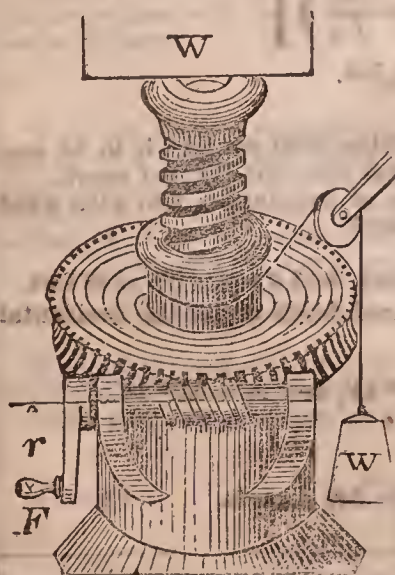
$$F : W = 4\pi^2 R r : P p.$$

$$F = \frac{W P r}{4\pi^2 R r}, \quad W = \frac{F 4\pi^2 R r}{P p}.$$

On the spur-wheel is a cylinder by which the weight W' is wound up, the formula will be, r' = radius of the cylinder, and

$$F : W' = P r' : 2\pi R r.$$

$$F = \frac{W' P r'}{2\pi R r}, \quad W = \frac{F 2\pi R r}{P r'}.$$



Example 4. Fig. 184. A Cylinder of cast iron, weighing $W = 5245$ pounds, is to be rolled up an inclined plane; the angles $v = 18^\circ 20'$ and $v' = 8^\circ 10'$

What force is required to keep the cylinder on the plane?

$$F = W \sin.(v+v') = \sin.5245 \times (18^\circ 20' + 8^\circ 10') = 2340 \text{ pounds.}$$

Example 5. Fig. 185. An iron ball which weighs 398 pounds, is tied to an inclined plane with a rope; the angle of the rope and the inclined plane is $v' = 16^\circ 40'$, and $v = 14^\circ 30'$. What force is acting on the rope?

$$F = \frac{W \sin.v}{\cos.v'} = \frac{398 \times \sin.14^\circ 30'}{\cos.16^\circ 40'} = 10.4 \text{ pounds.}$$

Example 6. Fig. 170. What force F is required to raise a weight $W = 8469$ pounds, by a double moveable pulley?

$$F = \frac{1}{2} W = \frac{1}{2} \times 8469 = 2117.25 \text{ pounds.}$$

Example 7. Fig. 173. How much weight can a force $F = 269$ pounds lift by three compound moveable pulleys?

$$W = 2^n F = 2^3 \times 269 = 2152 \text{ pounds, the answer.}$$

Screw.

Example 8. Fig. 189. What force is required to lift a weight $W = 16785$ pounds, by a screw, with a pitch $P = 0.125$ feet, the lever being $r = 5$ feet. 4 inches?

$$F = \frac{WP}{2\pi r} = \frac{16785 \times 0.125}{2 \times 3.14 \times 5.333} = 62.62 \text{ pounds, the answer.}$$

Including friction the force F will be

$$F = \frac{W(P + f d \pi)}{2\pi r}$$

Find the friction f on page 155. d diameter of the screw.

Wedge.

Example 9. Fig. 186. The head of the wedge $a = 3$ inches, and length $l = 16\frac{1}{2}$ inches; the resistance to be separated is $R = 4846$ pounds. Required the force $F = ?$ (Friction omitted.)

$$F = \frac{4846 \times 3}{16.5} = 881 \text{ pounds.}$$

Including friction the force F will be,

146

$$F = R \left[\frac{a}{l} + f \left(2 + \frac{a^2}{l s} \right) \right]$$

in which the friction f is to be found on page 155.

Catenaria.

Example 9. An iron chain 256 feet long, weighing 1560 pounds, is to be suspended between two points in the same horizontal line, but 196 feet apart.

How deep will the chain hang under the line of suspension, and with what force will the chain act at the points of suspension?

Figure and Formula 178, we have given,

$$W = \frac{1}{2} \times 1560 = 780 \text{ pounds, } l = \frac{1}{2} \times 256 = 128 \text{ feet, and } a = \frac{1}{2} \times 196 = 98 \text{ feet.}$$

$h = 0.6525 \sqrt{128^2 - 98^2} = 53.73$ feet, the required depth under the horizontal line.

$$\cot.v = \frac{2 \times 53.73}{98} = 1.096, \text{ or } v = 44^\circ 44', \text{ and } 2v = 89^\circ 28'.$$

The required force will be,

$$F = \frac{780 \times \sin.44^\circ 44'}{\sin.89^\circ 28'} = 549 \text{ pounds.}$$

DYNAMICS.

Velocity is a space passed through in a unit of time.

In machinery velocities are measured in feet per second; for Steamboats and Railroads in miles per hour.

Circular or angular velocity is the number of revolutions a revolving body makes per unit of time (minute.)

Velocity of Men.

A foot-soldier travels about 28 inches per step.

In common time 90 steps per minute = 3.5 feet per second = 2.4 miles per hour.

In quick time 110 " = 4.3 " = 3 "

Double quick time 140 " = 5.5 " = 3.75 "

A soldier occupies in the rank, a front of 20 inches, and 13 inches deep without knapsack; the interval between the ranks is 13 inches.

Average weight of men, 150 pounds each.

Five men can stand in a space of 1 square yard.

Example 1. A man walked 450 feet in 75 seconds. With what velocity did he walk?

$$\text{Formula 1.} \quad v = \frac{s}{t} = \frac{450}{75} = 6 \text{ feet per second.}$$

Example 2. A body moves 368 feet in $t = 8$ seconds. What velocity has it?

$$\text{Formula 1.} \quad v = \frac{s}{t} = \frac{368}{8} = 46 \text{ feet per second.}$$

Example 3. The radius of a wheel is 4 feet and 4 inches; it makes 131 revolutions per minute. What is the velocity of a point in the circumference?

$$\text{Formula 18.} \quad v = \frac{2 \pi r n}{60} = \frac{2 \times 3.14 \times 4.33 \times 131}{60} = 59.5 \text{ feet per second.}$$

Power is the product of force and velocity: that is, a force multiplied by the velocity with which it moves, is the power of the force. Force without velocity is no power.

Force and Power are two distinct quantities. Power can not be increased or diminished by mechanical means; but Force can be increased and diminished at libitum.

When a force is in motion, and increased or diminished by mechanical means, the alteration will be at the expense of the velocity, so that the power will remain the same.

Horse Power.

THE unit for the measurement of mechanical Powers is assumed to be the power of a horse; or a force of 33000 pounds, moved through a space of one foot in one minute, or 550 pounds moved 1 foot in 1 second. Another unit for measuring small powers is, one pound moved through a space of one foot in one second; this unit is called effect.

One horse power is = 550 effects.

One man's power is = 50 effects.

One horse power is = 11 men's power.

Example 4. A man draws up a bucket of water which weighs 52 pounds, from the bottom of a well 83 feet deep, which space the bucket passes in 43 seconds. With what effect is that man working?

$$\text{Formula 5.} \quad P = \frac{Fs}{t} = \frac{52 \times 83}{43} = 100 \text{ Effects.}$$

Dynamical Formula.

$$\text{Velocity } v = \frac{s}{t}, \quad - \quad - \quad 1, \quad \text{Horse Power } H = \frac{F v}{550}, \quad 11,$$

$$\text{Space } s = v t, \quad - \quad - \quad 2, \quad \text{Horse Power } H = \frac{F s}{550 t}, \quad - \quad 12,$$

$$\text{Time } t = \frac{s}{v}, \quad - \quad - \quad 3, \quad \text{Force } F = \frac{550 H}{v}, \quad 13,$$

$$\text{Power } P = v F, \quad - \quad - \quad 4, \quad \text{Force } F = \frac{550 H t}{s}, \quad 14,$$

$$\text{Power } P = \frac{F s}{t}, \quad - \quad - \quad 5, \quad \text{Velocity } v = \frac{550 H}{F}, \quad 15,$$

$$\text{Force } F = \frac{P}{v}, \quad - \quad - \quad 6, \quad \text{Space } s = \frac{550 H t}{F}, \quad 16,$$

$$\text{Force } F = \frac{P t}{s}, \quad - \quad - \quad 7, \quad \text{Time } t = \frac{F s}{550 H}, \quad 17,$$

$$\text{Velocity } v = \frac{P}{F}, \quad - \quad - \quad 8, \quad \text{Velocity } v = \frac{2 \pi r n}{60}, \quad - \quad 18,$$

$$\text{Space } s = \frac{P t}{F}, \quad - \quad - \quad 9, \quad \text{Effect } P = \frac{F 2 \pi r n}{60}, \quad 19,$$

$$\text{Time } t = \frac{F s}{P}, \quad - \quad 10, \quad H = \frac{F 2 \pi r n}{550 \times 60} = \frac{F r n}{5250}, \quad 20.$$

Letters denote.

P = Power in Effects.

H = Horse-power, number of.

v = Velocity of the force *F* in feet per second.

t = Time in seconds.

s = the space in feet which the force *F* passes through in the time *t*.

Circular Motion.

r = radius of the circle in feet.

n = number of revolutions per minute.

Power of Steam Engines.

When the power of a steam-engine is to be calculated, we have the following quantities given : namely,

a = area of the steam-cylinder piston in square inches.

d = diameter of the steam-cylinder in inches.

S = stroke of piston in feet.

$P =$ effectual pressure of steam per square inch.

n = number of double strokes of piston per minute.

f = a coefficient for frictions in the steam-engine.

Example 5. The piston in a steam engine moves a space of 90 feet in 25 seconds, with a force of 25000 pounds, (pressure of steam.)

How many horse power is the engine?

Formula 12. $H = \frac{Fs}{550 \cdot t} = \frac{25000 \times 90}{550 \times 25} = 163.6 \text{ horses.}$

The formula for the actual horse power of a steam-engine will appear, as,

$$H = \frac{2 \text{ a S P } n}{33000} (1 - f.) \quad . \quad . \quad . \quad . \quad 21.$$

For condensing-engines in good order, the friction can be estimated to $f = 0.32$

For high pressure,

By insertion of these values we have, for

Condensing Engines $H = \frac{a S P n}{24250}$, - - - - - - 22,

High Pressure Engines $H = \frac{a S P n}{19000}$, - - - - - 23,

For condensing-engines, the vacuum must be included in the pressure P .

Example 6. Required the horse-power of a steam-engine, of the following dimensions.

$a = 1017.8$ square inches, area of piston.

$S = 4$ feet stroke of piston.

Pressure in the boiler 35 pounds per square in.

Vacuum 11 pounds.

$P = 35 + 11 = 46$ pounds per square inch effectual pressure.

$n = 45$ revolutions per minute. Then,

Formula 22. $H = \frac{1017.8 \times 4 \times 46 \times 45}{24.50} = 347.5 \text{ horses.}$

Example 7. Required the horse power of a high pressure-engine with the following dimensions?

$a = 572.55$ sq. in. $s = 3$ feet, $p = 42$ pounds, and $n = 38$ revolutions.

Formula 23. $H = \frac{572.55 \times 3 \times 42 \times 58}{19000} = 220 \text{ horses.}$

If the horse power is given, the other quantities will be ascertained by

$$\mathbf{a} = \frac{Hk}{SPn}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 24, \quad \left| \quad P = \frac{Hk}{\mathbf{a}Sn}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 26,$$

$$S = \frac{Hk}{a H n}, \quad \dots \quad 25, \quad n = \frac{Hk}{a S p}, \quad \dots \quad 27.$$

k = the coefficient for high or low pressure, formula 22 and 23.

Nominal Horse Power.

$$H = \frac{d^2 \sqrt[3]{S}}{.47}. \text{ Established by Watt.}$$

OBSERVED RESULTS OF POWER.

Description of works.	Work hours per day.	Force. F	Velocity. V	Effects P	Horses. H
A man can raise a weight by a single fixed pulley,	6	50	0·8	40	0·072
A man working a crank,	8	20	2·5	50	0·090
A man on a tread-wheel, (horizontal),	8	144	0·5	72	0·130
A man in a tread-wheel, (axis 24° from vertical),	8	30	2·3	69	0·125
A man draws or pushes in a horizontal direction.	8	30	2	60	0·109
A man pulls up or down,	8	12	3·7	44·4	0·080
A man can bear on his back,	7	95	2·5	237·5	
A horse in a horse-mill, walking moderately,	8	106	3	318	0·577
do. do. do. running fast,	5	72	9	648	0·165
An ox do. do. walking moderately,	8	154	2	308	0·558
A mule do. do. do.	8	71	3	213	0·380
An ass do. do. do.	8	33	2·65	87·4	0·160

Flour Mills.

For every 100 pounds of fine flour ground per hour, requires	550	1·000
One pair of mill-stones of 4 feet diameter making 120 rev. per minute, can grind 5 bushels of wheat to fine flour per hour,	2400	4·86
Do. rye to coarse flour,	1600	2·91

Saw Mills, alternative.

For every 20 square feet sawed per hour, in dry oak, there requires,	550	1·000
Dry pine 30 square feet per hour,	550	1·000

Circular Saw.

A saw 2·5 feet in diameter; and making 270 revolutions per minute will saw 40 square feet in oak per hour, with	550	1·000
In dry spruce, 70 square feet per hour.	550	1·000

Threshing Machine.

Velocity of the feed rollers at the circumference 0·55 feet per second, diameter of threshing-cylinder 3·5 feet and $4\frac{1}{2}$ feet long, making 300 revolutions per minute, can thresh from 30 to 40 bushels of oats, and from 25 to 35 bushels of wheat, per hour.	2200	4·000
One man by a flail can thrash half a bushel per hour, (wheat.)	70	0·127

Paddle-Wheel Steamers. Observed Results.

Length in load line.	Beam.	Draft of water.	Tonnage.	Dia. & length of Paddles.	Miles per hour.	Hors. Power.
100	17	4·5	110	16×4·25	9	115
114	22·5	7·75	241	18·5×6·33	7·5	128
128	24	7	250	15·75×5·5	11·5	130
160	27	8·25	570	20×7·5	10	250
200	33	13	1100	29×9·25	14	573

Propeller Steamers. Observed Results.

Length in load line.	Beam.	Draft of water.	Tonnage of displacement.	Dia. & Pitch of Propeller.	Miles per hour.	Horse Power
112	22	7	208	8×18	7	76
230	32	16	1770	14×32	12	650
210	38	16·66	2150	14·5×36	11	500

DREDGING MACHINERY.

Letters denote. T = tons of materials excavated per hour. h = height in feet, to which the materials are raised above the bottom of the excavated channel. k = coefficient of the different materials.For very hard clay mixed with gravel, - - - $k = 0.1$ " hard pure clay, - - - $k = 0.07$ " common clay or sand, - - - $k = 0.05$ " soft clay or loose sand - - - $k = 0.04$ " very soft and loose do., - - - $k = 0.02$ H = horse power required for the excavation.

$$H = T \left(\frac{h}{700} + k \right), \text{ and } T = \frac{700 H}{h + 700k}.$$

Example. What power is required to excavate 160 tons of hard pure clay per hour, and raise it up 25 feet?For hard pure clay $k = 0.07$, then,

$$H = 160 \left(\frac{25}{700} + 0.07 \right) = 16.9, \text{ or } 17 \text{ horses, nearly.}$$

The velocity of the buckets should be about 1 foot per second.

COLLISION OF BODIES IN MOTION.

WHEN bodies in motion come in collision with each other, the sum of this concentrated momentum will be the same after the collision as before, but their velocities and sometimes their directions will differ.

On the accompanying page the bodies are supposed to move in the same straight line, and the formula illustrates the consequences after collision.

Letters denote. M and m = weight of the bodies in pounds. V and v = their respective velocities in feet per second. V' and v' = respective velocities of the bodies after impact.

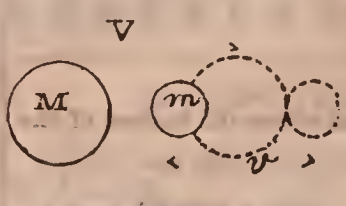
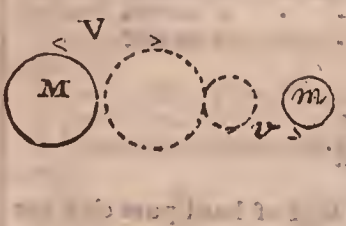
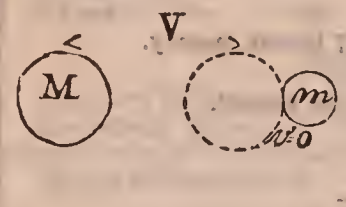
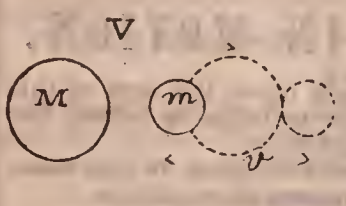
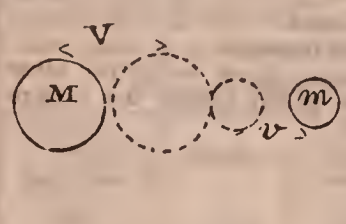
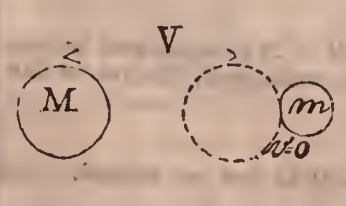
When the bodies are perfectly hard or non-elastic their velocities after impact will be common = v' . When the bodies are partly elastic the letters k and k' = coefficients for their elasticity.

For the body M , $k = \frac{MV}{M(V - V')}$

For the body m , $k' = \frac{mv}{m(v' - v)}$

Example 1. Fig. 193. The non-elastic body weighs $M = 25$ pounds, and moves at a velocity $V = 12$ feet per second; $m = 16$ pounds, and $v = 9$. Required the bodies' common velocities, $v' = ?$ after impact.

$$v' = \frac{MV + mv}{M + m} = \frac{25 \times 12 + 16 \times 9}{25 + 16} = 10.83 \text{ feet per second.}$$

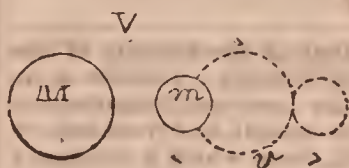
	<p>193.</p> <p><i>The bodies move in the same direction.</i></p> $v'(M+m) = MV + mv,$ $v' = \frac{MV + mv}{M+m}.$	The bodies perfectly hard.
	<p>194.</p> <p><i>The bodies move in opposite directions.</i></p> $v'(M+m) = \frac{MV - mv}{M+m},$ $v' = \frac{MV - mv}{M+m}.$	
	<p>195. <i>Only one body in motion.</i></p> $v'(M+m) = MV,$ $v' = \frac{MV}{M+m}.$	
	<p>196.</p> <p><i>The bodies move in the same direction.</i></p> $V' = \frac{V(M-m) - 2mv}{M+m},$ $v' = \frac{2MV - v(M-m)}{M+m}.$	The bodies perfectly elastic.
	<p>197.</p> <p><i>The bodies move in opposite direction.</i></p> $V' = \frac{V(M-m) - 2mv}{M+m},$ $v' = \frac{2VM + (M-m)v}{M+m}.$	
	<p>198. <i>Only one body in motion.</i></p> $V' = \frac{V(M-m)}{M+m}.$ $v' = \frac{2MV}{M+m}.$	

199.

The bodies move in the same direction.

$$V' = \frac{V(M - km) + vm(1+k)}{M+m},$$

$$v' = \frac{MV(1+k') + v(m - k'M)}{M+m}.$$

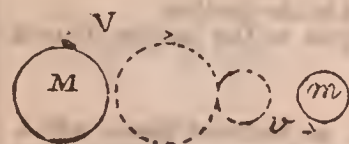


200.

The bodies move in opposite directions.

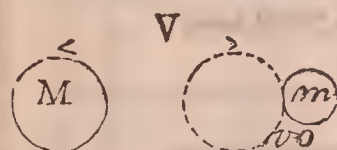
$$V' = \frac{V(M - km) - vm(1+k)}{M+m},$$

$$v' = \frac{MV(1+k') - v(m - k'M)}{M+m}.$$

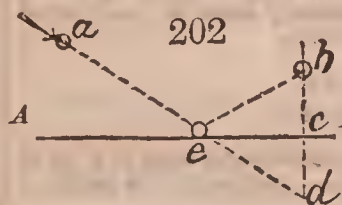
201. *Only one body in motion.*

$$V' = \frac{V(M - km)}{M+m},$$

$$v' = \frac{VM(1+k')}{M+m}.$$



When a moving body strikes a stationary elastic plane, its course of departure from the plane will be equal to its course of incident.



202

Problem. A body in a is to strike the plane AB so that it will depart to the given point b ; required its course of incident from a ?

Draw ld , at right angles through AB , make $cd = bc$ join a and d ; then ad is the course of incident, and eb the course of departure, and the body will strike in e .

Example 2. Fig. 197. The perfect elastic body $M = 84$ pounds, $V = 18$, $m = 48$, and $v = 27$. Required the velocity $V' = ?$ after impact with the body m .

$$V' = \frac{18(84 - 48) - 2 \times 48 \times 27}{84 + 48} = -23.64.$$

the negative sign denotes that the body will return after the collision with a velocity of 23.63 feet per second.

Example 3. Fig. 201. The partly elastic body $M = 38$ pounds and $V = 79$ feet per second, will strike the body in rest $m = 24$ pounds; what will be the velocity $v' = ?$ of the body m , its elasticity being $k' = 0.6$.

$$v' = \frac{79 \times 38(1 + 0.6)}{38 + 24} = 70.6 \text{ feet per second.}$$

F R I C T I O N .

THE resistance occasioned by Friction is independent of the velocity of motion; but the re-effect of friction is proportional to the velocity. Friction is independent of the extent of surface in contact when the pressure remains the same, but proportional to the pressure. This law was established from experiments by Arthur Morin in the years 1831-32 and 1833, from which a summary is contained in the accompanying Table.

Letters denote.

a = Fibres of the woods are parallel to themselves, and to the direction of motion.

b = Fibres at right-angles to fibres.

c = Fibres vertical on the fibres which are parallel to the motion.

d = Fibres parallel to themselves, but at right-angles to the motion, length by length.

e = Fibres vertical, end to end.

Example. A vessel of 800 tons is to be hauled up an inclined plane, which inclines $9^{\circ} 40'$ from the horizon; the plane is of oak, and greased with tallow. What power is required to haul her up?

The coefficient for oak on oak with continued motion is $f = 0.097$, say 0.1 , then,

$$800 \times \sin. 9^{\circ} 40' = 800 \times 0.16791 = 134.328 \text{ tons,}$$

the force required if there were no friction, and

$$800 \times \cos. 9^{\circ} 40' \times 0.1 = 800 \times 0.9858 \times 0.1 = 78.864 \text{ tons,}$$

the force required for the friction only, and

$$\begin{array}{r} 134.328 \\ 78.864 \\ \hline \end{array}$$

$$78.864$$

$$213.192 \text{ tons, the force required to haul her up.}$$

The effect lost by friction in axle and bearings is expressed simply by the formula

$$P = \frac{\pi d W n f}{12.60} = \frac{W d n f}{230},$$

in which W = the weight of pressure in the bearing, d = diameter on which the friction acts in inches, n = number of revolutions per minute, and f = coefficient of friction from the Table. In common machinery kept in good order the coefficient of friction can be assumed to $f = 0.065$. then

$$P = \frac{W d n}{353},$$

$$H = \frac{W d n}{1941500}$$

Example. The pressure on a steam-piston is 20000 pounds, and makes $n = 40$ double strokes per minute. Required the friction in the shaft of $d = 8$ inches?

$$H = \frac{20000 \times 8 \times 40}{1941500} = 3.3 \text{ horses, the loss by friction.}$$

Friction in Guides.

W = pressure on the steam piston in pounds.

S = stroke of piston in feet.

l = length of connecting rod in feet.

H = horse power of the friction.

$$H = \frac{W S n}{350000 \sqrt{5l^2 - S^2}}$$

Example. The pressure on a steam piston being $W = 30,000$ pounds, stroke $S = 4$ feet, length of connecting rod $l = 7$ feet, and making 50 revolutions per minute. Required the horse power of the friction $H = ?$

$$H = \frac{30000 \times 4 \times 50}{350000 \sqrt{5 \times 7^2 - 4^2}} = 1.13 \text{ horses.}$$

TABLE OF FRICTION FOR PLANE SURFACES IN CONTACT.

Kind of Materials in contact.		Lubricated with.	Coefficient in	
			Motion.	Starting.
Oak on Oak, - - - -	<i>a</i>	<i>o</i>	0.478	0.625
" " - - - -	"	tallow	0.097	0.160
" " - - - -	"	lard	0.067
" " - - - -	<i>b</i>	<i>o</i>	0.324	0.540
" " - - - -	"	unctuous	0.143	0.314
" " - - - -	"	tallow	0.083	0.254
" " - - - -	"	water,	0.25
" " - - - -	<i>d</i>	<i>o</i>	0.336
" " - - - -	<i>c</i>	<i>o</i>	0.192	0.271
" " - - - -	<i>e</i>	<i>o</i>	0.43
Cast-iron on Oak, - - -	<i>a</i>	<i>o</i>	0.400	0.570
" " - - - -	"	soap	0.214
" " - - - -	"	tallow	0.078	0.108
Wrought-iron on Oak, - -	"	<i>o</i>	0.252
" " - - - -	"	tallow	0.078
Wrought iron, together, -	<i>a</i>	<i>o</i>	0.138	0.137
" " - - - -	<i>a</i>	unctuous	0.177
" " - - - -	"	tallow	0.082
" " - - - -	"	olive oil	0.070	0.115
Wrought on cast-iron, - -	<i>a</i>	<i>o</i>	0.194	0.194
" " - - - -	"	unctuous	0.18	0.118
" " - - - -	"	tallow	0.103	0.10
" " - - - -	"	olive oil	0.066	0.100
Cast-iron on cast-iron, -	<i>a</i>	water	0.314	0.314
" " - - - -	"	soap	0.197
" " - - - -	"	tallow	0.100	0.100
" " - - - -	"	olive oil.	0.064
Wrought-iron on brass, - -	<i>a</i>	<i>o</i>	0.172
" " - - - -	"	unctuous	0.160
" " - - - -	"	tallow	0.103
" " - - - -	"	lard	0.075
" " - - - -	"	olive oil,	0.078
Cast-iron on brass, - - -	<i>a</i>	<i>o</i>	0.147	...
" " - - - -	"	unctuous	0.132
" " - - - -	"	tallow	0.103
" " - - - -	"	lard	0.075
" " - - - -	"	olive oil	0.078
Brass on brass, - - - -	<i>a</i>	<i>o</i>	0.201
" " - - - -	"	unctuous	0.134
" " - - - -	"	olive oil	0.053
Steel on cast-iron, - - -	"	<i>o</i>	0.202
" " - - - -	"	tallow	0.105
" " - - - -	"	lard	0.081
" " - - - -	<i>a</i>	olive oil	0.079	.

FRICTION OF AXLES IN MOTION.

Designation of surface in contact.	Dry or slightly greasy, or wet.	Oil, Tallow, or Hog's Lard.	
		Supplied in the ordinary manner.	The grease continually running.
Brass on Brass, - - -	0.079
" on cast-iron, - - -	0.072	0.049
Iron on Brass, - - -	0.251	0.075	0.054
" on cast-iron, - - -	0.075	0.054
Cast-iron on cast-iron, -	0.137	0.075	0.054
" on Brass, - - -	0.194	0.075	0.054
Iron on lignum-vitæ, - -	0.188	0.125
Cast-iron on " - - -	0.185	0.100	0.092
Lignum-vitæ on cast-iron.	0.116	0.170

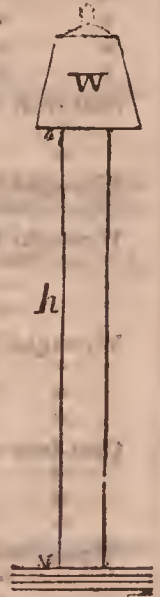
STRENGTH OF MATERIALS.

TABLE I., shows the weight a column can bear with *safety*; when the weight presses through the length of the column. The tabular number is the weight in pounds or tons per square inch on the transverse section of a column of a length less than 12 times its smallest thickness.

Table I.
RESISTANCE FOR COMPRESSION.

204

Kind of Materials.	Pounds.	Tons.
Oak, of good quality, - - - -	432	0.1885
Oak, common, - - - -	280	0.125
Spruce, red (Sapin rouge), - - -	540	0.241
“ white, (Sapin blanc), - - -	140	0.6256
Iron, wrought, - - - -	14400	6.48
Iron, cast, - - - -	28750	12.85
Basalt, - - - -	2875	1.285
Granite, hard, - - - -	1000	0.446
“ common, - - - -	575	0.256
Marble, hard, - - - -	1435	0.640
“ common, - - - -	431	0.192
Sandstone, hard, - - - -	1295	0.577
“ loose, - - - -	5.6	0.0025
Brick, good quality, - - - -	175	0.078
“ common, - - - -	58	0.0259
Lime-stone, of hardest kind, - - -	720	0.321
“ common, - - - -	432	0.193
Plaster-Paris, - - - -	86	0.0384
Mortar, good quality, and 18 months old,	58	0.0259
Do. common, - - - -	36	0.016



When the length or height of the column is more than 12 times its smallest thickness, divide the tabular weight by the corresponding number in this Table.

Length \times thickness	12	18	24	30	36	42	48	54	60
Divide by	1.2	1.6	2	2.8	4	5	6	8	12

Example. A building which is to weigh 2000 tons is to be supported by piles of Sapin rouge Spruce 18 feet in length, and 12 inches diameter. How many piles are required to support the building?

$$\frac{12^2 \times 0.785 \times 0.241}{1.6} = 17 \text{ tons, the weight which each pile can bear,}$$

and $\frac{2000}{17} = 118 \text{ piles.}$

To Find the Cohesive Strength.

RULE.—Multiply the cross-section of the materials in square inches by the tabular number in Table II., and the product is the cohesive strength.

Example An iron-bar has a cross-section of 2.27 sq. in. How many tons are required to tear it asunder, and how many pounds can it bear with safety?

English iron $2.27 \times 25 = 56.75$ tons, which will tear it asunder, and it will bear with safety

$$2.27 \times 14000 = 31780 \text{ pounds.}$$

Table II.
COHESIVE STRENGTH PER SQ. INCH OF CROSS-SECTION.

<i>Kind of Materials.</i>	<i>Just tear asunder.</i>		<i>With safety.</i>	
	<i>Pounds.</i>	<i>Tons.</i>	<i>Pounds.</i>	<i>Tons.</i>
Cast Steel, - - -	134256	59·93	33600	14·98
Blistered Steel, - - -	133152	59·43	33300	14·86
Steel, Shear, - - -	128632	56·97	32160	14·24
Iron, Swedish bar, - - -	65000	29·2	16260	7·3
“ Russian, - - -	59470	26·7	14900	6·7
“ English, - - -	56000	25·0	14000	6·25
“ common, over 2 in. sq.,	36000	16·0	9000	4·0
“ sheet, parallel rolling,	40000	17·85	10000	4·46
“ at right angles to roll,	34400	15·35	8600	3·84
Cast iron, good quality, -	45000	20·05	11250	5·00
“ inferior, - - -	18000	8·03	4500	2·0
Copper, cast, - - -	32500	14·37	8130	3·6
“ rolled, - - -	61200	27·2	15300	6·8
Tin, cast, - - -	5000	2·23	12500	0·56
Lead, cast, - - -	880	0·356	220	0·09
“ rolled, - - -	3320	1·48	830	0·37
Platinum, wire, - - -	53000	23·6	13250	5·9
Brass, common, - - -	45000	20·05	11250	5·0
<i>Wood.</i>				
Ash, - - - -	16000	7·14	4000	1·87
Beach, - - - -	11500	5·13	2875	1·28
Box, - - - -	20000	8·93	5000	2·23
Cedar, - - - -	11400	5·09	2850	1·27
Mahogany, - - - -	21000	9·38	5250	2·34
“ Spanish, - - - -	12000	5·36	3000	1·34
Oak, American white, -	11500	5·13	2875	1·28
“ English “ - - -	10000	4·46	2500	1·11
“ seasoned, - - -	13600	6·07	3400	1·52
Pine, pitch, - - -	12000	5·35	3000	1·34
“ Norway, - - -	13000	5·8	3250	1·45
Walnut, - - - -	7800	3·48	1950	0·87
Whalebone, - - - -	7600	3·40	1900	0·85
Hemp ropes, good, - -	6400	2·86	2130	0·95
Manilla ropes, - - -	3200	1·43	1100	0·49
Wire ropes, - - -	38000	17	12600	5·36
Iron chain, - - -	65000	29	21600	9·38
“ with cross pieces, -	90000	40	30000	13·4

205



To Ascertain the Strength of Cables.

Multiply the square of the circumference in inches by 120, and the product is the weight the cable will bear in pounds, with safety.

Strength of Hemp Rope, with Safety.

<i>Circumference.</i>	<i>Pounds.</i>	<i>Circumference.</i>	<i>Pounds.</i>	<i>Circumference.</i>	<i>Pounds.</i>
1·	200·	3· $\frac{1}{2}$	2450·	6·	7200·
1· $\frac{1}{4}$	312·5	3· $\frac{3}{4}$	2812·5	6· $\frac{1}{4}$	7812·5
1· $\frac{1}{2}$	450·	4·	3200·	6· $\frac{1}{2}$	8450·
1· $\frac{3}{4}$	612·5	4· $\frac{1}{4}$	3612·5	6· $\frac{3}{4}$	9112·5
2·	800·	4· $\frac{1}{2}$	4050·	7·	9800·
2· $\frac{1}{4}$	1012·5	4· $\frac{3}{4}$	4512·5	7· $\frac{1}{4}$	10512·5
2· $\frac{1}{2}$	1250·	5·	5000·	7· $\frac{1}{2}$	11250·
2· $\frac{3}{4}$	1512·5	5· $\frac{1}{4}$	5512·5	7· $\frac{3}{4}$	12012·5
3·	1800·	5· $\frac{1}{2}$	6050·	8·	12800·
3· $\frac{1}{4}$	2112·5	5· $\frac{3}{4}$	6612·5		

CABLES.

Strength of good Hemp Cable, with safety.

<i>Circumference.</i>	<i>Pounds.</i>	<i>Circumference.</i>	<i>Pounds.</i>	<i>Circumference.</i>	<i>Pounds.</i>
6	4320	10.25	12607.5	14.50	25230
6.25	4687.5	10.50	13230	14.75	26107.5
6.50	5070	10.75	13867.5	15	27000
6.75	5467.5	11	14520	15.25	27907.5
7	5880	11.25	15187.5	15.50	28830
7.25	6307.5	11.50	15870	15.75	29767.5
7.50	6750	11.75	16567.5	16	30720
7.75	7207.5	12	17280	16.25	31687.5
8	7680	12.25	18007.5	16.50	32670
8.25	8167.5	12.50	18750	16.75	33667.5
8.50	8670	12.75	19507.5	17	34680
8.75	9187.5	13	20280	17.25	35707.5
9	9720	13.25	21067.5	17.50	36750
9.25	10267.5	13.50	21870	17.75	37807.5
9.50	10830	13.75	22687.5	18	38880
9.75	11407.5	14	23520	18.25	39967.5
10	12000	14.25	24367.5		

Strength and Weight of Chains and Ropes.

<i>Iron Chains.</i>		<i>Hemp Ropes</i>		<i>Ultimate Strength</i>		<i>Absolute</i>	<i>Strength</i>
<i>Diameter in inches.</i>	<i>Weight per foot, lb.</i>	<i>Circumfer. in inches.</i>	<i>Weight per foot, lb.</i>	<i>Pounds.</i>	<i>Tons.</i>	<i>Pounds.</i>	<i>Tons.</i>
$\frac{1}{8}$	0.17	$1\frac{1}{4}$	0.07	920	0.41	306	0.14
$\frac{3}{8}$	0.38	2	0.18	2070	0.92	690	0.31
$\frac{1}{2}$	0.67	$2\frac{1}{2}$	0.25	3655	1.63	1218	0.54
$\frac{5}{8}$	1.08	3	0.36	5720	2.55	1906	0.85
$\frac{3}{4}$	1.55	$3\frac{3}{4}$	0.56	8200	3.66	2730	1.25
$\frac{7}{8}$	2.11	$4\frac{1}{4}$	0.72	11100	4.95	3700	1.65
$1\frac{1}{2}$	2.7	5	1	14220	6.36	4740	2.12
$1\frac{9}{16}$	3.42	$5\frac{3}{4}$	1.32	18000	8	6000	2.68
$1\frac{5}{8}$	4	$6\frac{1}{2}$	1.69	22200	9.9	7400	3.33
$1\frac{11}{16}$	4.84	$7\frac{1}{4}$	2.1	26000	11.6	8960	4
$1\frac{3}{4}$	5.75	8	2.33	30000	13.4	10000	4.5
$1\frac{13}{16}$	6	$8\frac{3}{4}$	2.84	34000	15.2	11300	5
$1\frac{7}{8}$	7.83	$9\frac{1}{2}$	3.30	38000	16.9	12660	5.6
$1\frac{15}{16}$	9.4	10	4.16	41000	18.3	13700	6.1
1 inch.	10.7	$10\frac{1}{2}$	4.6	44800	20	15000	6.7

To ascertain the weight of Cable-laid Ropes.

Multiply the square of the circumference in inches by .036, and the product is the weight in pounds of a foot in length.

To ascertain the Weight of Tarred Ropes and Cables.

Multiply the square of the circumference by 2.13, and divide by 9; the product is the weight of a fathom in pounds.

Or, multiply the square of the circumference by .04, and the product is the weight of a foot.

For the *ultimate strength*, divide the square of the circumference in inches by 5; the product is the weight in tons.

CABLES AND ANCHORS.

TABLE showing the size of Cables and Anchors proportioned to the Tonnage of Vessels.

Tonnage of Vessels.	Cables. Circumference in inches.	Chain Cables Diameter in inches.	Proof in tons.	Weight of Anchor in pounds.	Weight of Chain. fathom of	Weight of a fathom of Cables.
5	3.	$\cdot \frac{5}{16}$	$\frac{3}{4}$	56	$5\frac{1}{4}$	2.1
8	4.	$\cdot \frac{3}{8}$	$1\frac{3}{4}$	84	8.	4.
10	$4\frac{1}{2}$	$\cdot \frac{7}{16}$	$2\frac{1}{8}$	112	11.	4.6
15	$5\frac{1}{2}$	$\cdot \frac{1}{2}$	4.	168	14.	6.5
25	6.	$\cdot \frac{9}{16}$	5.	224	17.	8.4
40	$6\frac{1}{2}$	$\cdot \frac{5}{8}$	6.	336	24.	9.8
60	7.	$\cdot \frac{11}{16}$	7.	392	27.	11.4
75	$7\frac{1}{2}$	$\cdot \frac{3}{4}$	9.	532	30.	13.
100	8.	$\cdot \frac{13}{16}$	10.	616	36.	15.
130	9.	$\cdot \frac{7}{8}$	12.	700	42.	18.9
150	$9\frac{1}{2}$	$\cdot \frac{15}{16}$	14.	840	50.	21.
180	$10\frac{1}{4}$	1.	16.	952	56.	25.7
200	11.	$1\frac{1}{16}$	18.	1176	60.	28.2
240	12.	$1\frac{1}{8}$	20.	1400	70.	33.6
270	$12\frac{1}{2}$	$1\frac{3}{16}$	21.	1456	78.	36.4
320	$13\frac{1}{2}$	$1\frac{1}{4}$	$22\frac{1}{2}$	1680	86.	42.5
360	14.	$1\frac{5}{16}$	25.	1904	96.	45.7
400	$14\frac{1}{2}$	$1\frac{3}{8}$	27.	2072	104.	49.
440	$15\frac{1}{2}$	$1\frac{7}{16}$	30.	2240	115.	56.
480	16.	$1\frac{1}{2}$	33.	2408	125.	59.5
520	$16\frac{1}{2}$	$1\frac{9}{16}$	36.	2800	136.	63.4
570	17.	$1\frac{5}{8}$	39.	3360	144.	67.2
620	$17\frac{1}{2}$	$1\frac{11}{16}$	42.	3920	152.	71.1
680	18.	$1\frac{3}{4}$	45.	4200	161.	75.6
740	19.	$1\frac{13}{16}$	49.	4480	172.	84.2
820	20.	$1\frac{7}{8}$	52.	5600	184.	93.3
900	22.	$1\frac{15}{16}$	56.	6720	196.	112.9
1000	24.	2.	60.	7168	208.	134.6

The proof in the U. S. Naval Service is about $12\frac{1}{2}$ per cent. less than the above for the larger sizes, and from 25 to 30 per cent. for the smaller.

The results of experiments at the U. S. Navy Yard, Washington, D. C., give for the cohesive force of chain iron, per square inch, as follows:

Mean of experiments with good iron, - - - - - 41000 lbs.
 Mean of experiments with best iron, - - - - - 46000 lbs.

To find the weight of Castings, by the weight of Pine Patterns.

RULE.

Multiply the weight of the Pattern by $\left\{ \begin{array}{l} 12 \text{ for Cast Iron,} \\ 13 \text{ " Brass,} \\ 10 \text{ " Lead,} \\ 12.2 \text{ " Tin,} \\ 11.4 \text{ " Zinc,} \end{array} \right\}$ and the product is the weight of the Castings.

Reductions for Round Cores and Core-prints.

Rule. Multiply the square of the diameter by the length of the Core in inches, and the product by 0.017, is the weight of the pine core, to be deducted from the weight of the pattern.

Shrinking of Castings.

Pattern Makers' Rule should be for $\left\{ \begin{array}{l} \text{Cast Iron,} \\ \text{Brass,} \\ \text{Lead,} \\ \text{Tin,} \\ \text{Zinc,} \end{array} \right\} \begin{array}{l} \frac{1}{8} \\ \frac{3}{8} \\ \frac{1}{8} \\ \frac{1}{8} \\ \frac{3}{8} \end{array} \text{ of an inch} \left\{ \begin{array}{l} \text{longer per Linear} \\ \text{Foot.} \end{array} \right.$

Weight and Capacity of Balls.

Diameter in inches.	Capacity in cubic inches.	CAST IRON. Pounds.	LEAD. Pounds.
1.	·5235	·1365	·2147
1½	1·7671	·4607	·7248
2.	4·1887	1·0920	1·7180
2½	8·1812	2·1328	3·3554
3.	14·1371	3·6855	5·7982
3½	22·4492	5·8525	9·2073
4.	33·5103	8·7361	13·744
4½	47·7129	12·4387	19·569
5.	65·4498	17·0628	26·843
5½	87·1137	22·7206	35·729
6.	113·0973	29·4845	46·385
6½	143·7932	37·4528	58·976
7.	179·5943	46·8203	73·659
7½	220·8932	57·5870	90·598
8.	268·0825	69·8892	109·952
8½	321·5550	83·8396	131·383
9.	381·7034	99·5103	156·553
9½	448·9204	117·0338	184·121
10.	523·5987	136·5025	214·749
11.	696·9098	181·7648	285·832
12.	904·7784	235·8763	371·096
13.	1150·346	299·6230	471·806
14.	1436·754	374·5629	589·273
15.	1767·145	460·6959	724·781
16.	2144·660	559·1142	879·616
17.	2572·4 0	670·7168	1055·066
18.	3053·627	796·0825	1252·422
19.	3591·363	936·2708	1472·970
20.	4188·790	1092·0200	1717·995

TO FIND THE LATERAL STRENGTH.

RULE.—Find the coefficients k and x for the material on page 164. Insert this in the formula for the corresponding section of the materials, and the formula gives the absolute strength in pounds.

Expressions $\begin{cases} l, m, \text{ and } n = \text{feet.} \\ b, h, e, \text{ and } t = \text{inches.} \end{cases}$

Example 1. Fig. 208. A rectangular beam of oak fastened in a wall, projects out $l = 6$ feet, 4 inches, $h = 8$ inches, and $b = 5$ inches. How much weight can it bear on the end?

$$W = \frac{30 \times 5 \times 8^2}{6 \cdot 333} = 1509 \text{ pounds, with perfect safety.}$$

Example 2. Fig. 209. A beam of the section **T**, supported at each end is $l = 8 \cdot 75$ feet; dimensions of the section are $h = 10$, $b = 4$, $e = 1 \cdot 25$ and $t = 1 \cdot 5$ inches. The beam is of cast iron, for which $k = 150$ and $x = 2000$. Required how much weight can it bear on the middle $W = ?$ and how much at $n = 3$ feet from one end $W' = ?$

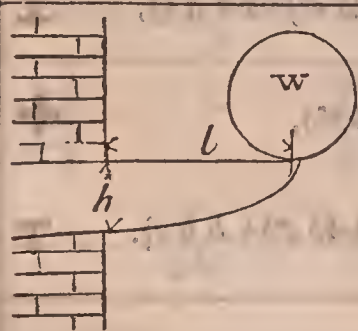
$$W = \frac{4}{l} [k(t h^2 + 0 \cdot 4 b e^2) + x b h e] = \frac{4}{8 \cdot 75} [150(1 \cdot 5 \times 10^2 + 0 \cdot 4 \times 4 \times 1 \cdot 25^2) + 2000 \times 4 \times 10 \times 1 \cdot 25] = 35976 \text{ pounds} = 16 \text{ tons on the middle with perfect safety, and}$$

$$W' = \frac{W l^2}{4 m n} = \frac{35976 \times 8 \cdot 75^2}{4 \times 5 \cdot 75 \times 3} = 39919 \text{ pounds} = 17 \cdot 8 \text{ tons,}$$

at 3 feet from the end.

The coefficient k here given is about one fourth of the extreme strength at which the material would break, which is considered the practical value when the force or load is acting continually for a term of years. When the force is acting at intervals, and greater per centage is desired, $\frac{1}{2}$ of the extreme strength can be relied upon, or $\frac{1}{3}$, if circumstances are pressing, but the latter never to be exceeded.

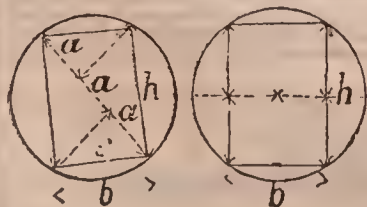
The weight of the materials itself has not here been taken into consideration, for which allowance must be made, if considerable.



206.

A beam fixed in one end and loaded at the other, should have the form of a Parabola in which

l = abscissa, and
 h = ordinate.



207.

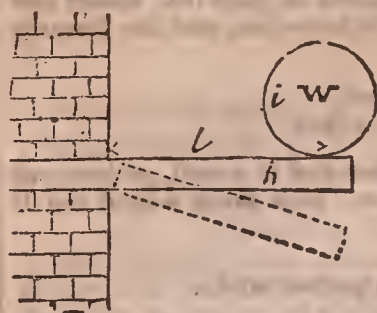
To cut out the stoutest rectangular beam from a log.

1st case, divide the diameter in 3 equal parts, and draw lines at right-angles as represented.

2nd, divide the diameter in 4 equal parts.

1, $h = 1 \cdot 414 b$, non-elastic.
2, $h = 1 \cdot 73 b$, elastic beams.

208.



$$W = \frac{k b h^2}{l},$$



$$W = \frac{1}{l} [k(t h^2 + 2b e^2) + x b h e],$$

I

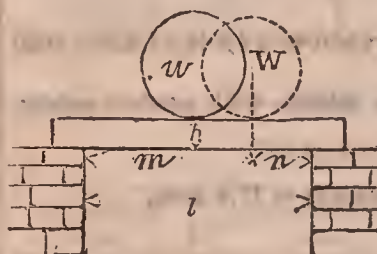
$$W = \frac{k}{l} (t h^2 + 2b e^2),$$

+

$$W = \frac{1}{l} [k(t h^2 + 0.4b e^2) + x b h e],$$

T

209.



$$W = \frac{4k b h^2}{l}, \quad W' = \frac{W l^2}{4m n},$$



$$W = \frac{4}{l} [k(t h^2 + 2b e^2) + x b h e],$$

I

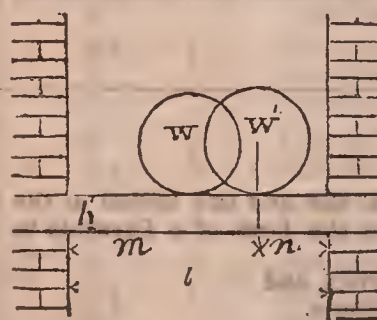
$$W = \frac{4k}{l} (t h^2 + 2b e^2),$$

+

$$W = \frac{4}{l} [k(t h^2 + 0.4b e^2) + x b h e].$$

T

210.



$$W = \frac{8k b h^2}{l}, \quad W' = \frac{W l^2}{4m n},$$



$$W = \frac{8}{l} [k(t h^2 + 2b e^2) + x b h e],$$

I

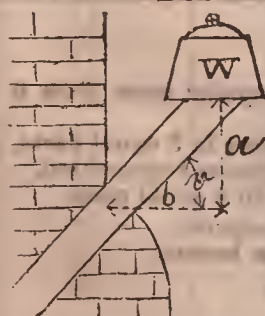
$$W = \frac{8k}{l} (t h^2 + 2b e^2),$$

+

$$W = \frac{8}{l} [k(t h^2 + 0.4b e^2) + b h e].$$

T

211.

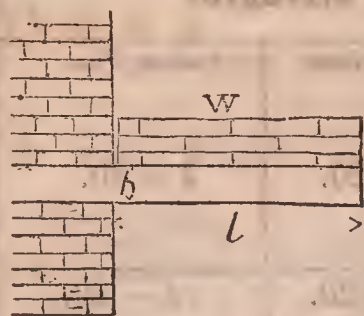


$$W = \frac{h^2 k b}{l \cos. v},$$

$$W = \frac{h^2 h q}{b'},$$

$$h = \sqrt{\frac{W b}{k b'}}$$

212.



$$W = \frac{2k b h^2}{l},$$



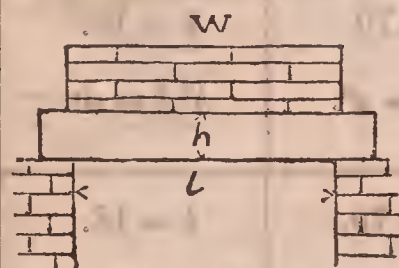
$$W = \frac{2}{l} [k(t h^2 + 2b e^2) + x h b h e], \quad \mathbf{I}$$

$$W = \frac{2k}{l} (t h^2 + 2b e^2),$$



$$W = \frac{2}{l} [k(t h^2 + 0.4b e^2) + x h b e], \quad \mathbf{T}$$

213.



$$W = \frac{8k b h^2}{l},$$



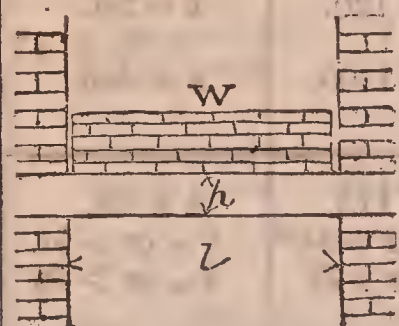
$$W = \frac{8}{l} [k(t h^2 + 2b e^2) + x b h e], \quad \mathbf{I}$$

$$W = \frac{8k}{l} (t h^2 + 2b e^2),$$



$$W = \frac{8}{l} [k(t h^2 + 0.4b e^2) + x b h e], \quad \mathbf{T}$$

214.



$$W = \frac{16k b h^2}{l},$$



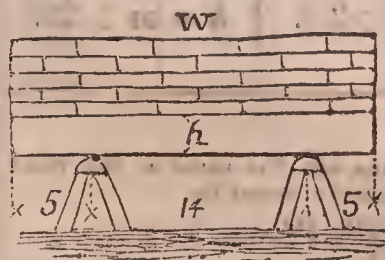
$$W = \frac{16}{l} [k(t h^2 + 2b e^2) + x b h e], \quad \mathbf{I}$$

$$W = \frac{16k}{l} (t h^2 + 2b e^2),$$



$$W = \frac{16}{l} [k(t h^2 + 2b e^2) + x b h e], \quad \mathbf{T}$$

215.



$$W = \frac{912k h^2 b}{25 l},$$

$$h = 5 \sqrt{\frac{W l}{912 k b}}.$$

$$x = \frac{14 l}{24}, \quad y = \frac{5 l}{24}.$$

Coefficients for the Lateral Strength.

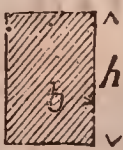


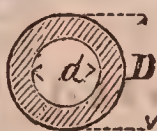
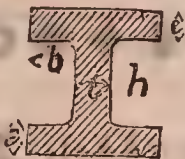

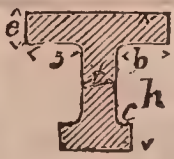
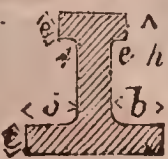
Transverse Section.	Cast Iron.	Wrought Iron.	Oak or Pine.
	$k = 150.$	$k = 120.$	$k = 30.$
	$k = 150.$ $b h^2 = s^3.$	$k = 120.$ $h^2 b = s^3.$	$k = 30.$ $h^2 b = s^3.$
	$k = 88.$ $h^2 b = d^3.$	$k = 70.$ $h^2 b = d^3.$	$k = 18.$ $h^2 b = d^3.$
	$k = 88.$ $h^2 b = D^3 - d^3.$	$k = 70.$ $h^2 b = D^3 - d^3.$	$k = 18.$ $h^2 b = D^3 - d^3.$
	$k = 150.$ $x = 1000.$	$k = 120.$ $x = 1500.$	$k = 30.$ $x = 400.$
	$k = 144.$ $b = \text{or} > 3e.$ $h = \text{or} > 3t.$	$k = 115.$ $b = \text{or} > 3e.$ $h = \text{or} > 3t.$	$k = 29.$ $b = \text{or} > 3e.$ $h = \text{or} > 3t.$
extension.  compression.	$k = 150.$ $x = 1000.$ $b = 5c.$ $h = \text{or} > 2 b.$	$k = 120.$ $x = 1500.$ $b = 5c.$ $h = \text{or} > 2 b.$	$k = 30.$ $x = 400.$ $b = 5c.$ $h = \text{or} > 2 b.$
compression.  extension.	The coefficients and proportions are the same as the next above, but the side marked extension must be up in the Figs. 208. 210. 212. 214. down in the Figs. 209. 213.		

Table III.**ABSOLUTE AND EXTREME STRENGTH OF MATERIALS.**

<i>Kind of Materials.</i>	<i>Coefficient k.</i>			
	<i>Continually for years.</i>	<i>At intervals.</i>	<i>Pressing Circumstances</i>	<i>Extreme. Break asunder.</i>
Iron wrought, - - -	120	162	240	488
“ cast, - - - - -	150	200	300	600
Steel, soft, - - - - -	175	233	350	700
Brass, - - - - -	58	75	113	226
Copper, - - - - -	53	71	106	212
Zinc, - - - - -	15	20	30	61
Tin, - - - - -	17	23	34	69
Lead, - - - - -	4	6	9	18
Ash, - - - - -	45	56	85	170
Hickory, - - - - -	67	90	135	270
Chestnut, sweet, - - -	42	56	85	170
Oak, white, - - - - -	50	66	100	200
“ English, - - - - -	25	33	50	100
“ Canadian, - - - - -	37	49	73	147
Pine, white, - - - - -	34	45	67	135
“ yellow, - - - - -	38	50	75	150
Teak, - - - - -	51	68	102	205

—→←—

BRICKS.

Dimensions.

Common brick $8 \times 4\frac{1}{4} \times 2\frac{1}{2}$ inches = 85 cubic inches.

Front brick $8\frac{1}{4} \times 4\frac{1}{2} \times 2\frac{1}{2}$ „ = 92.8 „ „

When laid in a wall with cement it occupies a space of

Common brick $8\frac{1}{4} \times 4\frac{1}{2} \times 2\frac{3}{4}$ inches = 102 cubic inches.

Front brick $8\frac{1}{2} \times 4\frac{3}{4} \times 2\frac{3}{4}$ „ = 111 „ „

Weight and Bulk of Bricks.

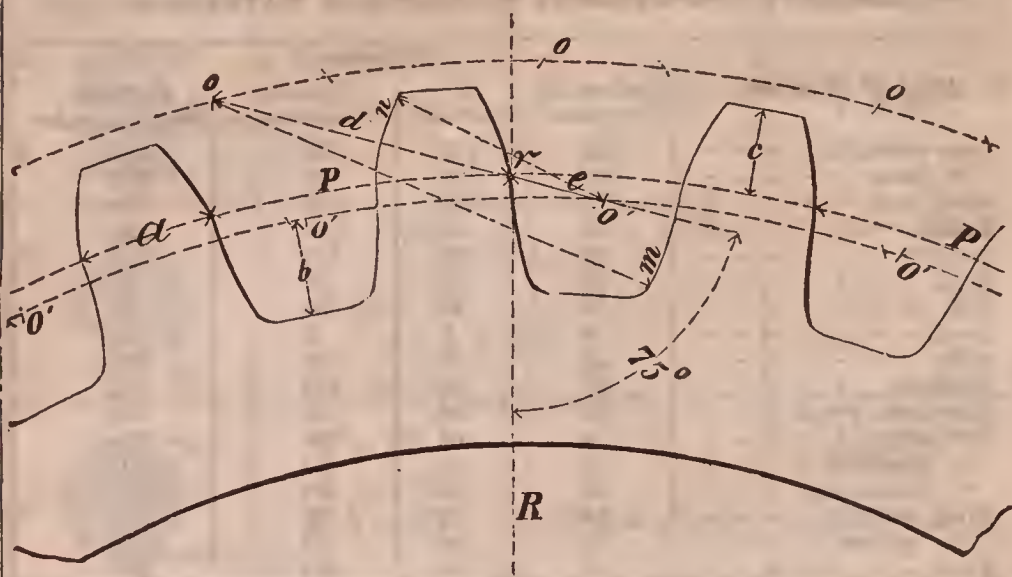
Tons.	Pounds.	Cub. feet.	Number of bricks,			
			by itself.		in wall with cement	
			C. Brick.	F. Brick.	C. Brick.	F. Brick.
1	2240	22.4	448	416.6	381	347
0.04464	100	1	20	18.6	17	15½
2.23	5000	50.00	1000	930	850	772
2.4	5376	53.76	1075	1000	914	834
2.62	5872	58.72	1130	1100	1000	913
2.88	6451	64.51	1240	1200	1100	1000

—→←—

WOOD.

A Cord of wood is 4 feet wide, 4 feet high, and 8 feet deep.
128 cubic feet.

GEARING.



Letters denote.

P = pitch,—the distances between the centres of two teeth in the pitch circle.

D = diameter

C = circumference

M = number of teeth

N = number of revolutions

d = diameter

c = circumference

m = number of teeth

n = number of revolutions

} of the wheel.

} of the pinion.

$$\text{Pitch} \left\{ \begin{array}{l} P = \frac{C}{M} \quad - \quad 1 \\ P = \frac{\pi D}{M} \quad - \quad 2 \end{array} \right.$$

$$\text{No. of teeth} \left\{ \begin{array}{l} M = \frac{C}{P} \quad - \quad 3 \\ M = \frac{\pi D}{P} \quad - \quad 4 \end{array} \right.$$

$$\text{Circum.} \left\{ \begin{array}{l} C = P M \quad - \quad 5 \\ C = \pi D \quad - \quad 6 \end{array} \right.$$

$$\text{Diameter} \left\{ \begin{array}{l} D = \frac{P M}{\pi} \quad - \quad 7 \\ D = \frac{C}{\pi} \quad - \quad 8 \end{array} \right.$$

$$D : d = C : c = M : m = n : N$$

Example 1. A wheel of $D = 40$ inches in diameter, is to have $M = 75$ teeth. Required the pitch $P = ?$

$$\text{Formula 2. Pitch } P = \frac{3.14 \times 40}{75} = 1.66 \text{ inches nearly.}$$

Example 2. The pitch of teeth in a wheel, is to be $P = 1.71$ inches, and having $M = 48$ teeth. Required the diameter $D = ?$ of the wheel.

$$\text{Formula 7. Diam. } D = \frac{1.71 \times 48}{3.14} = 26.14 \text{ in. of the pitch circle}$$

Construction of Teeth for Wheels.

Draw the radius Rr , and pitch circle PP . Through r draw the line oo' at an angle of 75° to the radius Rr .

Half the angle between two teeth in the	{	wheel, $v = \frac{180}{M}$	1
		pinion, $V = \frac{180}{m}$	2

$$D : d = \sin. V : \sin. v.$$

Diameter of the	{	wheel, $D = \frac{d \sin. V}{\sin. v}$	3
		pinion, $d = \frac{D \sin. v}{\sin. V}$	4

Pitch (chord) of teeth in the pitch circle	{	wheel, $P = D \sin. v$	5
		pinion, $P = d \sin. V$	6

Approximate pitch in the wheel $P = 0.028 D$	7
--	---	---	---	---

Number of teeth about	{	wheel, $M = \frac{3.14 D}{P}$	8
		pinion, $m = \frac{d M}{D}$	9

Thickness of tooth, $a = 0.46 P^*$	10
------------------------------------	---	---	---	---	----

Bottom clearance, $b = 0.4 P$	11
---------------------------------	---	---	---	---	----

Depth to pitch line, $c = 0.3 P$	12
------------------------------------	---	---	---	---	----

Distance $r o$,	$d = \frac{P(m+6)}{2(m-11)}$	13
------------------	--------------------------------	---	---	---	---	----

Distance $r o'$,	$e = 0.11 P \sqrt[3]{m}$	14
-------------------	--------------------------	---	---	---	---	----

* If a wheel of more than 80 teeth is to gear a pinion of less than 20 teeth, and the wheel and pinion are of the same kind of materials; take the thickness

of the tooth in the	{	wheel, $a = P \left(0.42 + \frac{m}{700} \right)$.	.	15
		pinion, $a = 0.5 P \left(1 - \frac{m}{350} \right)$.	.	16

A rack is to be considered as a wheel of 200 teeth.

Example with Plate IV.

Example. A wheel of $D = 48$ inches diameter is to gear a pinion about 8 revolutions to 1. Required a complete construction of the gearing?

Approximate pitch $P = 0.028 \times 48 = 1.34$ in. - - - 7

Number of teeth in the $\left\{ \begin{array}{l} \text{wheel, } M = \frac{3.14 \times 48}{1.34} = 112. \quad - \quad 8 \\ \text{pinion, } m = \frac{112}{8} = 14 \quad - \quad 9 \end{array} \right.$

Half the angle between two teeth in the $\left\{ \begin{array}{l} \text{wheel, } v = \frac{180}{112} = 1^\circ 36'. \sin = 0.028. \quad 1 \\ \text{pinion } V = \frac{180}{14} = 12^\circ 51'. \sin = 0.2224. \quad 2 \end{array} \right.$

Diameter of pinion $d = \frac{48 \times 0.028}{0.2224} = 6.043$ in. - 4

Draw the pitch circle for the wheel and pinion so that they tangent one another at r on a straight line between the centres of the circles.

Pitch in the gearing $P = 48 \times 0.028 = 1.344$ in. - 5

Take this chordal pitch in a pair of compasses, and set it off in the pitch circles.

Thickness of tooth $\left\{ \begin{array}{l} \text{wheel } a = 1.344 \left(0.42 + \frac{14}{700} \right) = 0.592 \text{ in.} \quad 15 \\ \text{pinion } a = 0.5 \quad 1.344 \left(1 - \frac{14}{350} \right) = 0.645 \text{ in.} \quad 16 \end{array} \right.$

Set off the thickness of tooth in the corresponding pitch circles.

Bottom clearance $b = 0.4 \times 1.344 = 0.5376$ in. - - 11

Depth to pitch line $c = 0.3 \quad 1.344 = 0.4032$ in. - - 12

Distances $r o$ and $r o'$ in the wheel $\left\{ \begin{array}{l} d = \frac{1.344(112 + 6)}{2(112 - 11)} = 0.7851 \text{ in.} \quad - \quad 13 \\ e = 0.11 \times 1.344 \sqrt{112} = 0.7126 \text{ in.} \quad 14 \end{array} \right.$

Set off these distances on the line $o o'$ from r , d beyond and e within the pitch circle for the wheel; then o is the centre and $o m$ radius for the flank m . o' the centre and $o' n$ radius for the face n . Draw circles through o and o' concentric with the pitch circle of the wheel.

Distances $r o$ and $r o'$ in the pinion $\left\{ \begin{array}{l} d = \frac{1.344(14 + 6)}{2(14 - 11)} = 4.48 \text{ in.} \quad - \quad 13 \\ e = 0.11 \times 1.344 \sqrt{14} = 0.356 \text{ in.} \quad 14 \end{array} \right.$

Proceed with the pinion similar as the wheel

On the plate is a scale of inches and decimals, which will be convenient for the above measurements.

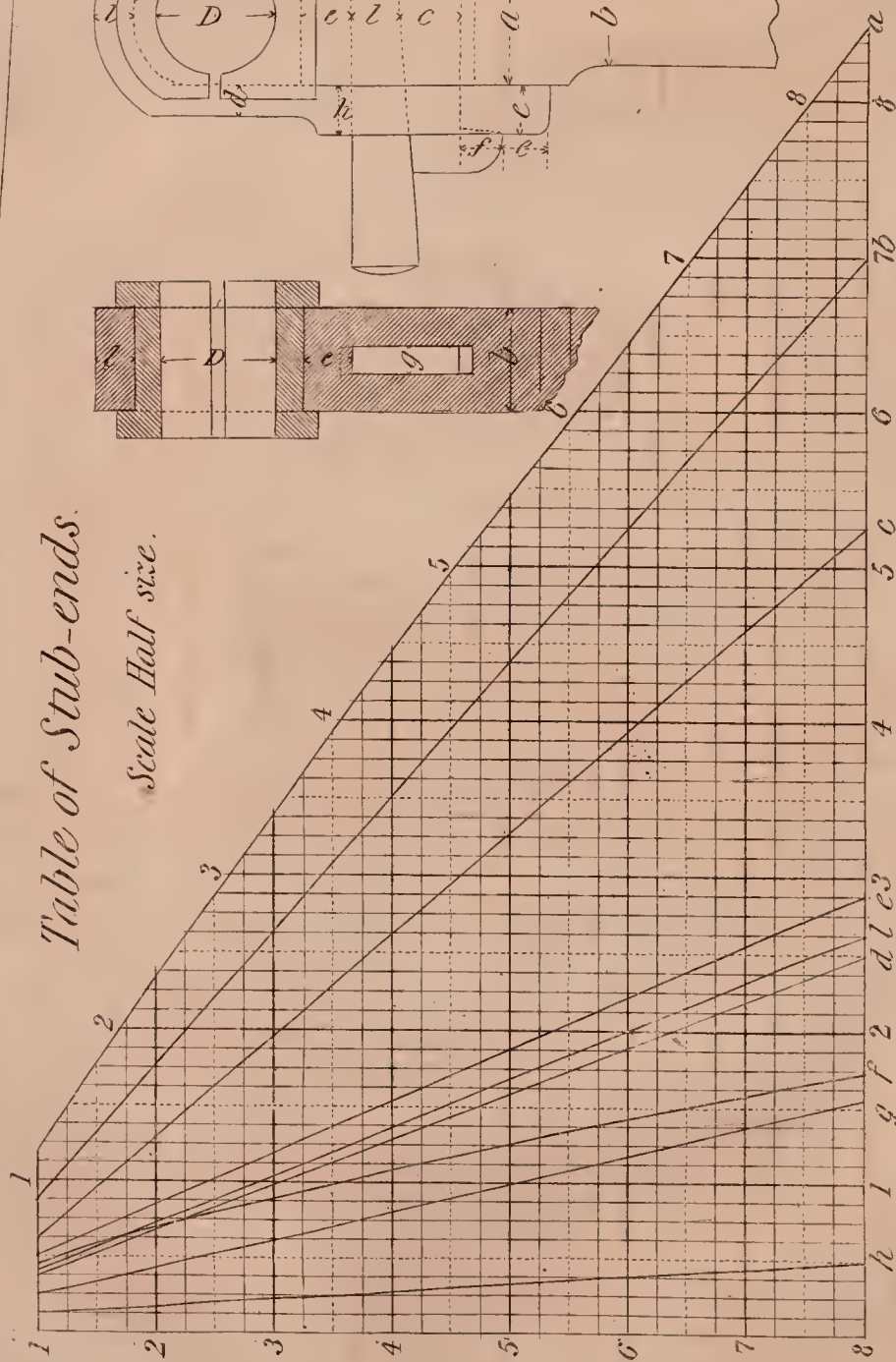


J. W. Nyström.

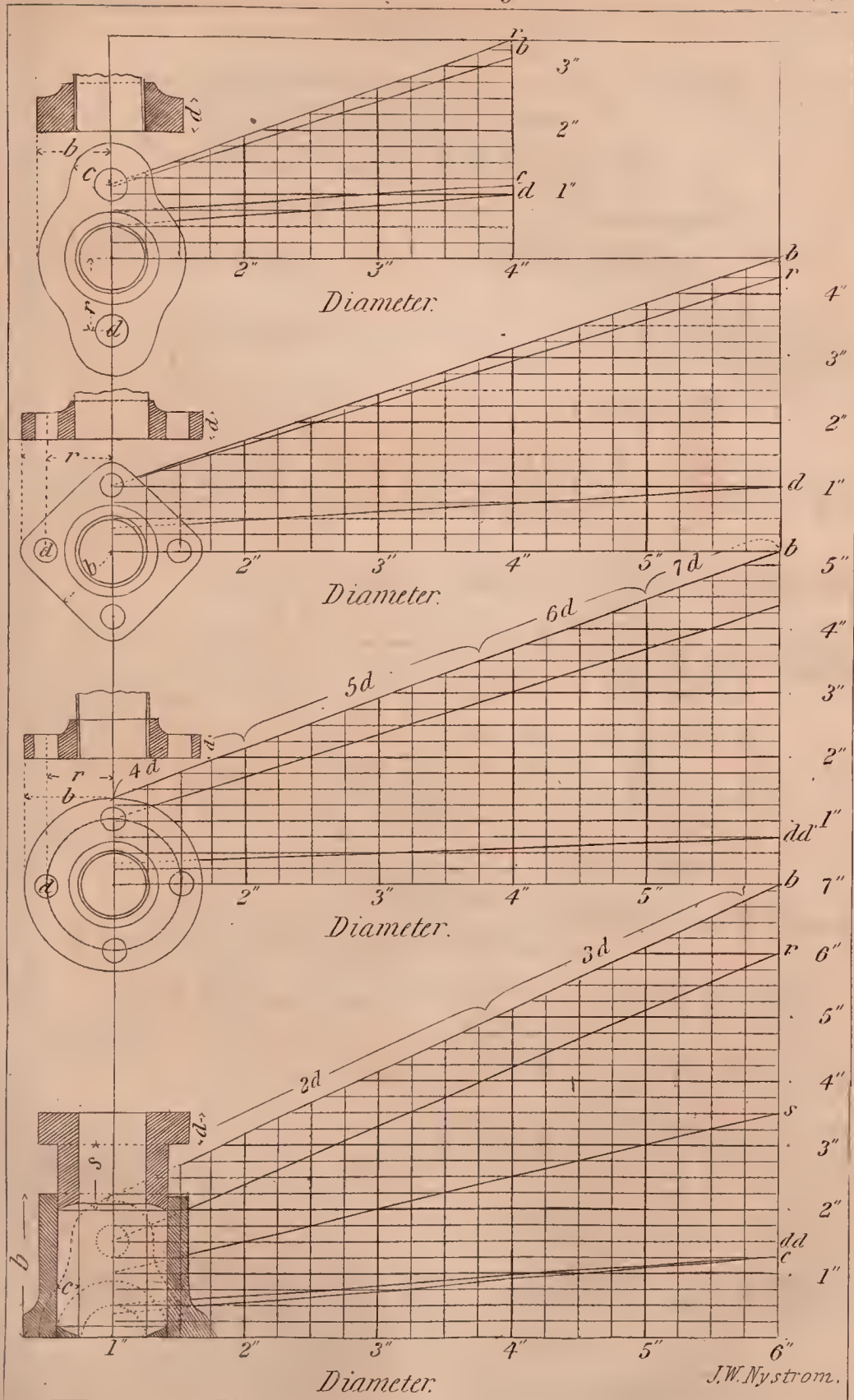
Angle of Key.

Table of Stub-ends.

Scale Half size.



Journals Diameter.



Strength of Teeth.*Letters denote.* S = strain on the teeth at the pitch-line in pounds. a = thickness, (see figure), } inches. h = breadth of the teeth, } v = velocity of the teeth in feet per second. H = Horse power transmitted by the teeth.

$$\text{Thickness} \begin{cases} a = 0.025\sqrt{S}, \\ a = 0.663\sqrt{\frac{H}{v}}, \end{cases}$$

$$\text{Strain} \begin{cases} S = 1600 a^2, \\ S = 300 h a. \end{cases}$$

$$\text{Pitch.....} \begin{cases} p = 0.0544\sqrt{S}, \\ p = 1.44\sqrt{\frac{H}{v}}, \end{cases}$$

$$\text{Horses} \begin{cases} H = 2.275 \sqrt{a v}, \\ H = 0.48 \sqrt{p v}. \end{cases}$$

Abrasion included in these formulas and the breadth $h = 2.5p$.

When great strain is required on the teeth, and the diameter or pitch of the wheel is limited, it is necessary to increase the breadth h proportionally, which will be thus,

$$h = \frac{S}{653p} = \frac{S}{300a} = \frac{Sm}{943D} = \frac{Hm}{1.347Dv} = \frac{H}{3.429av} = \frac{H}{0.934pv},$$

Example. The pinion-wheel on a propeller shaft is to be $D = 48$ inches in diameter, and to have $m = 36$ teeth; it is driven by a pair of engines $H = 450$ horses, and the propeller to make $n = 50$ revolutions per minute. Required the breadth of the teeth $h = ?$ The velocity at pitch circle will be,

$$v = \frac{3.14 \times 4 \times 50}{60} = 10.5 \text{ feet per second, nearly, and}$$

$$h = \frac{Hm}{1.347Dv} = \frac{450 \times 36}{1.347 \times 48 \times 10.5} = 23.88, \text{ or } 24 \text{ inches, nearly.}$$

To Find the Diameter of Axles and Shaft.*Letters denote,*

d = diameter in inches, in the bearing; and the length of the bearing $1.5d$.
 W = weight in pounds, acting in the bearing.

$$\text{Water-wheels} \begin{cases} d = \frac{\sqrt{W}}{18} \text{ of cast iron.} \\ d = \frac{\sqrt{W}}{21} \text{ of wrought iron.} \end{cases}$$

$$\text{Common Machinery in good order.} \begin{cases} d = \frac{\sqrt{W}}{24} \text{ of cast iron.} \\ d = \frac{\sqrt{W}}{28} \text{ of wrought iron.} \end{cases}$$

Example 1. A water wheel weighs 58,680 pounds, and supported in two bearings. Required the diameter of the wheel axles? The weight acting in each bearing will be $58680 : 2 = 29340$ pounds, and

$$\text{diameter } d = \frac{\sqrt{29340}}{21} = 8.15 \text{ inches of wrought iron.}$$

Example 2. Fig. 226, page 185. Required the diameter of the axle in the wheel, when the weights $P + Q = 4864$ pounds? If the wheel is supported in two bearings $W = 4864 : 2 = 2432$ pounds.

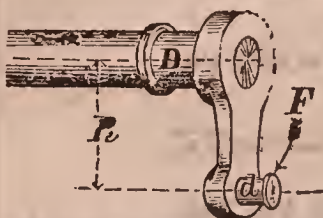
$$\text{diameter } d = \frac{\sqrt{2432}}{28} = 1.76 \text{ inches of wrought iron.}$$

Example 3. The pressure on the steam piston, in a walking beam engine is 25000 pounds. Required the diameter of the beam journals?

$$\text{diameter } d = \frac{\sqrt{25000}}{28} = 5.64 \text{ inches the centre one.}$$

$$d = \frac{\sqrt{12500}}{28} = 4 \text{ inches at the ends.}$$

In this example it is supposed that the beam is worked by a fork connecting rod.

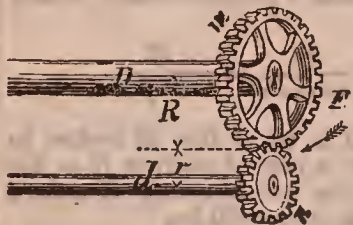


$$D = \frac{\sqrt[3]{F R}}{4} = 5 \sqrt[3]{\frac{H}{n}}$$

D = inches wrought iron.

R = radius of crank in feet.

F = force from the steam piston, lbs.



$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$D = 4.35 \sqrt[3]{\frac{H}{n}}$$

H = horse-power transmitted.

n = number revolutions per minute.

When an axle or shaft not only serves as a fulcrum, but effect is transmitted by the act of twisting it, the diameter is to be calculated as follow.

Example 4. The pressure on the piston in a steam engine is $F = 45,600$ pounds, applied direct on a crank of $R = 3$ feet radius. Required the diameter of the shaft and crank pin?

$$\text{Diameter of the shaft } D = \frac{\sqrt[3]{45600 \times 3}}{4} = 12.9 \text{ inches.}$$

$$\text{Diameter of the crank pin } d = \frac{\sqrt{45600}}{28} = 7.63 \text{ inches.}$$

Example 5. A steam engine of 368 horses is to make 32 revolutions per minute. Required the diameter of the main shaft?

$$\text{Diameter } D = 5 \sqrt[3]{\frac{368}{32}} = 11\frac{1}{4} \text{ inches.}$$

Example 6. A cog wheel of $R = 6.5$ feet radius is to gear with a pinion of $r = 1.25$ feet radius, and to transmit an effect of 231 horses with 42 revolutions per minute. Required the diameter of the wheel and pinion shafts? The force F is acting uniformly at the periphery.

$$\text{Diameter of wheel shaft } D = 4.35 \sqrt[3]{\frac{231}{42}} = 7.66 \text{ inches.}$$

$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$\text{Diameter of pinion shaft } d = 7.66 \sqrt[3]{\frac{1.25}{6.5}} = 4.41 \text{ inches.}$$

ALLOYS.

Letters denote.

A = Antimony, *B* = Bismuth, *C* = Copper, *G* = Gold, *I* = Iron, *L* = Lead, *N* = Nickel, *S* = Silver, *T* = Tin, and *Z* = Zinc.

Brass, yellow, - - - - -	2C, 1Z.
" rolled, - - - - -	32C, 10Z, 1·5T.
Brass-casting, common, - - - - -	20C, 1·25Z, 2·5T.
" hard, - - - - -	25C, 2Z, 4·5T.
Brass-Propellers, (large), - - - - -	8C, 0·5Z, 1T.
Gun-metal, - - - - -	8C, 1T.
Copper-flanges, for pipes, - - - - -	9C, 1Z, 0·26T.
Brass that bears soldering well, - - - - -	2C, 0·75Z.
Muntz's metal can be rolled and worked at red heat, - - - - -	6C, 4Z.
Statuary, - - - - -	91·4C, 5·53Z, 1·7T, 1·37L.
German Silver, - - - - -	20C, 15·8N, 12·7Z, 1·3I.
Frick's Imitative Silver, - - - - -	53·39C, 17·4N, 13Z.
Medals, - - - - -	100C, 8Z.
Pinchbeck, - - - - -	5C, 1Z.
Chinese Silver, - - - - -	65·2C, 19·5Z, 13N, 2·5S, 12 cobalt of I.
Britannia metal, - - - - -	1Z, 1A }
When fused add, - - - - -	1A, 1B } 1Z, 2A, 1B.
Babbitt's anti-Attrition metal, - - - - -	25T, 2A 0·5C.
<i>The Tin of the best quality of Banca, is to be added gradually to the melted composition.</i>	
Bell-metal, large, - - - - -	3C, 1T.
" small, - - - - -	4G, 1T.

Gold Metal.

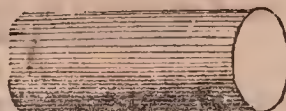
$x = 21C, 13T$ }
 $y = 62C, 9Z,$ } To be melted separately.
 Gold = $71y + 9x$, this makes a brilliant composition.

Solders.

Newton's fusible alloys, - - - - -	8B, 5L, 3T, melts at 212°.
Rose's " " - - - - -	2B, 1L, 1T, " 201°.
A more fusible composition - - - - -	5B; 3L, 2T, " 199°.
Tin solder, coarse, - - - - -	1T, 3L, " 500°.
" ordinary, - - - - -	2T, 1L, " 360°.
Soft Spelter-solder, for common brass works, - - - - -	1C, 1Z.
Hard " for iron, - - - - -	2C, 1Z.
Solder for Steel, - - - - -	19S, 3C, 1Z.
Solder for fine brass works, - - - - -	1S, 8C, 8Z.
Pewterer's soft solder, - - - - -	2B, 4L, 3T.
" " - - - - -	1B, 1L, 2T.
Gold Solder, - - - - -	24G, 2S, 1C.
Silver solder, hard, - - - - -	4S, 1C.
" soft, - - - - -	2S, 1 brass wire.

Tempering Steel.

	Tem. Fah.
Yellow, very faint, for lancets - - - - -	430°
" pale straw, for razors scalpels - - - - -	450°
" full, for penknives and chisels for hard cast iron - - - - -	470°
Brown, for scissors and chisels for wrought iron - - - - -	490°
Red, for carpenter tools in general - - - - -	510°
Purple, for fine watch springs and table knives - - - - -	530°
Blue, bright, for swords, lock springs - - - - -	550°
" full, for daggers, fine saws, needles - - - - -	560°
" dark, for common saws - - - - -	600°



Side in inches.	Weight in pounds.	Side in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.
$\frac{1}{16}$	0·013	$3\frac{5}{8}$	44·418	$\frac{1}{16}$	0·010	$3\frac{5}{8}$	34·886
$\frac{1}{8}$	0·53	$3\frac{3}{4}$	47·534	$\frac{1}{8}$	0·041	$3\frac{3}{4}$	37·332
$\frac{3}{16}$	0·118	$3\frac{7}{8}$	50·756	$\frac{3}{16}$	0·119	$3\frac{7}{8}$	39·864
$\frac{1}{4}$	0·211	4	54·084	$\frac{1}{4}$	0·165	4	42·464
$\frac{3}{8}$	0·475	$4\frac{1}{8}$	57·517	$\frac{3}{8}$	0·373	$4\frac{1}{8}$	45·174
$\frac{1}{2}$	0·845	$4\frac{1}{4}$	61·055	$\frac{1}{2}$	0·663	$4\frac{1}{4}$	47·952
$\frac{5}{8}$	1·320	$4\frac{3}{8}$	64·700	$\frac{5}{8}$	1·043	$4\frac{3}{8}$	50·815
$\frac{3}{4}$	1·901	$4\frac{1}{2}$	68·448	$\frac{3}{4}$	1·493	$4\frac{1}{2}$	53·760
$\frac{7}{8}$	2·588	$4\frac{5}{8}$	72·305	$\frac{7}{8}$	2·032	$4\frac{5}{8}$	56·788
1	3·380	$4\frac{3}{4}$	76·264	1	2·654	$4\frac{3}{4}$	59·900
$1\frac{1}{8}$	4·278	$4\frac{7}{8}$	80·333	$1\frac{1}{8}$	3·360	$4\frac{7}{8}$	63·094
$1\frac{1}{4}$	5·280	5	84·480	$1\frac{1}{4}$	4·172	5	66·752
$1\frac{3}{8}$	6·390	$5\frac{1}{8}$	88·784	$1\frac{3}{8}$	5·019	$5\frac{1}{8}$	69·731
$1\frac{1}{2}$	7·604	$5\frac{1}{4}$	93·168	$1\frac{1}{2}$	5·972	$5\frac{1}{4}$	73·172
$1\frac{5}{8}$	8·926	$5\frac{3}{8}$	97·657	$1\frac{5}{8}$	7·010	$5\frac{3}{8}$	76·700
$1\frac{3}{4}$	10·325	$5\frac{1}{2}$	102·24	$1\frac{3}{4}$	8·128	$5\frac{1}{2}$	80·304
$1\frac{7}{8}$	11·883	$5\frac{5}{8}$	106·95	$1\frac{7}{8}$	9·333	$5\frac{5}{8}$	84·001
2	13·520	$5\frac{3}{4}$	111·75	2	10·616	$5\frac{3}{4}$	87·776
$2\frac{1}{8}$	15·263	$5\frac{7}{8}$	116·67	$2\frac{1}{8}$	11·988	$5\frac{7}{8}$	91·634
$2\frac{1}{4}$	17·112	6	121·66	$2\frac{1}{4}$	13·440	6	95·552
$2\frac{3}{8}$	19·066	$6\frac{1}{4}$	132·04	$2\frac{3}{8}$	14·975	$6\frac{1}{4}$	103·70
$2\frac{1}{2}$	21·120	$6\frac{1}{2}$	142·82	$2\frac{1}{2}$	16·688	$6\frac{1}{2}$	112·16
$2\frac{5}{8}$	23·292	$6\frac{3}{4}$	154·01	$2\frac{5}{8}$	18·293	$6\frac{3}{4}$	120·96
$2\frac{3}{4}$	25·56	7	165·63	$2\frac{3}{4}$	20·076	7	130·05
$2\frac{7}{8}$	27·939	$7\frac{1}{2}$	190·14	$2\frac{7}{8}$	21·944	$7\frac{1}{2}$	149·33
3	30·416	8	216·34	3	23·888	8	169·85
$3\frac{1}{8}$	33·010	$8\frac{1}{2}$	244·22	$3\frac{1}{8}$	25·926	$8\frac{1}{2}$	191·81
$3\frac{1}{4}$	35·704	9	273·79	$3\frac{1}{4}$	28·040	9	215·04
$3\frac{3}{8}$	38·503	10	337·92	$3\frac{3}{8}$	30·240	10	266·29
$3\frac{1}{2}$	41·408	12	486·66	$3\frac{1}{2}$	32·512	12	382·21

Cements for Cast Iron.

Two ounces Sal-ammoniac, one ounce Sulphur and sixteen ounces of borings or filings of cast Iron, to be mixed well in a mortar, and kept dry. When required for use take one part of this powder to twenty parts of clear iron borings or filings, mixed thoroughly in a mortar, make the mixture into a stiff paste with a little water, and then it is ready for use. A little fine grindstone sand improves the cement.

Or one ounce of Sal-ammoniac to one hundred weight of Iron borings. No heat allowed to it.

The Cubic contents of the joint in inches, divided by five, is the weight of dry borings in pounds Avoir. required to make cement to fill the joint nearly.

Cement for Stone and Brick work.

Two parts Ashes, three of Clay, and one of Sand, mixed with oil, will resist weather equal to marble.

Brown Mortar.

One part Thomaston lime, two of Sand, and a small quantity of Hair.

Hydraulic Mortar.

Three parts of Lime, four Puzzolana, one Smithy Ashes, two of Sand, and four parts of rolled stone or shingles.

Diameter inches.	Thickness 16th in.	Copper pipe pounds.	Iron pipe pounds.	Diameter inches.	Thickness 16th in.	Copper pipe pounds.	Iron pipe pounds.
5	3	12.50	10.96	9½	4	30.59	26.81
5	4	16.88	14.78	10	4	32.21	28.21
5¼	3	13.15	11.52	10½	4	33.94	29.70
5¼	4	17.75	15.55	11	4	35.20	30.84
5½	3	13.63	11.94	11½	4	36.94	32.35
5½	4	18.39	16.07	12	4	38.45	33.67
5¾	3	14.25	12.48	13	4	41.45	36.30
5¾	4	19.25	16.86	14	4	44.64	39.11
6	3	14.76	12.94	14	5	55.88	43.97
6	4	19.91	17.43	15	4	47.64	41.74
6¼	3	15.36	13.46	15	5	59.59	52.20
6¼	4	20.75	18.16	16	4	50.75	44.45
6½	3	15.90	13.93	16	5	63.47	55.60
6½	4	21.41	18.75	17	4	53.86	47.15
6¾	3	16.50	14.45	17	5	67.34	59.00
6¾	4	22.25	19.70	18	4	57.04	50.00
7	3	17.03	14.93	18	5	71.26	62.41
7	4	22.93	20.07	19	4	60.14	52.65
7¼	3	17.65	15.45	19	5	75.23	65.90
7¼	4	23.74	20.79	20	4	62.51	54.74
7½	3	18.32	16.05	20	5	78.21	68.5
7½	4	24.45	21.40	21	5	82.98	72.64
7¾	3	18.95	16.60	22	5	86.77	76.00
7¾	4	25.28	22.13	23	5	90.57	79.34
8	3	29.42	17.03	24	5	94.31	82.60
8	4	25.96	22.72	26	5	101.9	89.32
8½	3	20.58	18.03	28	5	109.4	95.68
8½	4	27.47	24.07	30	5	117.0	102.4
9	4	28.98	25.38	36	5	140.0	122.5

Weight of Cast Iron Cylinders per Foot.

Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.
¾	1.39	2¾	18.74	4¾	55.92	7½	139.4
7⁄8	1.88	2⅞	20.48	4⅞	58.72	7¾	148.87
1 in.	2.47	3 in.	22.35	5 in.	61.96	8 in.	158.63
1⅛	3.13	3⅛	24.20	5⅛	64.66	8¼	168.15
1¼	3.87	3¼	26.18	5¼	68.31	8½	179.1
1⅜	4.68	3⅜	28.23	5⅜	71.00	8¾	189.0
1½	5.57	3½	30.36	5½	74.98	9 in.	200.8
1⅝	6.54	3⅝	32.57	5⅝	78.65	9¼	211.12
1¾	7.59	3¾	34.85	5¾	81.95	9½	223.7
1⅞	8.71	3⅞	37.21	5⅞	85.81	9¾	235.3
2 in.	9.91	4 in.	39.66	6 in.	89.23	10 in.	247.9
1⅞	11.19	4⅞	41.80	6¼	96.82	10½	273.27
2¼	12.54	4¼	44.77	6½	104.7	11 in.	299.9
2⅜	13.98	4⅜	47.00	6¾	112.9	11½	327.8
2½	15.49	4½	50.19	7 in.	112.4	12 in.	356.9
2⅝	17.08	4⅝	52.71	7¼	130.28	13 in.	418.9

Thickness of Metal.

Bore.	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
1	3.06	5.05					
1 $\frac{1}{4}$	3.67	6.00					
1 $\frac{1}{2}$	6.89	6.89	9.81				
1 $\frac{3}{4}$		7.80	11.04				
2		8.74	12.23	16.0			
2 $\frac{1}{4}$		9.65	13.48	17.52			
2 $\frac{1}{2}$		10.57	14.66	19.05	23.8		
2 $\frac{3}{4}$		11.54	15.91	20.59	25.68		
3		12.28	17.15	22.15	27.56	33.30	39.31
3 $\frac{1}{4}$		13.24	18.40	23.72	29.64	35.46	41.77
3 $\frac{1}{2}$		14.20	19.66	25.27	31.20	37.63	44.23
3 $\frac{3}{4}$		15.50	20.90	26.83	33.07	39.77	46.68
4		16.80	22.05	28.28	34.94	41.92	49.14
4 $\frac{1}{4}$		17.41	23.35	29.85	36.73	44.05	51.57
4 $\frac{1}{2}$		18.00	24.49	31.40	38.58	46.19	54.00
4 $\frac{3}{4}$		18.89	25.70	32.91	40.43	48.34	56.45
5		19.79	26.94	34.34	42.28	50.50	58.90
5 $\frac{1}{4}$		21.54	29.40	37.44	45.94	54.81	63.82
6		23.42	31.82	40.56	49.60	58.96	68.70
6 $\frac{1}{4}$		25.26	34.32	43.68	53.30	63.18	73.40
7		27.15	36.66	46.80	56.96	67.60	78.39
7 $\frac{1}{4}$		28.92	39.22	49.92	60.48	71.76	83.28
8		30.76	41.64	52.68	64.27	76.12	88.20
8 $\frac{1}{4}$		32.82	44.11	56.16	68.00	80.50	93.28
9		34.45	46.50	58.92	71.70	84.70	97.98
9 $\frac{1}{4}$		36.26	48.98	62.02	75.32	88.98	102.9
10		38.15	51.46	65.08	78.99	93.24	108.8
10 $\frac{1}{4}$			53.88	68.14	82.68	97.44	112.7
11			56.34	71.19	86.40	101.83	117.6
11 $\frac{1}{4}$			58.82	74.28	90.06	106.1	122.6
12			61.26	77.36	93.70	110.5	127.4
12 $\frac{1}{4}$			63.70	80.40	97.40	114.7	132.5
13			66.14	83.46	101.1	118.9	137.3
13 $\frac{1}{4}$			68.64	86.55	104.8	123.3	137.28
14			71.07	89.61	108.46	127.6	147.0
14 $\frac{1}{4}$			73.72	92.66	112.1	131.9	151.9
15			75.96	95.72	115.8	136.2	156.8
15 $\frac{1}{4}$			78.40	98.78	119.5	140.4	161.8
16			80.87	101.8	123.1	144.8	166.6
16 $\frac{1}{4}$			83.30	104.8	126.8	149.0	171.6
17			85.73	107.9	130.5	153.3	176.6
17 $\frac{1}{4}$			88.23	111.1	134.2	157.6	181.3
18				114.1	137.8	169.9	186.2
19				120.2	145.2	170.5	195.9
20				126.3	152.5	179.0	205.8
21				132.5	159.8	187.6	215.5
22				138.6	167.2	196.5	225.4
23				144.8	174.6	204.8	235.3
24				150.8	181.9	213.28	245.1
26					196.6	230.6	264.7
28					211.3	247.6	284.3
30					226.2	264.8	303.9

Weight of Flat Rolled Iron per Foot,

Look first for the thickness, and follow that line until the column where the breadth is on the top; is the number of pounds per foot.

Look first for the thickness, and follow that line until the column where the breadth is on the top; is the number of pounds per foot.

Weight of Flat Rolled Iron per Foot,

RULE.—Look first for the thickness, and follow the line until the column where the breadth is on the top, is the number of pounds per foot.

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5½											
5											
4½											
4											
3½											
3											
2½											
2											
1½											
1											
¾											
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1/8											
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1/32											
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1/6070											

Weight of Flat Rolled Iron per Foot,

The thickness is in the first column, and the breadth in the top line.

	2½	2⅝	2⅞	3	3⅛	3¼	3½	3⅝	3⅞	3⅞	4	4¼	4½	4¾	5	5¼	5½	5¾	6		
2⅝	20.06	21.07	22.07	23.07	24.07	25.07	26.08	27.18	28.09	28.84	29.59	30.84	32.10	34.11	36.12	38.12	40.13	42.13	44.14	45.95	47.76
2¾	19.01	19.96	20.91	21.86	22.81	23.76	24.71	25.66	26.61	27.56	28.51	29.46	30.41	32.32	34.22	36.12	38.02	39.92	41.82	43.32	44.82
2⅞	17.95	18.85	19.75	20.65	21.54	22.44	23.34	24.23	25.13	25.78	26.43	27.57	28.72	30.52	32.31	34.11	35.90	37.70	39.49	41.09	42.69
3	16.89	17.74	18.58	19.43	20.28	21.12	21.97	22.81	23.65	24.50	25.35	26.19	27.04	28.72	30.41	32.10	33.79	35.48	37.17	38.86	39.55
3⅛	15.84	16.63	17.42	18.22	19.01	19.80	20.59	21.38	22.18	22.97	23.76	24.55	25.35	26.92	28.51	30.09	31.68	33.26	34.85	36.43	38.01
3¼	14.78	15.52	16.26	17.00	17.74	18.48	19.22	19.96	20.70	21.44	22.18	22.92	23.66	25.13	26.61	28.09	29.57	31.05	32.53	34.01	36.49
3½	13.73	14.42	15.10	15.79	16.47	17.16	17.85	18.53	19.22	19.90	20.59	21.28	21.97	23.33	24.71	26.08	27.46	28.83	30.20	31.53	32.86
3⅝	12.67	13.31	13.94	14.57	15.21	15.84	16.47	17.10	17.74	18.37	19.01	19.64	20.28	21.54	22.81	24.08	25.35	26.61	27.88	29.15	30.42
3⅞	11.62	12.20	12.78	13.36	13.94	14.52	15.10	15.68	16.26	16.84	17.42	18.00	18.59	19.74	20.91	22.07	23.23	24.39	25.55	26.72	27.90
4	10.56	11.09	11.62	12.14	12.67	13.20	13.73	14.25	14.78	15.31	15.84	16.37	16.90	17.95	19.01	20.07	21.12	22.18	23.23	24.29	25.35
4⅛	9.504	9.981	10.45	10.93	11.41	11.88	12.36	12.83	13.31	13.79	14.27	14.74	15.21	16.15	17.11	18.06	19.01	19.96	20.91	21.86	22.81
4¼	8.448	8.872	9.294	9.716	10.14	10.55	10.98	11.40	11.83	12.20	12.67	13.09	13.52	14.36	15.21	16.05	16.90	17.74	18.59	19.43	20.27
4½	7.392	7.763	8.132	8.502	8.871	9.240	9.610	9.980	10.35	10.72	11.09	11.46	11.83	12.56	13.31	14.04	14.78	15.52	16.26	17.00	17.74
4⅞	6.336	6.654	6.970	7.287	7.604	7.920	8.237	8.554	8.871	9.188	9.505	9.822	10.14	10.77	11.41	12.04	12.67	13.31	13.94	14.57	15.20
5	5.280	5.545	5.808	6.072	6.337	6.601	6.865	7.129	7.393	7.657	7.921	8.183	8.445	8.975	9.507	10.03	10.56	11.09	11.61	12.14	12.67
5⅛	4.224	4.436	4.647	4.858	5.069	5.280	5.492	5.703	5.914	6.125	6.336	6.547	6.759	7.181	7.604	8.026	8.449	8.871	9.294	9.716	10.23
5¼	3.168	3.372	3.485	3.644	3.802	3.960	4.119	4.277	4.436	4.594	4.752	4.910	5.069	5.386	5.703	6.019	6.386	6.653	6.970	7.287	7.594
5½	2.112	2.218	2.320	2.429	2.535	2.640	2.746	2.851	2.957	3.062	3.168	3.274	3.380	3.591	3.802	4.013	4.224	4.436	4.647	4.858	5.069
5⅞	1.584	1.663	1.741	1.822	1.901	1.980	2.059	2.138	2.218	2.297	2.376	2.455	2.535	2.693	2.851	3.009	3.168	3.327	3.485	3.643	3.801
6	1.056	1.109	1.162	1.215	1.267	1.320	1.373	1.426	1.479	1.531	1.584	1.637	1.690	1.795	1.901	2.006	2.112	2.218	2.323	2.429	2.535

Weight Per Square Foot in Pounds.







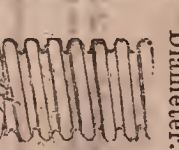
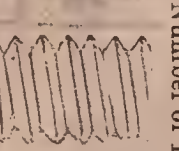

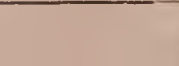
Thickness in inches.	Cast Iron.	Wrought or Sheet Iron.	Sheet Copper.	Sheet Lead.	Sheet Zinc
$\frac{1}{16}$	2.346	2.517	2.890	3.694	2.320
$\frac{1}{8}$	4.693	5.035	5.781	7.382	4.642
$\frac{3}{16}$	7.039	7.552	8.672	11.074	6.961
$\frac{1}{4}$	9.386	10.070	11.562	14.765	9.275
$\frac{5}{16}$	11.733	12.588	14.453	18.456	11.61
$\frac{3}{8}$	14.079	15.106	17.344	22.148	13.93
$\frac{7}{16}$	16.426	17.623	20.234	25.839	16.23
$\frac{1}{2}$	18.773	20.141	23.125	29.530	18.55
$\frac{9}{16}$	21.119	22.659	26.016	33.222	20.87
$\frac{5}{8}$	23.466	25.176	28.906	36.913	23.19
$1\frac{1}{16}$	25.812	27.694	31.797	40.604	25.53
$\frac{3}{4}$	28.159	30.211	34.688	44.296	27.85
$1\frac{1}{8}$	30.505	32.729	37.578	47.987	30.17
$1\frac{1}{4}$	32.852	35.247	40.469	51.678	32.47
$1\frac{3}{8}$	35.199	37.764	43.359	55.370	34.81
$1\frac{1}{2}$	37.545	40.282	46.250	59.061	37.13
$1\frac{5}{8}$	42.238	45.317	52.031	66.444	41.78
$1\frac{3}{4}$	46.931	50.352	57.813	73.826	46.42
$1\frac{7}{8}$	51.625	55.387	63.594	81.208	51.04
$1\frac{1}{2}$	56.317	60.422	69.375	88.592	55.68
$1\frac{5}{8}$	61.011	65.458	75.156	95.975	60.35
$1\frac{3}{4}$	65.704	70.493	80.938	103.358	65.00
$1\frac{7}{8}$	70.397	75.528	86.719	110.740	69.61
2	75.090	80.563	92.500	118.128	74.25

Weight of Copper Rods or Bolts per Foot,

Diameter. Inches.	Weight. Pounds.	Diameter. Inches.	Weight. Pounds.	Diameter. Inches.	Weight. Pounds.	Diameter Inches.	Weight. Pounds.
$\frac{1}{4}$	0.1892	1	3.0270	$1\frac{1}{8}$	10.642	$3\frac{3}{8}$	34.487
$\frac{5}{16}$	0.2956	$1\frac{1}{16}$	3.4170	2	12.108	$3\frac{1}{2}$	37.081
$\frac{3}{8}$	0.4256	$1\frac{3}{16}$	3.8912	$2\frac{1}{8}$	13.668	$3\frac{7}{8}$	39.737
$\frac{7}{16}$	0.5794	$1\frac{1}{2}$	4.2688	$2\frac{1}{4}$	15.325	$3\frac{1}{2}$	42.568
$\frac{1}{2}$	0.7567	$1\frac{5}{8}$	4.7298	$2\frac{3}{8}$	17.075	4	45.455
$\frac{9}{16}$	0.9578	$1\frac{3}{4}$	5.2140	$2\frac{1}{2}$	18.916		48.433
$\frac{5}{8}$	1.1824	$1\frac{7}{8}$	5.7228	$2\frac{7}{8}$	20.856	$4\frac{1}{4}$	53.550
$1\frac{1}{16}$	1.4307	2	6.2547	3	22.891	$4\frac{1}{2}$	61.321
$\frac{3}{4}$	1.7027	$2\frac{1}{8}$	6.8109	$3\frac{1}{8}$	25.019	$4\frac{3}{4}$	68.312
$1\frac{1}{8}$	1.9982	$2\frac{1}{4}$	7.3898	$3\frac{3}{8}$	27.243	5	76.130
$1\frac{1}{4}$	2.3176	$2\frac{3}{8}$	7.9931	$3\frac{1}{2}$	29.559	$5\frac{1}{2}$	91.550
$1\frac{3}{8}$	2.6605	$2\frac{1}{2}$	9.2702	$3\frac{3}{4}$	31.972	6	109.

		<i>Weight per Square Foot in Pounds.</i>				
Thickness by the Gauge.		Thickness in Inches.	Sheet and Boiler Iron.	Sheet Cast Steel.	Sheet Copper.	Thickness in Inches.
No. 0		0.340	13.7	14.0	15.6	$\frac{11}{32}$
" 1		0.300	12.1	12.4	13.8	$\frac{5}{16}$
" 2		0.284	11.4	11.7	13.0	$\frac{9}{32}$
" 3		0.259	10.4	10.6	11.9	$\frac{1}{2}$
" 4		0.238	9.60	9.80	11.0	$\frac{5}{16}$
" 5		0.220	8.85	9.02	10.1	"
" 6		0.203	8.17	8.33	9.32	"
" 7		0.180	7.24	7.38	8.25	$\frac{3}{16}$
" 8		0.165	6.65	6.78	7.59	"
" 9		0.148	5.96	6.08	6.80	$\frac{5}{32}$
" 10		0.134	5.40	5.51	6.16	"
" 11		0.120	4.83	4.93	5.51	$\frac{1}{8}$
" 12		0.109	4.40	4.50	5.02	"
" 13		0.095	3.83	3.91	4.37	$\frac{2}{32}$
" 14		0.083	3.34	3.41	3.81	"
" 15		0.072	2.90	2.96	3.31	$\frac{1}{16}$
" 16		0.065	2.62	2.67	3.00	"
" 17		0.058	2.34	2.39	2.67	"
" 18		0.049	1.97	2.01	2.25	"
" 19		0.042	1.69	1.72	1.93	$\frac{2}{64}$
" 20		0.035	1.41	1.42	1.61	"
" 21		0.032	1.29	1.31	1.47	"
" 22		0.028	1.13	1.15	1.29	$\frac{1}{32}$
" 23		0.025	1.00	1.02	1.14	"
" 24		0.022	0.885	0.903	1.01	"
" 25		0.020	0.805	0.820	0.918	"
" 26		0.018	0.724	0.738	0.826	$\frac{1}{64}$
" 27		0.016	0.644	0.657	0.735	"
" 28		0.014	0.563	0.574	0.642	
" 29		0.013	0.523	0.533	0.597	
" 30		0.012	0.483	0.493	0.551	
" 31		0.010	0.402	0.410	0.480	
" 32		0.009	0.362	0.370	0.420	
" 33		0.008	0.322	0.328	0.370	
" 34		0.007	0.282	0.288	0.323	
" 35		0.005	0.230	0.235	0.262	
" 36		0.004	0.170	0.173	0.194	

Proportion of Bolts and Nuts. Number of threads per In.

Diameter of Bolt.			Diameter.		Number of Threads per Inch.	
	3 inch.	$\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				
		$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$ 2				

Velocity at the end.	Space fall- en through	Time in seconds	Velocity at the end.	Space fall- en through	Time in seconds.	Velocity at the end.	Space fall- en through	Time in seconds.
V	S	T	V	S	T	V	S	T
0.1	·00015	0.0031	5.1	·40388	0.158	11	1.8789	0.342
0.2	·00062	0.0062	5.2	·41987	0.162	12	2.0652	0.373
0.3	·00139	0.0093	5.3	·43618	0.165	13	2.2624	0.405
0.4	·00248	0.0124	5.4	·45279	0.168	14	3.0435	0.436
0.5	·00388	0.0155	5.5	·46972	0.171	15	3.4938	0.467
0.6	·00559	0.0187	5.6	·48695	0.174	16	3.9751	0.498
0.7	·00761	0.0218	5.7	·50450	0.177	17	4.4876	0.530
0.8	·00994	0.0230	5.8	·52236	0.181	18	5.0310	0.560
0.9	·01257	0.0280	5.9	·55057	0.184	19	5.6056	0.591
1.	·01552	0.0311	6.	·55900	0.187	20	6.2112	0.622
1.1	·01879	0.0342	6.1	·57779	0.190	21	6.8478	0.654
1.2	·02065	0.0373	6.2	·59689	0.193	22	7.5155	0.685
1.3	·02624	0.0404	6.3	·61630	0.196	23	8.2143	0.716
1.4	·03043	0.0436	6.4	·63602	0.199	24	8.9441	0.747
1.5	·03493	0.0467	6.5	·65606	0.202	25	9.7049	0.778
1.6	·03975	0.05	6.6	·67639	0.205	26	10.497	0.810
1.7	·04487	0.052	6.7	·69705	0.209	27	11.320	0.840
1.8	·05031	0.0556	6.8	·71801	0.212	28	12.174	0.872
1.9	·05605	0.0591	6.9	·73928	0.215	29	13.059	0.903
2.	·06211	0.0623	7.	·76087	0.218	30	13.975	0.933
2.1	·06847	0.0654	7.1	·78276	0.221	31	14.922	0.965
2.2	·07515	0.0685	7.2	·80497	0.224	32	15.900	0.996
2.3	·08214	0.0717	7.3	·82748	0.227	33	16.910	1.025
2.4	·08944	0.0747	7.4	·85031	0.231	34	18.789	1.058
2.5	·09705	0.0780	7.5	·87344	0.234	35	19.022	1.091
2.6	·10497	0.0810	7.6	·89689	0.237	36	20.124	1.120
2.7	·11320	0.0841	7.7	·92065	0.240	37	21.258	1.151
2.8	·12174	0.0872	7.8	·94472	0.243	38	22.422	1.184
2.9	·13059	0.0903	7.9	·96910	0.246	39	23.618	1.213
3.	·13975	0.0934	8.	·99379	0.250	40	24.844	1.243
3.1	·14922	0.0966	8.1	1.0187	0.253	41	26.102	1.276
3.2	·15900	0.0997	8.2	1.0441	0.256	42	27.391	1.308
3.3	·16910	0.1025	8.3	1.0697	0.259	43	28.57	1.338
3.4	·18788	0.1059	8.4	1.0956	0.262	44	30.062	1.370
3.5	·19022	0.1092	8.5	1.1218	0.265	45	31.444	1.400
3.6	·20124	0.1121	8.6	1.1484	0.268	46	32.857	1.431
3.7	·21257	0.1152	8.7	1.1753	0.271	47	34.301	1.463
3.8	·22422	0.1185	8.8	1.2015	0.274	48	35.776	1.495
3.9	·23618	0.1214	8.9	1.2299	0.278	49	37.282	1.525
4.	·24844	0.1246	9.	1.2577	0.281	50	38.820	1.555
4.1	·26102	0.1278	9.1	1.2858	0.283	51	40.388	1.588
4.2	·27391	0.1309	9.2	1.3143	0.287	52	41.987	1.619
4.3	·28571	0.1339	9.3	1.3430	0.290	53	43.618	1.650
4.4	·30062	0.1371	9.4	1.3720	0.293	54	45.279	1.680
4.5	·31444	0.1403	9.5	1.4041	0.296	55	46.972	1.711
4.6	·32857	0.1433	9.6	1.4310	0.300	56	48.695	1.742
4.7	·34301	0.1465	9.7	1.4610	0.302	57	50.450	1.774
4.8	·35776	0.1496	9.8	1.4913	0.306	58	52.236	1.805
4.9	·37282	0.1526	9.9	1.5219	0.309	59	55.058	1.835
5.	·38820	0.1559	10	1.5528	0.312	60	55.900	1.868

GRAVITATION.

Gravity or **Gravitation** is a mutual faculty which all bodies in nature possess, to attract one another; or *Gravity* is the force by which all bodies tend to approach each other. A large body attracting a comparatively very small one, and their distance apart being inconsiderable, the force of gravity in the small body will be very sensible compared with that in the large one; such is the case with the body, our earth, attracting small bodies on or near her surface.

Gravitation is not periodical, it acts continually *ever* and *ever*. A body placed unsupported at a distance from the earth, the *force of gravity* is instantly operating to draw it down, and then we say, "the body fell down." If it were possible to withdraw the attraction between the body and the earth, it would not fall down, but remain unsupported in the space where it was placed;—giving the body a motion upwards it would continue that, and never come back to the earth again.

Law of Gravity.

The force of Gravity is proportional to the mass of the attracting bodies, and inverse as the square of their distance apart.

This law was discovered by Sir Isaac Newton. It is this law that supports the condition of the whole universe, and enables us to calculate the distances, motions and masses, &c., of the heavenly bodies.

The unit or measure of force of gravity is assumed to be the velocity a falling body has obtained at the end of the first second it falls; this unit is commonly denoted by the letter g ; its value at the level of the sea in New York is $g = 32.166$ feet per second, in vacuum. The space fallen through in the first second is $\frac{1}{2}g = 16.083$ feet.

This value augments with the latitude, and abates with the elevation above the level of the sea.

l = latitude, h = height in feet above the level of the sea, and r = radius of the earth in feet, at the given latitude l .

$$r = 20887510(1 + 0.00164 \cos.2l),$$

$$g = 32.16954(1 - 0.00284 \cos.2l)\left(1 - \frac{2h}{r}\right)$$

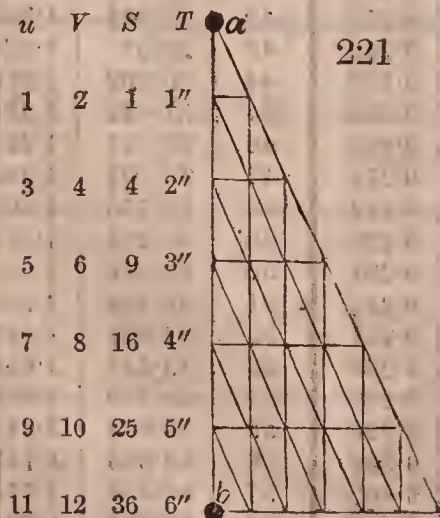
Letters denote.

S = the space in feet, which the falling body passes through in the time T .

u = the space in feet, which the body falls in the T th second.

V = velocity in feet per second, of the falling body at the end of the time T .

T = time in seconds the body is falling.



The accompanying Diagram is a good illustration of the acceleration of a falling body. The body is supposed to fall from a to b , every small triangle represents the space 16.08 feet which the body falls in the first second; when the body has reached the line 3" seconds, it will be found that it has passed 9 triangles, and $9 \times 16.08 = 144.72$ feet the space which a body will fall in 3" seconds. The number of triangles between each line is the space u which the body has fallen in that second. Between 3" and 4" are 7 triangles and $7 \times 16.08 = 112.56$ feet, the space fallen through in the fourth second. Under the line 3" will be found 6 triangles, which represents the velocity V the body has obtained at the end of the third second or $6 \times 16.08 = 96.48$ feet per second. For every successive second the body will gain two triangles or $2 \times 16.08 = 32.16$ feet per second.

Formulas for Accelerated Motion.

$$V = g T = \frac{2 S}{T} = \sqrt{2g S} = 8.02 \sqrt{S}, \quad - \quad - \quad - \quad 1,$$

$$S = \frac{g T^2}{2} = \frac{V T}{2} = \frac{V^2}{2g} = \frac{V^2}{64.33}, \quad - \quad - \quad - \quad 2,$$

$$T = \frac{V}{g} = \frac{2 S}{V} = \sqrt{\frac{2 S}{g}} = \frac{V S}{4.01}, \quad - \quad - \quad - \quad 3,$$

$$u = g(T - \frac{1}{2}), \quad T = \frac{u}{g} + \frac{1}{2}, \quad - \quad - \quad - \quad 4.$$

Example 1. A body is dropped at a height of 98 feet. What velocity will it have when it reaches the ground, and what time will it take to fall down?

Formula 1. $V = 8.02 \sqrt{S} = 8.02 \sqrt{98} = 79.39$ feet per second.

Formula 3. $T = \frac{\sqrt{S}}{4.01} = \frac{\sqrt{98}}{4.01} = 2.46$ seconds.

Example 2. A body was dropped at the opening of a hole in a rock, and reached the bottom after 3.5 seconds. How deep was the hole?

Formula 2. $S = \frac{g T^2}{2} = \frac{32.16 \times 3.5^2}{2} = 196.98$ feet.

Retarded Motion.

A body thrown up vertically will obtain inversely the same motion as when it falls down, because it is the same force that acts upon it, and causes *retarded motion* when it ascends, and *accelerated motion* when it descends.

V = the velocity at which the body starts to ascend.

v = velocity at the end of the line t .

T = time in seconds in which the body will ascend.

t = any time less than T .

S = height in feet to which the body will ascend.

s = the space it ascends in the time t .

Formulas for Retarded Motion.

$$v = V - g t = \frac{s}{t} - \frac{g t}{2}, \quad - \quad - \quad - \quad 5,$$

$$s = V t - \frac{g t^2}{2} = t v + \frac{g t^2}{2}, \quad - \quad - \quad - \quad 6,$$

$$V = v + g t = \frac{s}{t} + \frac{g t}{2}, \quad - \quad - \quad - \quad 7,$$

$$t = \frac{V - v}{g} = \frac{V}{g} - \sqrt{\frac{V^2}{g^2} - \frac{2 S}{g}}, \quad - \quad - \quad - \quad 8.$$

Formulas for T and S , is the same as for accelerated motion.

Example 3. A ball starts to ascend with a velocity of 135 feet per second. At what velocity will it strike an object 60 feet above? Find the time t , by the Formula 8.

$$t = \frac{135}{32.16} - \sqrt{\frac{135^2}{32.16^2} - \frac{2 \times 60}{32.16}} = 0.41 \text{ seconds, until it strikes, and from}$$

Formula 5, we have,

$$v = 135 - 32.16 \times 0.41 = 121.83 \text{ feet, per second.}$$

Example 4. A ball thrown up vertically from a cannon, occupied 9 seconds, until it arrived at the same place it started from. How high up was the ball, and at what velocity did it start?

One half of 9 = $4\frac{1}{2}$ seconds. Formula 2.

$$S = \frac{32.16 \times 4.5^2}{2} = 326 \text{ feet high.}$$

$$V = 32.16 \times 4.5 = 144.7 \text{ feet per second.}$$

If a cannon ball be shot from *A*, in the direction *AB*, at an angle *BAC* to the horizon, there are two forces acting on the ball at the same time, namely,—the force of gunpowder, which would propel the ball uniformly in the direction *AB*, and the force of gravity which only acts to draw the ball down at an accelerated motion; these two different (uniform and accelerated) motions will cause the ball to move in a curved line, (Parabola) *AaC*. Fig. 225.

V = velocity of the ball at *A*. *W* = weight of the ball in pounds.

S = the greatest height of ball over the horizontal line *AC*.

t = time from *A* to *C*, via *a*. *p* = pounds of powder in the charge.

b = the distance from *A* to *C*, called *horizontal range*.

$$V = 2800 \sqrt{\frac{p}{W}}, \quad p = \frac{W V^2}{7840000}, \quad b = 87.06 \sin. x \cos. x \frac{p}{W}$$

Example 5. The cannon being loaded sufficiently to give the ball a velocity of 900 feet per second, the angle $x = 45^\circ$. Required the distance *b* = ? and the time *t* = ?

$$b = \frac{900^2 \times \sin. 45^\circ \times \cos. 45^\circ}{32.16} = 1259 \text{ feet, the distance from } A \text{ to } C.$$

It will be observed that the distance *b* will be longest when the angle x is 45° , because the product of *sine* and *cosine* is greatest for that angle. $\sin. 45^\circ \times \cos. 45^\circ = 0.5$.

Example 5. What time will it take for a ball to roll 38 feet on an inclined plane angle, $x = 12^\circ 20'$, and what velocity has it at 38 feet from the starting point.

$$T = \sqrt{\frac{2S}{g \sin. x}} = \sqrt{\frac{2 \times 38}{32.16 \times \sin. 12^\circ 20'}} = 3.33 \text{ seconds.}$$

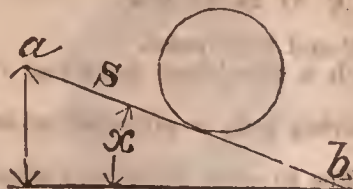
$$V = g T \sin. x = 32.16 \times 3.33 \times \sin. 12^\circ 20' = 22.8 \text{ feet per second.}$$

Power Concentrated in Moving Bodies.

It is highly important to distinguish between power simply, and power when concentrated in a moving body. The former is the *force multiplied by its velocity*,—but the power concentrated in a moving body is equal to the *weight of the body multiplied by the square of its velocity*, and the product divided by the *acceleratrix g*,—or the power concentrated in a moving body is equal to the power expended in giving it the motion.

Example. A sledge weighing 20 pounds, strikes a nail with a velocity of 12 feet per second. With what effect did it strike?

$$P = \frac{20 \times 12^2}{32.16} = 89.55 \text{ effects.}$$

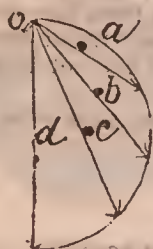


222.

$$V = g T \sin.x = \sqrt{2g S \sin.x},$$

$$S = \frac{g T^2}{2 \sin.x} = \frac{V^2}{2 g \sin.x},$$

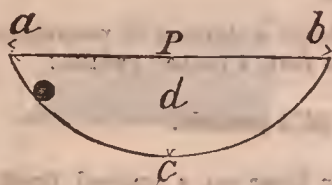
$$T = \frac{V}{g \sin.x} = \sqrt{\frac{2S}{g \sin.x}}.$$



223.

A body will fall from *o* the distances *a, b, c,* and *d,* in equal times.

$$T = \sqrt{\frac{2d}{g}}.$$

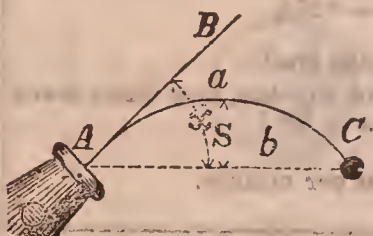


224.

A body will fall from *a* to *b* via *c* in the short-
est time, if the curve is a Cycloid.

$S = 4d$, the length of the Cycloid.

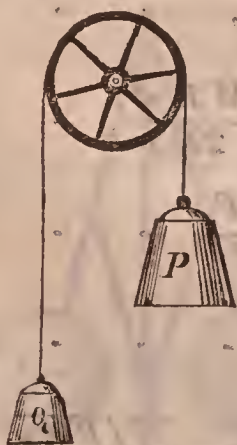
$$T = \pi \sqrt{\frac{d}{2g}} = \pi \sqrt{\frac{p}{2\pi g}}.$$



225.

$$b = \frac{V^2 \sin.x \cos.x}{g},$$

$$T = \frac{V \sin.x}{g}, \quad S = \frac{V \sin.^2x}{2g}.$$



226.

$$S = g \frac{T^2 F}{2M} = \frac{V^2 M}{2g F},$$

$$V = g T \frac{F}{M} = \sqrt{\frac{2g S F}{M}},$$

$$T = \frac{V M}{g F} = \sqrt{\frac{2S M}{g F}},$$

$$F = \frac{V M}{g T} = \frac{2S M}{g T^2},$$

$$M = P + Q, \text{ and } F = P - Q.$$

CENTRIFUGAL FORCE.

Central Forces are of two kinds, *centrifugal* and *centripetal*.

Centrifugal Force is the tendency which a revolving body has to depart from its centre of motion.

Centripetal Force is that by which a revolving body is attracted or attached to its centre of motion.

The *Centrifugal* and *Centripetal* forces are opposites to each other, and when equal the body revolves in a circle; but when they differ the body will revolve in other curved lines, as the Ellipse, the Parabola, &c., according to the nature of the difference in the forces. If the *centrifugal* force is 0 while the other is acting, the body will move straight to the centre of motion; and if the *centripetal* force is 0 while the other is acting, the body will depart from the circle in a straight line, tangent to the circle in the point where the *centripetal* force ceased to act. The *central* forces are distinct from the force that has set the body in motion.

If the *centrifugal* force be made use of to produce an effect, such effect will be at the expense of the one producing the rotary motion.

Letters denote.

F = Centrifugal force in pounds.

M = the Mass or weight of the revolving body in pounds.

v = Velocity of the revolving body, in feet per second.

R = Radii of the circle in which the body revolves, in feet.

n = number of revolutions per minute.

Example 1. Required the *centrifugal* force of a body weighing 63 pounds, and making 163 revolutions per minute, in a circle of 4 feet, 4 inches radius?

$$F = \frac{M R n^2}{2933} = \frac{63 \times 4.33 \times 163^2}{2933} = 2475 \text{ pounds.}$$

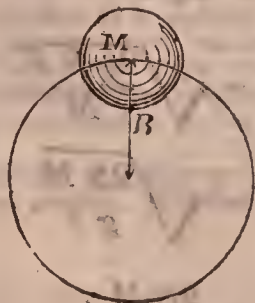
Example 2. A Railroad train runs 43 miles per hour on a curved track of 115 feet radii. What should be the obliquity of the track?

$$\tan x = \frac{\text{Miles}^2}{69 R} = \frac{43^2}{69 \times 115} = 0.233,$$

or $x = 13^\circ 10'$, the obliquity of the track.

Example 3. A governor having its arms $l = 1$ foot, 6 inches, how many revolutions must it make per minute to form an angle $x = 30^\circ$?

$$n = \frac{54.16}{\sqrt{1.5 \times \cos 30^\circ}} = 47.5 \text{ revolutions per minute.}$$



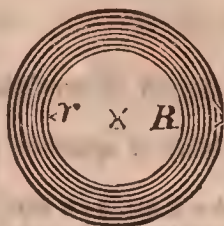
$$227. \quad F = \frac{M v^2}{g R} = \frac{M v^2}{32.16 R}, \quad - \quad - \quad 1,$$

$$F = \frac{4 M R \pi^2 n^2}{60^2 g} = \frac{M R n^2}{2933}, \quad - \quad - \quad 2,$$

$$M = \frac{F g R}{v^2} = \frac{2933 F}{M n^2}, \quad - \quad - \quad 3,$$

$$R = \frac{M v}{F g} = \frac{2933 F}{M n^2}, \quad - \quad - \quad 4,$$

$$n = \sqrt{\frac{2933 F}{M R}}, \quad v = \sqrt{\frac{F R g}{M}}, \quad 5,$$



228.

Centrifugal force of a ring.

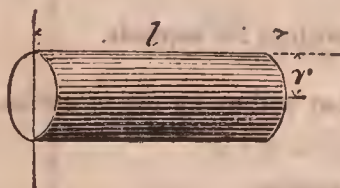
$$F = \frac{M n^2 \sqrt{R^2 + r^2}}{4150}.$$



229.

Centrifugal force of a grinding stone, circle-plane, cylinder, rotating round this centre.

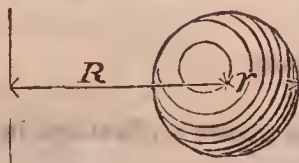
$$F = \frac{M R n^2}{4150}.$$



230.

Centrifugal force of a cylinder rotating round the diameter of its base.

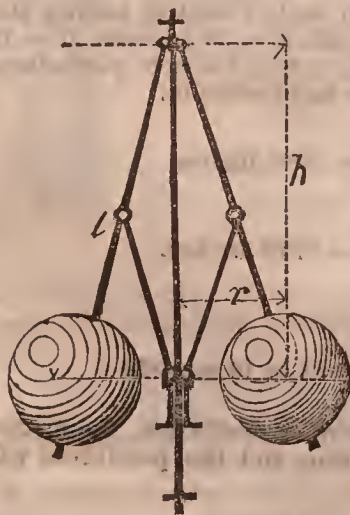
$$F = \frac{M n^2 \sqrt{4 l^2 + 3 r^2}}{10260}.$$



231.

Centrifugal force of a ball, (centre of gyration included.)

$$F = \frac{M n^2 \sqrt{R^2 + \frac{2}{5} r^2}}{2933},$$



232.

Governor.

$$n = \frac{60}{2\pi} \sqrt{\frac{g}{h}} = \frac{54.16}{\sqrt{h}} = \frac{54.16}{\sqrt{l \cos x}},$$

$$h = \frac{2933}{n^2}, \quad l = \frac{2933}{n^2 \cos x} = \frac{h}{\cos x},$$

$$\cos x = \frac{2933}{n^2 l} = \frac{h}{l}, \quad r = \sqrt{l^2 - h^2}.$$

P E N D U L U M .

Simple Pendulum is a material point under the action of gravitation, and suspended at a fixed point by a line of no weight.

Compound Pendulum is a suspended rod and body of sensible magnitude, fixed as the simple pendulum.

Centre of Oscillation is a point in which if all the matter in the compound pendulum were there collected, it would make a simple pendulum oscillate at the same times.

Angle of Oscillation is the space a pendulum describes when in motion.

The velocity of an oscillating body through the vertical position, is equal to the velocity a body would obtain by falling vertically the distance *versed sine* of half the angle of oscillation.

Letters denote.

l = length of the simple pendulum, or the distance between the centre of suspension, and centre of oscillation in inches.

t = time in seconds for n oscillations.

n = number of single oscillations in the time t .

Example 1. Required the length of a pendulum that will vibrate seconds? here $n = 1$, and $t = 1''$.

$$l = 39.109 \frac{t^2}{n^2} = 39.109 \text{ inches, the length of a pendulum for seconds.}$$

Example 2. Require the length of a pendulum that will make 180 vibrations per minute? here $t = 60''$ and $n = 180$.

$$l = \frac{39.109 t^2}{n^2} = \frac{39.109 \times 60^2}{180^2} = 4.346 \text{ inches.}$$

Example 3. How many vibrations will a pendulum of 25 inches length make in 8 seconds?

$$n = \frac{6.254 t}{\sqrt{l}} = \frac{6.254 \times 8}{\sqrt{25}} = 10 \text{ vibrations.}$$

Example 4. A pendulum is 137.67 inches long and makes 8 vibrations in 15 seconds. Required the unit or acceleratrix $g = ?$

$$g = \frac{0.8225 l n^2}{t^2} = \frac{0.8225 \times 137.67 \times 8^2}{15^2} = 32.209.$$

Example 5. A compound pendulum of two iron balls P and Q , having the centre of suspension between themselves: see Fig. 238. $P = 38$ pounds, $Q = 12$ pounds, $a = 25$ inches, and $b = 18$ inches. How long is the simple pendulum, and how many vibrations will the pendulum make in 10 seconds?

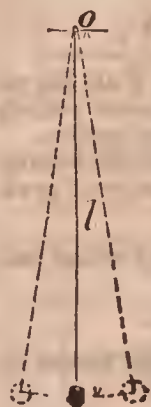
$$x = \frac{a P - b Q}{P + Q} = \frac{25 \times 38 - 18 \times 12}{38 + 12} = 14.68 \text{ inches.}$$

$$l = \frac{a^2 P + b^2 Q}{x(P + Q)} = \frac{25^2 \times 38 + 18^2 \times 12}{14.68(38 + 12)} = 37.68 \text{ inches,}$$

the length of the single pendulum.

$$n = \frac{6.254 t}{\sqrt{l}} = \frac{6.254 \times 10}{\sqrt{37.68}} = 10.193 \text{ vibrations in 10 seconds.}$$

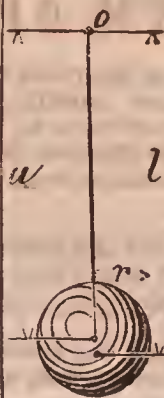
If a compound pendulum is hung up at its centre of oscillation, the former centre of suspension will be the centre of oscillation, and the pendulum will oscillate the same time.


 233. *Simple Pendulum.*

$$l = \frac{12g}{\pi^2 n^2} = \frac{39.1t^2}{n^2},$$

$$t = \frac{n\sqrt{l}}{6.25},$$

$$n = \frac{6.254t}{\sqrt{l}},$$



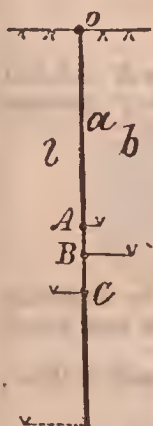
236.

$$g = \frac{l\pi^2 n^2}{12t^2},$$

$$g = \frac{0.8225l n^2}{t^2},$$

o = centre of suspension.

$$l = a + \frac{2r}{5a}.$$



34.

A = centre of gravity.

B = centre of gyration.

C = centre of oscillation.

$$a : b = b : l,$$

$$b = \sqrt{al} = 1.1432a,$$

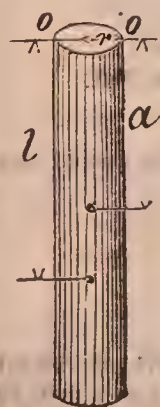
$$l = 1\frac{1}{3}a.$$



237.

$$l = \frac{a^2 P + b^2 Q}{a P + b Q}.$$

P and Q expressed in pounds, or cubic contents.



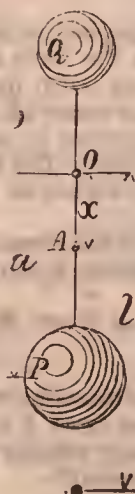
235.

Compound Pendulum.

r = radius of cylinder.

$$l = \frac{16a^2 + 3r^2}{12a},$$

$$l = \frac{4a}{3} + \frac{r^2}{4a}.$$



238.

$$x = \frac{a P - b Q}{P + Q},$$

$$l = \frac{a^2 P + b^2 Q}{x(P + Q)}.$$

Length of a Pendulum vibrating seconds at the level of the sea, in various places.

At the Equator, lat. $0^\circ 0' 0''$	-	-	-	-	-	39.0152 inches.
" Washington, lat. $38^\circ 53' 23''$	-	-	-	-	-	39.0958 "
" New York, lat. $40^\circ 42' 40''$	-	-	-	-	-	39.1017 "
" London, lat. $51^\circ 31'$	-	-	-	-	-	39.1393 "
" lat. 45°	-	-	-	-	-	39.1270 "
" Stockholm, lat. $59^\circ 21' 30''$	-	-	-	-	-	39.1845 "

$$l = 39.127 - 0.09982 \cos.2 \text{ lat. for seconds.}$$

CENTRE OF GYRATION.

Centre of Gyration is a point in revolving bodies in which, if all the revolving matters were there contained, it would obtain equal *angular velocity* from, and sustain equal resistance to, the force that gives it a rotary motion.

The *centre of gyration* in different bodies will be found by the accompanying formulas, in which x = distance from the centre of motion to the centre of gyration.

Example 1. Fig. 239. Find the *centre of gyration* in a bar, rotating round one of its ends; its length is 7 feet, 3 inches?

$$x = 0.5775 \times 7.25 = 4.13 \text{ feet, from the centre of motion.}$$

Example 2. Fig. 245. Find the *centre of gyration* of a cone, rotating round its vertex, its height being $h = 3.3$ feet, and $R = 8$ inches = 0.666 feet.

$$x = \sqrt{\frac{12h^2 + 3R^2}{20}} = \sqrt{\frac{12 \times 3.3^2 + 3 \times 0.666^2}{20}} = 2.569 \text{ feet}$$

from the centre of motion.

Example 3. Fig. 249. A ring or fly wheel having its outer radius $R = 6$ feet 4 inches, the inner radius $r = 5$ feet 8 inches. Required its *centre of gyration* $x = ?$ from the centre of motion.

$$x = \sqrt{\frac{R^2 + r^2}{2}} = \sqrt{\frac{6.33^2 + 5.66^2}{2}} = 6 \text{ feet.}$$

CONCLUSIONS.

The object of finding the *centre of gyration* of revolving bodies is to ascertain what effect is necessary to give a mass a certain angular velocity; or how much effect is concentrated in a body having a certain angular velocity.

Angular velocity is the number of revolutions a body makes in a unit of time, it is herein denoted by the letter n .

Letters denote.

P = power in effects.

H = horse-power.

F = the Force which is applied to rotate a body, in pounds.

s = the radius on which the force acts, in feet.

M = Mass of the revolving body, in pounds.

x = the distance from the centre of motion to centre of gyration, in feet.

T = time the force F is applied in seconds.

N = number of revolutions in the time T .

n = angular velocity or number of revolutions per minute, at the end of the time T .

$g = 32.166$ *acceleratrix* of the force of gravity.

G = *acceleratrix* of the force F , then,

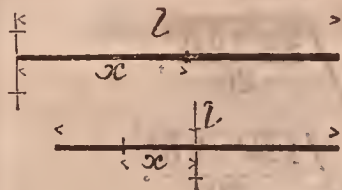
$$G : g = F s^2 : M x^2, \text{ or } G = \frac{g F s^2}{M x^2}.$$

Example 4. Fig. 249. In connection with the preceding example (3) the fly-wheel weighs 7400 pounds. What force F must be applied at the radius $r = 2$ feet, to give the fly wheel an *angular velocity* of $n = 128$ revolutions per minute, at the end of the time $T = 40$ seconds?

$$\text{Formula 6. } F = \frac{n M x^2}{153.5 g T^2} = \frac{128 \times 7400 \times 6^2}{153.5 \times 40^2} = 2773 \text{ pounds.}$$

How many revolutions did the wheel make in the 40 seconds?

$$\text{Formula 9. } N = \frac{2.56 T^2 F s}{M x^2} = \frac{2.56 \times 40^2 \times 2773 \times 2}{7400 \times 6^2} = 85.27 \text{ revolutions.}$$

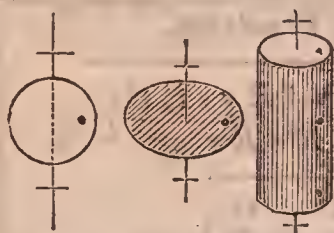


239.

A line or bar.

$$x = 0.5775l,$$

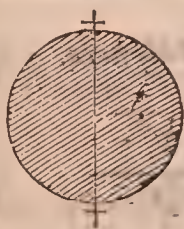
$$x = 0.2887l.$$



240.

*A circumference round its diameter,**A circle-plane round its centre,**A cylinder round its axis.*

$$x = 0.7071r.$$



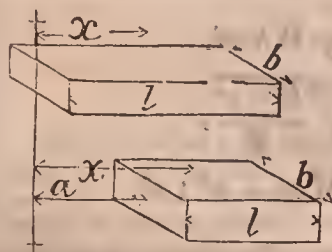
241.

A circle-plane round its diameter.

$$x = 0.5r.$$



242.

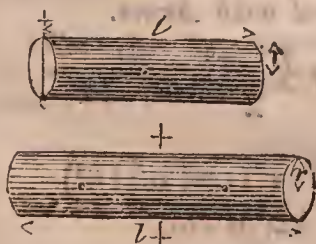
*A Sphere round its diameter.*Convex surface, $x = 0.8165r,$ Solid, $x = 0.6324r.$ 

243.

Parallelopiped.

$$x = \sqrt{\frac{4l^2 + b^2}{12}},$$

$$x = \sqrt{\frac{4l^2 + b^2}{12} + a^2 + a l}.$$

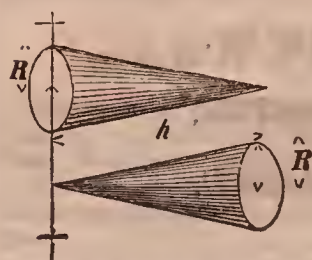


244.

Cylinder.

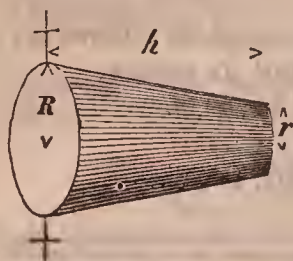
$$x = \sqrt{\frac{4l^2 + 3r^2}{12}},$$

$$x = \sqrt{\frac{l^2 + 3r^2}{12}}.$$

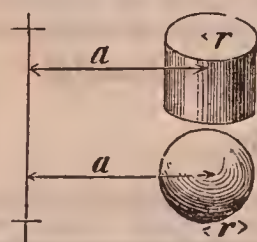
245. *Cone.*

$$x = \sqrt{\frac{2h^2 + 3R^2}{20}},$$

$$x = \sqrt{\frac{12h^2 + 3R^2}{20}}.$$

246. *Conic Frustrum.*

$$x = \sqrt{\frac{h}{10} \left(\frac{R^3 + 3Rr + Rr^2}{R^2 + Rr + r^2} \right) + \frac{3}{20} \left(\frac{R^5 - r^5}{R^2 - r^2} \right)}$$

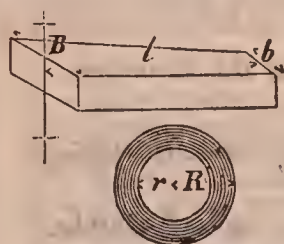


247.

Cylinder and Sphere.

$$x = \sqrt{a^2 + \frac{1}{2}r^2},$$

$$x = \sqrt{a^2 + \frac{2}{5}r^2}.$$

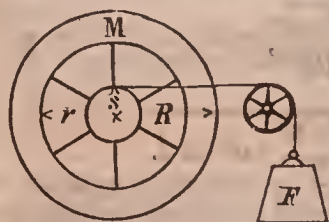


248.

Wedge and Ring.

$$x = 0.204 \sqrt{12l^2 + B^2 + b^2},$$

$$x = \sqrt{\frac{R^2 + r^2}{2}}.$$

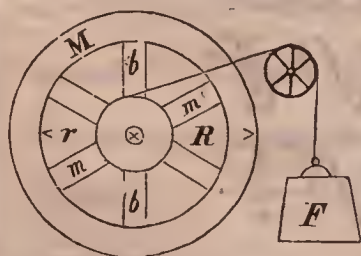


249.

Fly Wheel.

$$x = \sqrt{\frac{R^2 + r^2}{2}},$$

$$F G : M g = x^2 : s^2.$$

250. *Fly Wheel with Arms.*

$$x^2(M+m) = M \frac{R^2 + r^2}{2} + m \frac{4r^2 + b^2}{12},$$

$$x = \sqrt{\frac{6M(R^2 + r^2) + m(4r^2 + b^2)}{12(M+m)}}.$$

Formulas for Force and Power of Acceleration.

$$T = \sqrt{\frac{4\pi s N}{G}}, \quad - \quad 1,$$

$$N = \frac{2.56 T^2 F s}{M x^2}, \quad - \quad - \quad 9,$$

$$T = x \sqrt{\frac{4\pi N M}{g F s}}, \quad 2,$$

$$n = \frac{153.5 T F s}{M x^2}, \quad - \quad - \quad 10,$$

$$T = \frac{2\pi s n}{60G}, \quad - \quad - \quad 3,$$

$$P = \frac{n^2 M x^2}{244T}, \quad - \quad - \quad 11,$$

$$T = \frac{4\pi n M x^2}{60g F s}, \quad - \quad - \quad 4,$$

$$M = \frac{244 T P}{n^2 x^2}, \quad - \quad - \quad 12,$$

$$F = \frac{N M x^2}{2.56 T^2 s}, \quad - \quad - \quad 5,$$

$$T = \frac{M x^2 n^2}{244P}, \quad - \quad - \quad 13,$$

$$F = \frac{n M x^2}{153.5 T s}, \quad - \quad - \quad 6,$$

$$n = \sqrt{\frac{244 T P}{M x^2}}, \quad - \quad 14,$$

$$s = \frac{N M x^2}{2.56 T^2 F}, \quad - \quad - \quad 7,$$

$$H = \frac{n^2 M x^2}{134100 T}, \quad - \quad - \quad 15,$$

$$s = \frac{n M x^2}{153.5 T F}, \quad - \quad - \quad 8,$$

$$M = \frac{134100 T H}{n^2 x^2}, \quad 16,$$

Fly-Wheels. Weight of.

The weight of a fly-wheel will be determined by the formula 16 in which the time $T = 130$ seconds, the time in which the fly-wheel would concentrate the same power as the steam-engine. When the works or resistance is very irregular it will be better to take the time $T = 170$. The centre of gyration (including ring and arms,) can in practice be assumed at $x = r$ the inner radius of the ring.

Example 5. Required the weight of a fly-wheel for ordinary work, the steam engine being 56 horse power, making 42 revolutions per minute, and the inner radius $r = 10$ feet?

$$M = \frac{134100 T H}{n^2 r^2} = \frac{134100 \times 130 \times 56}{42^2 \times 10^2} = 5535 \text{ pounds.}$$

CENTRE OF PERCUSSION.

Centre of Percussion is a point in which the momentums of a moving body are concentrated. *Centre of Percussion* is the same as *centre of oscillation*, and to be calculated by the same formulas.

Take an iron bar in one hand, and strike heavily over a sharp edge, if the *centre of percussion* of the bar strikes over the edge, the whole momentum will there be discharged, but if it strikes at a distance from the *centre of percussion* a part of the momentum will be discharged in the hand, and a shock felt.

It is sometimes of great importance to properly place the *centre of percussion*. If it is dislocated, the moving body not only fails to properly transmit its effect, but the lost momentum acts to wear out the machinery.



CENTRE OF GRAVITY.

Centre of Gravity is a point around which the momentums of all matters (under the action of the force of gravity) in a body, or system of bodies, are equally divided.

A body or system of bodies suspended at its *centre of gravity*, will be in equilibrium in all positions.

A body or system of bodies, suspended in a point out of its *centre of gravity*, will hang with its *centre of gravity* vertical under the point of suspension.

A body or system of bodies suspended in a point out of its *centre of gravity*, and having two different positions, the two vertical lines through the point of suspension will meet in the *centre of gravity*; thus if a plane be hung up in two different positions, the vertical lines *a*, *b*, and *c*, *d*, will meet in the *centre of gravity* *o*.

z = distance to the centre of gravity as noted in the figures.

Example 1. The radius of a circle being 3 feet, how far is its centre of gravity from the centre of the half circle?

$$z = 0.6367 \times 3 = 1.91 \text{ feet.}$$

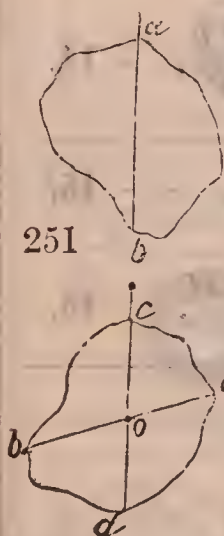
Example 2. How far from the bottom of a cylindric shell, open at one end, is its centre of gravity? The cylinder is 4 feet long, radius *r* = 0.8 feet.

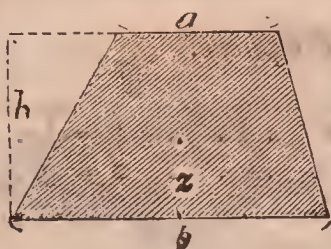
$$z = \frac{h}{r+2h} = \frac{4}{0.8+2 \times 4} = 0.625 \text{ feet.}$$

Example 3. Fig. 264. An irregular figure weighing *P* = 138 pounds, is suspended between a fulcrum and a weight, *l* = 5.6 feet, *W* = 57 pounds. Required the distance to the centre of gravity *z* = ?

$$z = \frac{57 \times 5.6}{138} = 2.31 \text{ feet.}$$

251

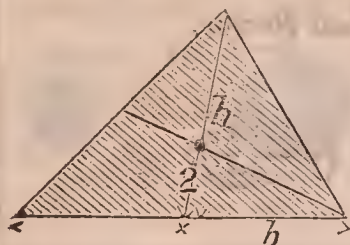




252.

Quadrangle.—*a* and *b* parallel.

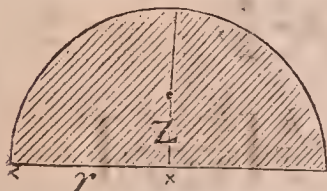
$$z = \frac{h}{2} - \frac{h}{6} \left(\frac{b-a}{b+a} \right).$$



253.

Triangle.

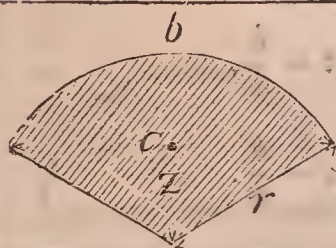
$$z = \frac{h}{3}.$$



254.

Half a circle plane or Elliptic plane.

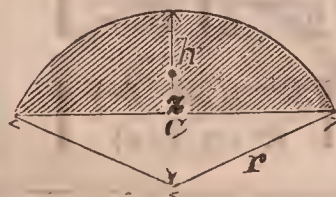
$$z = 0.424r.$$



255.

Circle sector.

$$z = \frac{2c}{3b} r.$$

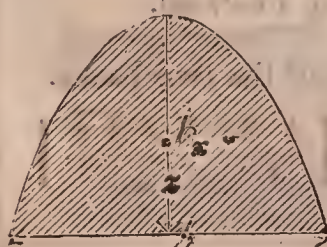


256.

Circle Segment. **a** = area.

$$z = \frac{c^3}{12a}.$$

$$x = h + z - r.$$

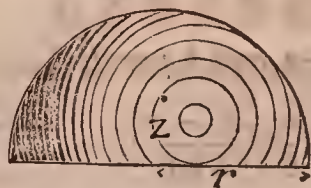


257.

Parabola.

$$z = \frac{2h}{5}.$$

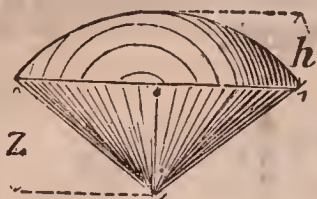
For half a Parabola $x = \frac{3}{8} b.$



258.

Half Sphere.

Convex surface $z = \frac{1}{2}r$.
 Solid $z = \frac{3}{8}r$.



259.

Spherical Sector.

Solid, $z = \frac{1}{4} \left(r - \frac{h}{2} \right)$.

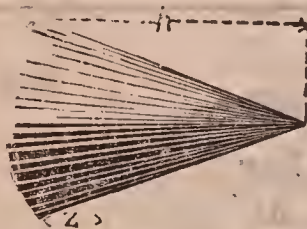


260.

Spherical Segment.

Convex surface $z = \frac{h}{2}$,

Solid $z = \frac{h}{2} \cdot \left[\frac{2r^2 + h^2}{3r^2 + h^2} \right]$

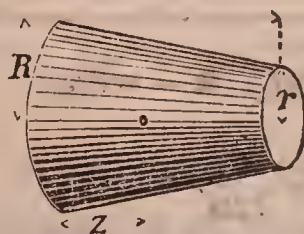


261.

Cone.

Convex surface $z = \frac{h}{3}$,

Solid $z = \frac{h}{4}$.

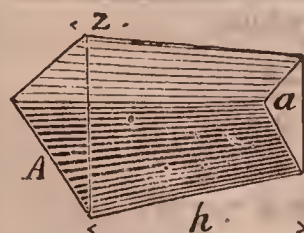


262.

Conic Frustrum.

Con. sur. $z = \frac{h}{2} - \frac{h}{6} \left[\frac{R-r}{R+r} \right]$

Solid $z = \frac{h}{4} \cdot \left[\frac{R^2 + r(2R+3r)}{R^2 + r(R+r)} \right]$

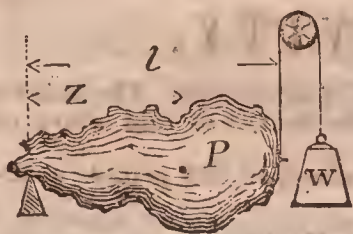


263.

Pyramidal Frustrum.

A and a = area of the two bases.

Solid $z = \frac{h}{4} \left[\frac{A + 3a + 2\sqrt{Aa}}{A + a + \sqrt{Aa}} \right]$

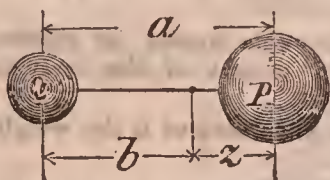


264.

Irregular Figure.

$$P : W = l : z,$$

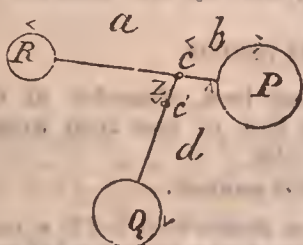
$$z = \frac{W l}{P}.$$



265.

To find the Centre of Gravity of two bodies, P and Q.

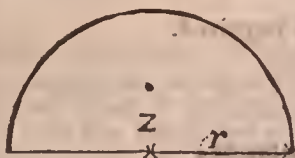
$$z = \frac{Q a}{P+Q}, \quad b = \frac{P a}{P+Q}.$$



266.

To find the Centre of Gravity of a system of bodies.

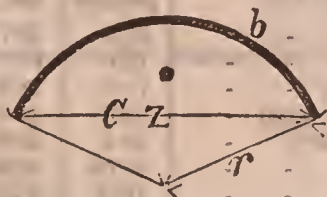
$$b = \frac{R a}{P+R}, \quad z = \frac{Q d}{P+R+Q}.$$



267.

Half a circumference of a Circle or Ellipse.

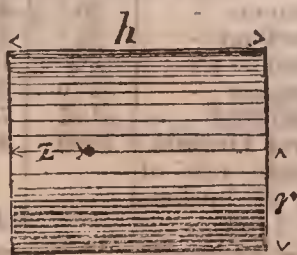
$$z = 0.6367r.$$



268.

Circle arc or Elliptic arc.

$$z = \frac{c r}{b} = \frac{c(c^2 + 4h^2)}{8h b}.$$



269.

Cylindric Surface with a bottom in one end.

$$z = \frac{h}{r+2h}.$$

SPECIFIC GRAVITY.

Specific Gravity is the comparative density of substances. The unit for measuring the specific gravity is assumed to be the density of rain water, or distilled water.

One cubic foot of distilled water weighs 1000 ounces, or 62·5 pounds avoirdupois.

To Find the Weight of a Body.

RULE 1. Multiply the contents of the body in cubic feet by 62·5, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 2. Multiply the contents of the body in cubic inches by 0·03616, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 3. Divide the specific gravity by 0·016 and the quotient is the weight of a cubic foot.

Example 1. A bottle full of mercury is 3 inches, inside diameter, and 6 inches high. How much mercury is there in the bottle in pounds?

One cubic inch of mercury weighs 0·491 pounds, and by the formula for Fig. 119 we have the

$$\text{weight} = 0\cdot491 \times 0\cdot785 \times 3^2 \times 6 = 20\cdot85 \text{ pounds.}$$

Example 2. Required the weight of a cone of cast iron, diameter at the base $d = 1\cdot33$ feet, height $h = 4$ feet? One cubic foot of cast iron weighs 450·5 pounds, and by formula for Fig. 117 we have the

$$\text{weight} = 450\cdot5 \times 0\cdot2616 \times 1\cdot33^2 \times 4 = 834 \text{ pounds.}$$

Example 3. The section area of the lower hole in a steam boat is 245 square feet; how much space must be taken in the length of the hole for 131 tons of anthracite coal?

Anthracite coal are 42·3 cubic feet per ton.

$$\text{length} = \frac{42\cdot3 \times 131}{245} = 22\cdot6 \text{ feet, the space required.}$$

Weight and Bulk of Substances,

Names of Substances.	Cubic feet in pounds.	Cubic foot per ton.	Names of Substances.	Cubic feet in pounds.	Cubic foot per ton.
Cast iron, - -	450·5	4·97	Sand, - - -	94·5	23·7
Wrought iron, -	486·6	4·60	Granite, - - -	139	16·1
Steel, - - -	489·8	4·57	Earth, loose, - -	78·6	28·5
Copper, - - -	555·	4·03	Water, salt, (sea) -	64·3	34·8
Lead, - - -	707·7	3·16	“ fresh - - -	62·5	35·9
Brass, - - -	537·7	4·16	Ice, - - -	58·08	38·56
Tin, - - -	456	4·91	Gold, - - -	1013	2·21
Pine, white - -	29·56	75·6	Silver, - - -	551	4·07
“ yellow, - -	33·81	66·2	Coal, Anthracite -	53	42·3
Mahogany, - -	66·4	33·8	“ Bituminous -	50	44·8
Marble, common, -	141·0	15·9	“ Cumberland -	53	42·3
Mill-stone, - -	130	17·2	“ Charcoal -	18·2	123
Oak, live - -	70	32·0	Coke, Midlothian -	32·70	68·5
“ white, - -	45·2	49·5	“ Cumberland -	31·57	70·9
Clay, - - -	101·3	22·1	“ Natural Virginia	46·64	48·3
Cotton Bales, - -			Conventional rate of		
Brick, - - -	100	22·4	Stone coal, 28 bushels		
Plaster Paris, - -	105	21·3	(5 pecks) = 1 ton. -		43·56

To Find the Specific Gravity. W = weight of a body in the air. w = weight of the body (heavier than water) immersed in water. S = specific gravity of the body. Then,

$$W - w : W = 1 : S. \quad S = \frac{W}{W - w}, \quad 1,$$

Example 4. Required the specific gravity of a piece of iron-ore weighing 6.345 pounds in the air, and 4.935 pounds in water, $S = ?$

$$S = \frac{6.345}{6.345 - 4.935} = 4.5 \text{ the specific gravity.}$$

When the body is lighter than water, annex to it a heavier body that is able to sink the lighter one.

 S = specific gravity of the heavier annexed body. s = specific gravity of the lighter body. W = weight of the two bodies in air. w = weight of the two bodies in water. V = weight of the heavier body in air. v = weight of the lighter body in air.

$$s = \frac{v}{W - w - \frac{V}{S}}, \quad 2,$$

Example 5. To a piece of wood, which weighs $v = 14$ pounds in the air, is annexed a piece of cast-iron $V = 28$ pounds; the two bodies together weigh $w = 11.7$ pounds in water. Required the specific gravity of the wood?

$$W = V + v = 28 + 14 = 42 \text{ pounds.}$$

$$S = 7.2 \text{ specific gravity of cast-iron.}$$

$$\text{Formula 2.} \quad S = \frac{14}{42 - 11.7 - \frac{28}{7.2}} = 0.529, \text{ the specific}$$

gravity of the wood, (Poplar White Spanish.)

A simple way to obtain the specific gravity of woods, is to form it to a parallel rod, and place it vertically in water, then when in equilibrium, the immersed end is to the whole rod as the specific gravity is to 1.

Example 6. A cylinder of wood is 6 feet, 3 inches long, when immersed vertically in water it will sink 3 feet, 9 inches by its own weight. Required its specific gravity.

$$3.75 : 6.25 = S : 1, \quad S = \frac{3.75}{6.25} = 0.600.$$

To discover the Adulteration in Metals. or to find the proportions of two Ingredients in a Compound.

$$V = \frac{W - s(W - w)}{1 - \frac{s}{S}}, \quad 3,$$

Example 7. A metal compounded of silver and gold weighs $W = 6$ pounds in the air, and in water $w = 6.636$ pounds. Require the proportions of silver and gold?

$$S = 19.36 \text{ specific gravity of gold.}$$

$$s = 10.51 \text{ specific gravity of silver.}$$

$$\text{weight } V = \frac{6 - 10.51(6 - 5.636)}{1 - \frac{10.51}{19.36}} = 4.755 \text{ pounds of gold.}$$

$$\text{and } 1.245 \text{ pounds of silver.}$$

<i>Names of Substances.</i>	<i>Specific gravity.</i>	<i>Weight per cubic inch.</i>	<i>Names of Substances.</i>	<i>Specific gravity.</i>	<i>Weight per cubic inch.</i>
Metals.					
Platinum, rolled - - -	22.669	.798	Alabaster, white - -	2.730	.0387
“ wire, - - -	21.042	.761	“ yellow - - -	2.699	.0974
“ hammered, - - -	20.337	.736	Coral, red - - - -	2.700	.0974
“ purified, - - -	19.50	.706	Granite, Susquehanna	2.704	.0976
“ crude, grains	15.602	.565	“ Quincy - - -	2.652	.0958
Gold, hammered - - -	19.361	.700	“ Patapsco - - -	2.640	.0954
“ pure cast - - -	19.258	.697	“ Scotch - - - -	2.625	.0948
“ 22 carats fine - -	17.486	.733	Marble, white Italian	2.708	.0978
“ 20 “ “ - - -	15.702	.568	“ common - - -	2.686	.0968
Mercury, solid at - 40°	15.632	.566	Tale, black - - - -	2.900	.0105
“ at +32° Fahr.	13.619	.493	Quartz, - - - - -	2.660	.0962
“ “ 60° “ - - -	13.580	.491	Slate, - - - - -	2.672	.0965
“ “ 212° “ - - -	13.375	.484	Pearl, oriental - - -	2.650	.0957
Lead, pure - - - -	11.330	.410	Shale, - - - - -	2.600	.0940
“ hammered - - - -	11.388	.412	Flint, white - - - -	2.594	.0936
Silver, hammered - - -	10.511	.381	“ black - - - - -	2.582	.0933
“ pure - - - - -	10.474	.379	stone, common - - -	2.520	.0910
Bismuth, - - - - -	9.823	.355	“ Bristol - - - - -	2.510	.0906
Red Lead, - - - - -	8.940	.324	“ Mill - - - - -	2.484	.0897
Cinober, - - - - -	8.098	.293	“ Paving - - - - -	2.416	.0873
Manganese, - - - - -	8.030	.290	Gypsum, opaque - - -	2.168	.0783
Copper, wire and rolled	8.878	.321	Grindstone, - - - -	2.143	.0775
“ pure - - - - -	8.788	.318	Salt, common - - - -	2.130	.0770
Bronze, gun metal - -	8.700	.315	Saltpetre, - - - - -	2.090	.0755
Brass, common - - - -	7.820	.282	Sulphur, native - - -	2.033	.0735
Steel, cast steel - - -	7.919	.286	Common soil, - - - -	1.984	.0717
“ common soft - - -	7.833	.283	Rotten stone, - - - -	1.981	.0416
“ hardened & temp.	7.818	.283	Clay, - - - - -	1.930	.0698
Iron, pure - - - - -	7.768	.281	Brick, - - - - -	1.900	.0686
“ wrought and rolled	7.780	.282	Nitre, - - - - -	1.900	.0636
“ hammered - - - -	7.789	.282	Plaster Paris, - - -	1.872	.0677
“ cast-iron - - - -	7.207	.261	Ivory, - - - - -	2.473	.0894
Tin, from Böhmen - - -	7.312	.265		1.822	.0659
“ English - - - - -	7.291	.264	Sand, - - - - -	1.800	.0651
Zinc, rolled - - - - -	7.191	.260	Phosphorus, - - - -	1.770	.0640
“ cast - - - - -	6.861	.248	Borax, - - - - -	1.714	.0620
Antimony, - - - - -	6.712	.244	Coal, Anthracite - -	1.640	.0593
Chrom, - - - - -	5.900	.213		1.436	.0592
Arsenic, - - - - -	5.763	.208	“ Maryland - - - -	1.355	.0490
Stones and Earths.			“ Scotch - - - - -	1.300	.0470
Topaz, oriental - - -	4.011	.145	“ New Castle - - -	1.270	.0460
Emery, - - - - -	4.000	.144	“ Bituminous - - -	1.270	.0460
Diamond, - - - - -	3.521	.127	Charcoal, triturated -	1.380	.0500
Limestone, green - - -	3.180	.115	Earth, loose - - - -	1.500	.0542
“ white - - - - -	3.156	.114	Amber, - - - - -	1.078	.0387
Asbestos, starry - - -	3.073	.111	Pimstone, - - - - -	1.647	.0596
Glass, flint - - - - -	2.933	.106	Lime, quick - - - -	0.804	.0291
“ white - - - - -	2.892	.104	Charcoal, - - - - -	0.441	.0160
“ bottle - - - - -	2.732	.0987	Woods (Dry.)		
“ green - - - - -	2.642	.0954	Alder, - - - - -	.800	.0289
Marble, Parian - - - -	2.838	.103	Apple-tree, - - - -	.793	.0287
“ African - - - - -	2.708	.0978	Ash, the trunk - - -	.845	.0306
“ Egyptian - - - - -	2.668	.0964	Bay-tree, - - - - -	.822	.0297
Mica, - - - - -	2.800	.1000	Beech, - - - - -	.852	.0308
Hone, white razor - - -	2.838	.104	Box, French - - - -	.912	.0330
Chalk, - - - - -	2.784	.100	“ Dutch - - - - -	1.323	.0480
Porphyry, - - - - -	2.765	.0999	“ Brazilian red - - -	1.031	.0373
Spar, green - - - - -	2.704	.0976	Cedar, wild - - - -	.596	.0219
“ blue - - - - -	2.693	.0971	“ Palestine - - - -	.613	.0222

<i>Names of Substances.</i>	<i>Specific gravity.</i>	<i>Weight per cubic inch.</i>	<i>Names of Substances.</i>	<i>Specific gravity.</i>	<i>Weight per cubic inch.</i>
Cedar, Indian -	1.315	.0476	Oil, Linseed -	.940	.0340
" American -	.561	.0203	" Olive -	.915	.0331
Citron, -	.726	.0263	" Turpentine -	.870	.0314
Cocoa-wood, -	1.040	.0376	" Whale -	.932	.0337
Cherry-tree, -	.715	.0259	Proof Spirit, -	.925	.0334
Cork, -	.240	.0087	Vinegar, -	1.080	.0390
Cypress, Spanish -	.644	.0233	Water, distilled	1.000	.0361
Ebony, American -	1.331	.0481	" Sea -	1.026	.0371
" Indian -	1.209	.0437	" Dead sea -	1.240	.0448
Elder-tree, -	.695	.0252	Wine, -	.992	.0359
Elm, trunk of -	.671	.0243	" Port -	.997	.0361
Filbert-tree, -	.600	.0217			
Fir, male -	.550	.0199	Miscellaneous.		
" female -	.498	.0180	Asphaltum, -	.905	.0327
Hazel, -	.600	.0217	Beeswax, -	1.650	.0597
Jasmine, Spanish -	.770	.0279	Butter, -	.965	.0349
Juniper-tree, -	.556	.0201	Camphor, -	.942	.0341
Lemon-tree, -	.703	.0254	India rubber, -	.988	.0357
Lignum-vitæ, -	1.333	.0482	Fat of Beef, -	.933	.0338
Linden-tree, -	.604	.0219	" Hogs, -	.923	.0334
Log-wood, -	.913	.0331	" Mutton, -	.936	.0338
Mastic-tree -	.849	.0307	Gamboge, -	.923	.0334
Mahogany, -	1.063	.0385	Gunpowder, loose -	1.222	.0442
Maple, -	.750	.0271	" shaken	.900	.0325
Medlar, -	.944	.0342	" solid -	1.000	.0361
Mulberry -	.897	.0324	Gum Arabic, -	1.550	.0561
Oak, heart of, 60 old	1.170	.0423	Indigo, -	1.800	.0650
Orange-tree, -	.705	.0255	Lard, -	1.452	.0525
Pear-tree, -	.661	.0239	Mastic, -	1.009	.0365
Pomegranate-tree, -	1.354	.0490	Spermaceti, -	.947	.0343
Poplar, -	.383	.0138	Sugar, -	1.074	.0388
" white Spanish	.529	.0191	Tallow, sheep -	.943	.0341
Plum-tree, -	.785	.0284	" calf -	1.605	.0580
Quince-tree, -	.705	.0255	" ox, -	.924	.0334
Sassafras, -	.482	.0174	Atmospheric air, -	.934	.0338
Spruce, -	.500	.0181		.923	.0334
" old -	.460	.0166		.001243
Pine, yellow	.660	.0239			
" white -	.554	.0200			
Vine, -	1.327	.0480	Gases. Vapours.		<i>Weight</i>
Walnut, -	.671	.0243	Atmospheric air, -	1.000	<i>cub. ft.</i>
Yew, Dutch -	.788	.0285	Ammoniacal gas, -	.500	<i>grains.</i>
" Spanish -	.807	.0292	Carbonic acid, -	1.527	527.0
			Carbonic acid, -	.972	262.7
Liquids.			Carburetted hydrogen,	.972	805.3
Acid, Acetic -	1.062	.0384	Chlorine, -	2.500	512.7
" Nitric -	1.217	.0440	Chlorocarbonous acid,	3.472	1316
" Sulphuric -	1.841	.0666	Chloroprussic acid,	2.152	1828
" Muriatic -	1.200	.0434	Fluoboric acid,	2.371	1134
" Fluoric -	1.500	.0542	Hydriodic acid, -	4.346	1250
" Phosphoric -	1.558	.0563	Hydrogen, -	.069	2290
Alcohol, commercial	.833	.0301	Oxygen, -	1.104	36.33
" pure -	.792	.0287	Sulphuretted hydrogen,	1.777	581.8
Ammoniac, liquid -	.897	.0324	Nitrogen, -	.972	9370
Beer, lager -	1.034	.0374	Vapour of Alcohol,	1.613	512.0
Champagne, -	.997	.0360	" turpentine spir.,	.5013	851.0
Cider, -	1.018	.0361	" water, -	.623	2642
Ether, sulphuric -	.739	.0267	Smoke of bitumin. coal,	.102	53.80
Egg, -	1.090	.0394	" wood, -	.90	474.0
Honey, -	1.450	.0524	Steam at 212° -	.488	257.3
Human blood, -	1.054	.0381			
Milk, -	1.032	.0373			

HYDROMETER.

A BODY wholly immersed in a liquid will lose as much of its weight, as the weight of the liquid it displaces.

A floating body will displace its own weight of the liquid in which it floats.

A cylindrical rod of wood or some light materials, being set down in two liquids, *A* and *B*, of different specific gravities, when in equilibrium it will sink to the mark *a* in the liquid *A*, and to *b* in the liquid *B*; then the specific gravity of $A : B = b, c : a, c$, or inverse as the immersed part of the rod. This is the principle upon which a hydrometer is constructed.

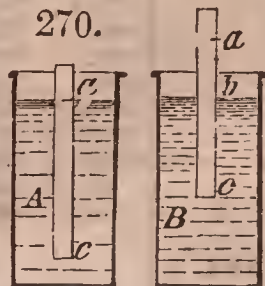
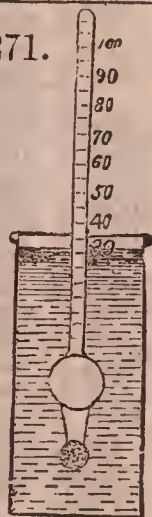


Table showing the comparative Scales of Guy Lussac and Baumé, with the Specific Gravity and Proof, at the temperature of 60° Fahr.

271.



Guy Lussac's.	Baumés	Specific Grav.	Proof.
100	46	·796	100
95	40	·815	92
90	36	·833	82
85	33	·848	72
80	31	·863	62
75	28	·876	52
70	26	·889	42
65	24	·901	32
60	23	·912	22
55	21	·923	12
50	19	·933	0 Proof.
45	18	·942	8
40	17	·951	18
35	16	·958	29
30	15	·964	35
25	14	·970	48

HYDROSTATICS.

Letters denote.

A and **a** = areas of the pressed surfaces in square feet.

P and **p** = hydrostatic pressure in pounds.

d = depth of the centre of gravity of **A** or **a** under the surface of the liquids in feet.

S = specific gravity of the liquid.

Example 1. Fig. 272. The plane **A** = 3·3 square feet, at a depth of **d** = 6 feet under the surface of fresh water. Required the pressure **P** = ? Specific gravity of fresh water **S** = 1.

$$P = 62\cdot5 A d = 62\cdot5 \times 3\cdot3 \times 6 = 1237\cdot5 \text{ pounds.}$$

Example 2. Fig. 275. The area of the pistons **A** = 8·5 square feet, **a** = 0·02 square feet, **l** = 4 feet, **e** = 9 inches, and **F** = 18 pounds. Required the pressure **P** = ?

$$P = \frac{F l A}{e a} = \frac{18 \times 4 \times 8\cdot5}{0\cdot75 \times 0\cdot02} = 40800 \text{ pounds.}$$

It must be distinguished that the centre of pressure and centre of gravity of the planes, are two different points; the centre of pressure is below the centre of gravity, when the plane is inclined or vertical.

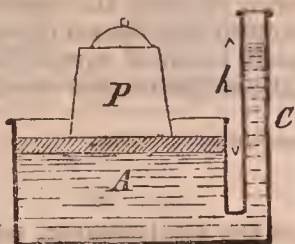


272.

$$P = 62.5 S A d,$$

$$A = \frac{P}{62.5 S d},$$

$$d = \frac{P}{62.5 S A}.$$

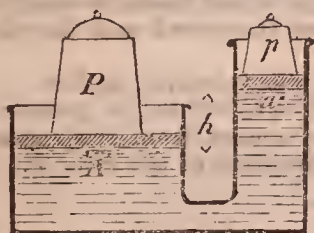


273.

The Hydrostatic paradox.

The pressure P is independent of the width of column C .

$$P = 62.5 S A h. \text{ (same as above.)}$$

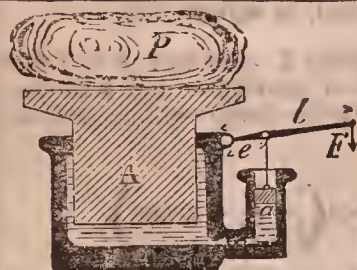


274.

$$P = A \left(62.5 S h + \frac{p}{a} \right)$$

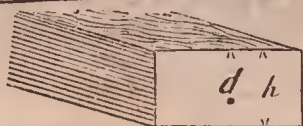
$$p = a \left(\frac{P}{A} - 62.5 S h \right),$$

$$h = \frac{P a - p A}{62.5 S A a}$$

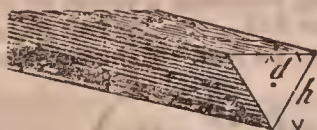
275. *Bramah's Hydraulic Press.*

$$P = \frac{F l A}{e a}, \quad A = \frac{P e a}{F l},$$

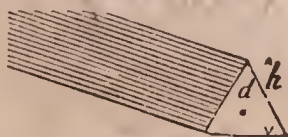
$$F = \frac{P e a}{A}, \quad a = \frac{F A l}{P e}.$$



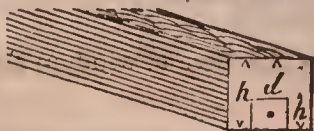
276. *Centre of Pressure of a rectangle, the upper edge at the surface of the liquid $d = \frac{2}{3} h$.*



277. *Centre of Pressure of a triangle, the base being at the surface of the liquid, $d = \frac{1}{2} h$.*



278. *Centre of Pressure of a triangle, the vertex being at the surface of the liquid. $d = \frac{3}{4} h$.*



279.

$$d = \frac{h}{3} + \sqrt{4(h-h)^2 + h^2}.$$

STABILITY OF FLOATING BODIES.

We have before said that a floating body *displaces its own weight* of the liquid in which it floats.

A floating body is in equilibrium when its centre of gravity is in the same vertical line, as the centre of gravity of its displacement.

The *momentum of stability* of a floating body, is equal to the weight of the displacement multiplied by the horizontal distance between the two centres of gravity of the body and the displacement.

Metacentrum is the point where the vertical line drawn through the centre of gravity of the body when in equilibrium, crosses the vertical line drawn through the centre of gravity of the displacement when the body is out of equilibrium.

When the centre of gravity is below metacentrum, the momentum of stability is positive, but negative when above.

A boat will capsize when its centre of gravity passes metacentrum.

Letters denote.

L , B and d = Length, Breadth, and depth in feet, of the displacement, when the floating body is a parabolipipedon.

Q = weight of the displacement of a vessel, in pounds.

P = weight of a cubic foot of the liquid in which the vessel floats, in pounds.

a = distance between the two centres of gravity of the vessel and the displacement when in equilibrium, in feet.

b = the horizontal distance between the same centres of gravity, in feet.

F = the force which acts to careen the vessel out of equilibrium, in pounds.

l = the lever on which the force F acts, in feet, measured from the centre of gravity of the displacement at right-angles to the direction of the force F .

Fl = momentum of stability.

c = centre of gravity of the vessel.

m = metacentrum.

\mathcal{X} = the greatest immersed section area of the vessel, in square feet.

Example 1. Fig. 281. A vessel being of the dimensions $L = 20$ feet, $B = 7$ feet 3 inches, $d = 3$ feet. Distance between the centres of gravity — $a = 8$ inches, $h = 4$ feet. Required the momentum of stability in fresh water, $Fl = ?$
 $P = 62.5$.

$$Fl = PLd2h\left(\frac{B^2}{12d} - a\right) = 62.5 \times 20 \times 3 \times 2 \times 4 \left(\frac{7.25^2}{12 \times 3} - 0.666\right)$$

= 23820 momentums of stability. Let the force be applied on a lever $l = 18$ feet. Required the force $F = ?$

$$F = \frac{23820}{18} = 1326.6 \text{ pounds.}$$

Example 2 Fig. 283. What force ($F = ?$) is required to careen a vessel of 800 tons? the greatest immersed section $\mathcal{X} = 314$ square feet. $B = 28$ feet beam, and the distance between the centres of gravity $+a = 1.4$ feet. To be careened to an angle $v = 30^\circ$.

$$Fl = Q \sin.v \left(\frac{B^2}{14\mathcal{X}} + a \right) = 800 \times 2240 \times \sin.30^\circ \left(\frac{28^2}{14 \times 314} + 1.4 \right)$$

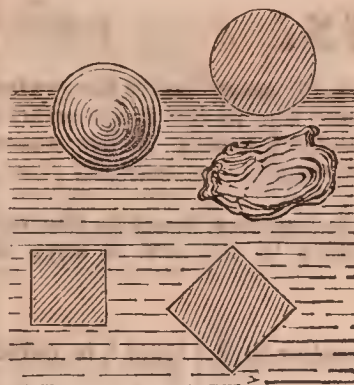
= 5716480 momentums of stability.

The lever on which the force F acts $l = 25$ feet.

Then,
$$F = \frac{5716480}{25} = 228659.2 \text{ pounds, } -102 \text{ tons.}$$

In the same manner the careen can be calculated by the force of wind on the sails.

280.



The momentum of stability of a ball or cylinder is $= 0$, because its centre of gravity will always be in the same vertical line, as the centre of gravity of its displacement. The momentum of stability of any solid and wholly immersed body, is $= 0$, because the two centres of gravity will be in one point:

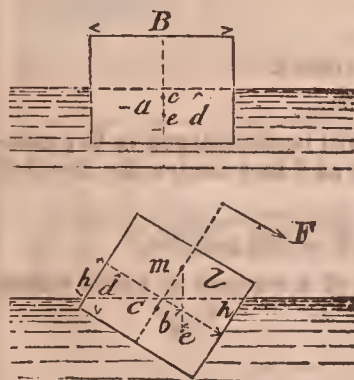
281.

Parallelopipedon.

$$Fl = P B L d b = Q b,$$

$$Fl = P L d 2 h \left(\frac{B^2}{12 d} \pm a \right),$$

$$b = \frac{Fl}{P B L d} = \frac{Fl}{Q}.$$

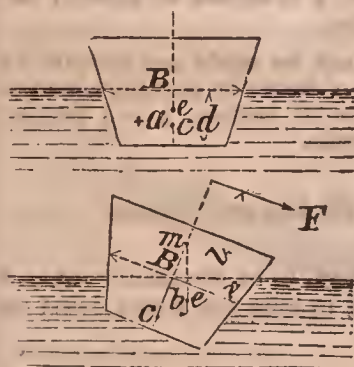


282.

Prismatic Vessel.

$$Fl = P \mathcal{X} L \sin.v \left(\frac{B^3}{12 \mathcal{X}} + a \right),$$

$$\sin.v = Q \frac{Fl}{\left(\frac{B^3}{12 \mathcal{X}} + a \right)}.$$



283.

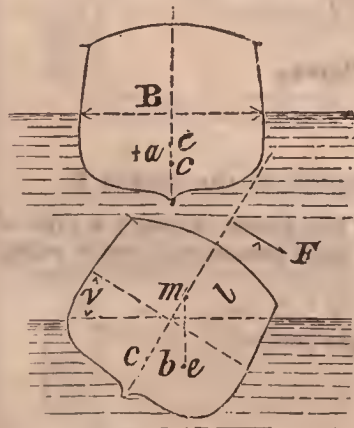
Sailing Vessel.

$$Fl = Q \sin.v \left(\frac{B^3}{12 \mathcal{X}} + a \right),$$

$$\pm a = \frac{Fl}{Q \sin.v} - \frac{B^3}{12 \mathcal{X}},$$

$$b = \frac{Fl}{Q}, \quad c m = \frac{b}{\sin.v} = \frac{B^3}{12 \mathcal{X}} \pm a.$$

For very sharp vessels take the coefficient $12=16$.



HYDRAULICS.

Let the vessel *A*, Fig. 284, be kept constantly full of water up to the water line *w*. In two horizontal faces lower than the water line *w*, are made orifices *a* and *a'*, through which the water will pass up vertical nearly to the water line *w*. Omitting the resistance of air, &c., the jet should theoretically reach the water line *w*; practically it reaches 0.967*h*.

It is evident that the velocity of the *jet* through the orifices, must be the velocity due to a body falling the height *h*, according to the law of force of gravity.

Letters denote.

Q = actual quantity of water discharged per second or in the time *t*, in cubic feet.

h = head, or height of water over the orifice.

t = operating time in seconds.

a = area of the orifice in square feet.

m = the coefficient for contraction. (See Fig. 299)

G = gallon of 231 cubic inches discharged in the time *t*.

V = velocity through the orifice in feet per second.

Example 1. Fig. 284. How many gallons of water will be discharged in five minutes, through an orifice of 0.025 square feet, applied at 8 feet under the level of the water?

$$G = 37.75 a t \sqrt{h} = 37.75 \times 0.025 \times 5 \times 60 \sqrt{8} = 800 \text{ gallons.}$$

Fig. 285. The weight *P* can represent the weight of a column of water whose

$$\text{height} = \frac{P}{62.5 A} = \frac{h'}{0.967}, \text{ acting on the area } A.$$

Fig. 286. *n* = number of down strokes per minute, *s* = stroke of piston; the air vessel *C* = 6*A s* at the pressure of the atmosphere.

Example 2. Fig. 286. How many double strokes must be made per minute by the lever of a hand pump, to throw up 22 cubic feet of water 18 feet high, in the time of 8 minutes and 15 seconds; the levers *l* = 30 inches, *e* = 8 inches, *s* = 0.6 feet, *F* = 20 pounds? $8 \times 60 + 15 = 495$ seconds.

$$n = \frac{3630 Q h' e}{t s F l} = \frac{3630 \times 22 \times 18 \times 8}{495 \times 0.6 \times 20 \times 30} = 64.5 \text{ strokes per minute.}$$

Example 3. Fig. 294. A vessel of rectangular form is of dimensions *A* = 6 square feet, the height *h* = 5 feet. What time will it take the water level to sink 2 feet, when the orifice *a* = 0.212 square feet.

$$t = \frac{A (h - h')}{2.52 a (\sqrt{h} + \sqrt{h'})} = \frac{6(5 - 3)}{2.52 \times 0.212 (\sqrt{5} + \sqrt{3})} = 5.66.$$

Motion of Water in Pipes,

Letters denote.

L = extreme length of the pipe in feet.

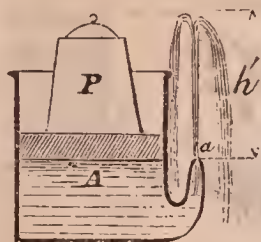
d = inside diameter in feet, and uniform throughout the length *L*.

Example 4. Fig. 287. What will be the velocity of the water through a pipe of 0.45 feet inside diameter, and *L* = 68 feet long, the head pressure of water being *h* = 8 feet?

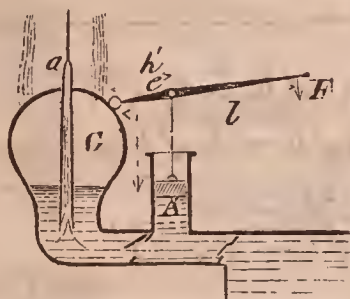
$$V = 48 \sqrt{\frac{0.45 \times 8}{68 + 50 \times 0.45}} = 9.6 \text{ feet per second.}$$



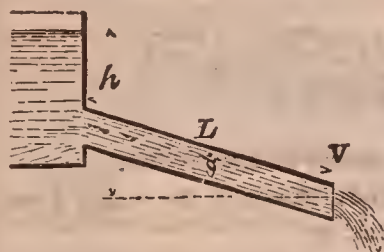
284. $V = 8.02 \sqrt{h}$
 $Q = m a 18.02 \sqrt{h} = 5.05 a t \sqrt{h}$
 $Q = 37.77 a t \sqrt{h}, \quad m = 0.63,$
 $h = \frac{Q^2}{25.5 a^2 t^2}, \quad \text{jet} = 0.967 h$



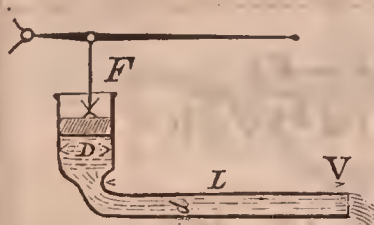
285. $V = 1.015 \sqrt{\frac{P}{A}}, \quad Q = a t \sqrt{\frac{P}{A}}$
 $G = 7.5 a t \sqrt{\frac{P}{A}}, \quad h' = \frac{P}{60.5 A}$



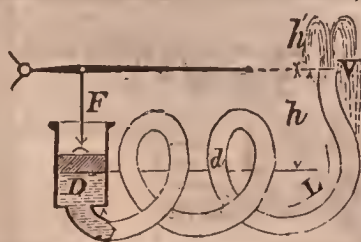
286. $Q = a t \sqrt{\frac{F l}{A e}}, \quad n = \frac{3630 Q h' e}{t s F l}$
 $G = 7.5 a t \sqrt{\frac{F l}{A e}}, \quad h' = \frac{F l}{60.5 A e}$



287. *Motion of Water in Pipes.*
 $V = 48 \sqrt{\frac{d h}{L + 50 d}}, \quad Q = 37.7 d^2 \sqrt{\frac{d h}{L + 50 d}}$
 $d = 0.24 \sqrt[5]{\frac{L Q^2}{h}}, \quad h = \frac{Q^2 (L + 50 d)}{142 d^5}$



288. *Motion of Water in Pipes.*
 $V = 6.86 \sqrt{\frac{d F}{D(L + 50 d)}}, \quad Q = 5.38 d^2 \sqrt{\frac{d F}{D(L + 50 d)}}$
 $d = 1.68 \sqrt[5]{\frac{L D Q^2}{F}}, \quad F = \frac{Q^2 D (L + 50 d)}{2.9 d^5}$



289. $V = 6.86 \sqrt{\frac{d (F - 49 D^2 h)}{D(L + 50 d)}}, \quad Q = 5.38 V d^2 t$
 $h' = \frac{V^2}{66.5}, \quad h' = \frac{D \sqrt{s n}}{57.65 d}$

290.

Weirs.

$$Q = k b t. \quad \text{See Table for Weirs.}$$

$$t = \frac{Q}{k b}, \quad b = \frac{Q}{k t},$$

291.

$$Q = 5.35 m b h t \sqrt{h},$$

$$G = 40 m b h t \sqrt{h},$$

$$t = \frac{Q}{5.35 m b h \sqrt{h}},$$

292.

$$Q = 5.35 m b t (h \sqrt{h} - h' \sqrt{h'}),$$

$$G = 40 m b t (h \sqrt{h} - h' \sqrt{h'}),$$

$$t = \frac{Q}{5.35 m b (h \sqrt{h} - h' \sqrt{h'})},$$

293.

$$t = \frac{0.95 n A (\sqrt{h} - \sqrt{h'})}{b \sqrt{h h'}},$$

A area of the vessel in square feet.

t time in seconds, in which the water level will sink the space $h - h'$.

294.

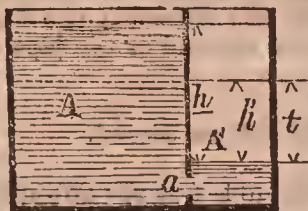
$$t = \frac{A (h - h')}{4 m a (\sqrt{h} + \sqrt{h'})},$$

$$Q = 4 m a t (\sqrt{h} + \sqrt{h'}),$$

295.

$$t = \frac{A}{3.85 a m} (\sqrt{h} - \sqrt{h'}),$$

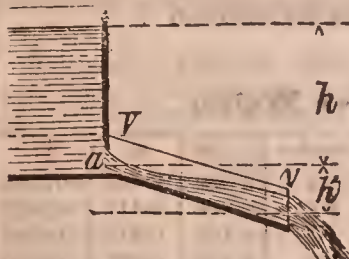
$$a = \frac{A \sqrt{h}}{3.85 t' m}, \quad t' = \frac{A \sqrt{h}}{3.85 a m},$$



296.

$$t = \frac{A A' \sqrt{h}}{13.7 m a \sqrt{A + A'}}$$

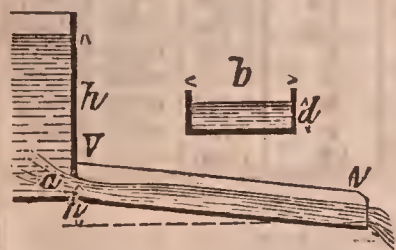
$$h = \frac{A h'}{A + A'}$$

297. *Short Drain.*

$$V = \frac{8.02 \sqrt{h}}{\sqrt{1 + \left(\frac{1}{m} - 1\right)^2}}$$

$$v = \sqrt{V^2 + 32.1 h'}$$

$$Q = a m V t. \text{ from } V \text{ to } v \text{ about } 6 \sqrt{a}$$

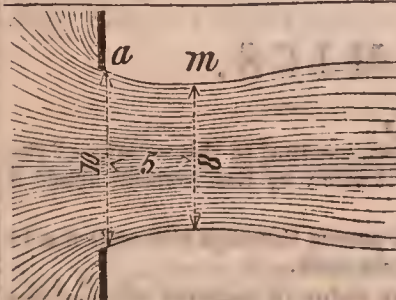
298. *Long Drain.*

$$V = \frac{8.02 \sqrt{h}}{\sqrt{1 + \left(\frac{1}{m} - 1\right)^2}}$$

$$v = \sqrt{V^2 + 64 \cdot 32 h' - 0.007 \frac{s^2 V^2}{a}}$$

$$s = b + 2d$$

$$l = V \text{ to } v, \text{ feet.}$$

299. *Proportions of the contracted Vein.*

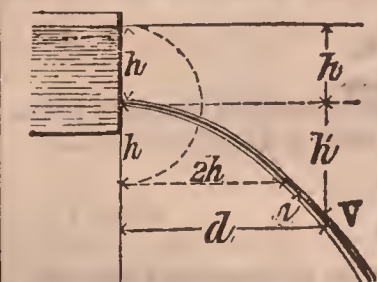
$$a : m = 10^2 : 8^2. \quad m = 0.64 a.$$

$$m = 0.64 \text{ when contracted on 4 sides.}$$

$$m = 0.72 \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{3 sides.}$$

$$m = 0.8 \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{2 sides.}$$

$$m = 0.9 \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{1 side.}$$



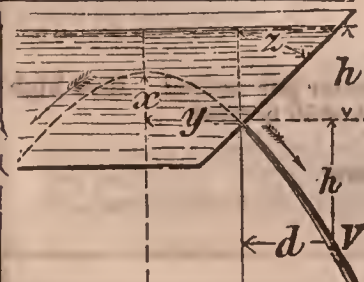
300.

The form of the Vein is a Parabola.

$$d = 2 \sqrt{h h'}, \quad V = 8 \sqrt{h + h'}$$

$$Q = 8 m a t \sqrt{h},$$

$$\tan v = \frac{2 h'}{d}, \quad h = \frac{d^2}{4 h'}$$



301.

$$x = \sin^2 z h \quad y = \sin. z \cos. z h.$$

$$d = 2 \sqrt{(h' + x)(h - x) - y}$$

$$Q = 8 m a t \sqrt{h} \quad V = 8 \sqrt{h + h'}$$

Example 5. Fig. 289. Required the velocity and quantity of water discharged in a long pipe or hose of $L = 135$ feet long, and $d = 0.17$ feet, attached to a hand-pump of $D = 0.2$ feet in diameter $P = 44$ pounds, and the end of the pipe elevated $h = 20$ feet above the piston D ?

$$V = 6.86 \sqrt{\frac{0.17(44 - 49 \times 0.2^2 \times 20)}{0.2(135 + 50 \times 0.17)}} = 1.95 \text{ feet per second.}$$

$$Q = 1.95 \times 5.38 \times 0.2^2 = 0.042 \text{ per second} \times 60 = 2.52 \text{ cubic feet per minute.}$$

$s = 0.8$ feet the stroke of piston, we shall have

$$n = \frac{2.52}{0.8 \times 0.785 \times 0.2^2} = 100 \text{ strokes per minute.}$$

Table for Water flowing over Weirs.

This Table is set up from careful experiments on a large scale, and is suited for *weirs* only. See Fig. 290.

RULE. Multiply the width b in feet, of the weir by the coefficient k , and the product is the quantity of water discharged per second, in cubic feet. h is the height as represented by Fig. 290. The width b should be $b > h$.

Example 6. How much water will flow over a weir of $b = 5$ feet, $h = 0.5$ feet in one minute?

$$Q = k b t = 1.1295 \times 5 \times 60 = 338.35 \text{ cubic feet.}$$

<i>h. inches.</i>	<i>h. feet.</i>	<i>m.</i>	<i>k.</i>
0.4	0.033	0.424	0.01365
0.8	0.066	0.417	0.05452
1.2	0.100	0.412	0.10592
1.6	0.133	0.407	0.16616
2.4	0.200	0.401	0.29171
3.2	0.266	0.397	0.44480
4.	0.333	0.395	0.63111
6.	0.500	0.393	1.1295
8.	0.666	0.390	1.7464
9.	0.750	0.385	2.0331
12	1.000	0.376	3.1350

HYDRODYNAMICS.

Water Power.

THE natural effect concentrated in a fall of water, is equal to the weight of the quantity of water passed through per second multiplied by the vertical space it falls.

Fig. 297. Let Q be the quantity of water which passes through the orifice a in the time $t = 1''$ second, in cubic feet of 62.5 pounds each.

h = the vertical space the water falls; then the value or natural effect of the fall is at the orifice a .

$$P = 62.5 Q h, \text{ effects.}$$

But, $Q = 5.06 a \sqrt{h}$, then we have

$$P = 315.5 a h \sqrt{h}.$$

This will be in horse-power,

$$H = 0.573 a h \sqrt{h}, \quad h = \frac{1}{1.07} \sqrt[3]{\frac{H^2}{a^2}},$$

$$H = 0.1134 Q h, \quad h = \frac{H}{0.1134 Q}.$$

Example 1. In a creek passes 18 cubic feet of water per second. How high must that creek be dammed up to produce an effect of 10 horses?

$$h = \frac{10}{0.1134 \times 18} = 4.9 \text{ feet, the answer.}$$

W A T E R - W H E E L S .

Water-wheels are of two essential kinds, namely, *Vertical* and *Horizontal*.

The *Vertical* are subdivided into

Overshot-wheels, Undershot-wheels, Breast-wheels, and High-breast and Low-breast wheels.

The *Horizontal* are with *Floats, Screw-wheels, Turbin, Reaction-wheels, &c.*

Waterwheels do not transmit in full the natural effect concentrated in a fall of water; under most favourable circumstances 80 per cent. has been utilized, but under poor arrangements only 20 per cent. may be expected.

Example 1. Fig. 302. The vertical section of the immersed floats of an undershot-wheel in a mid-stream is $a = 27$ square feet, velocity of the stream $V = 8.6$, and $v = 4$ feet per second. Required the horse-power of the wheel $H = ?$

$$H = \frac{a v}{200} (V - v)^2 = \frac{27 \times 4}{200} (8.6 - 4)^2 = 11.4 \text{ horses.}$$

Example 2. Fig. 307. On a breast-wheel is acting $Q = 88$ cubic feet of water per second, the head $h = 8$ feet, velocity of the wheel at the centre of the buckets $v = 5$ feet per second; the water strikes the buckets at an angle $u = 80^\circ$ and velocity $V = 7$ feet per second. Required the horse-power of the wheel, $H = ?$

$$H = \frac{88}{11.4} \left(8 + \frac{5}{25} (7 \times \cos. 80^\circ - 5) \right) = 65 \text{ horses.}$$

Example 3. Required the effect of Poncelet's wheel, the head $h = 4$ feet, and the orifice $a = 5$ square feet, the velocity of the wheel at the centre of pressure of the floats is $v = 6.78$ feet per second?

$$V = 6.91 \sqrt{4} = 13.82 \text{ feet per second.}$$

$$Q = 6.5 \times 5 \times \sqrt{4} = 65 \text{ cubic feet per second.}$$

$$H = \frac{65 \times 6.78}{197} (13.82 - 6.78) = 15.8 \text{ horses.}$$

Example 4. Fig. 309. A saw-mill wheel is to be built under a fall of $h = 18$ feet, and to make $n = 110$ revolutions per minute. Required the proper diameter of the wheel.

$$D = \frac{100}{110} \sqrt{18} = 3.857 \text{ feet,}$$

at the centre of pressure of the buckets.

$$\text{Velocity } V = 8 \sqrt{18} = 33.94 \text{ feet per second.}$$

$$\text{Velocity } v = \frac{3.14 \times 3.857 \times 110}{60} = 22.2 \text{ feet per second.}$$

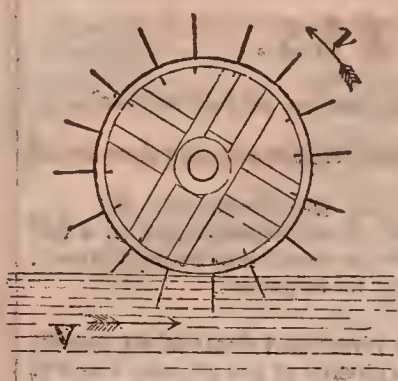
The fall discharged 30 cubic feet of water per second. Required the horse-power of the wheel. $H = ?$

$$H = \frac{30 \times 22.2}{200} (33.94 - 22.2) = 39 \text{ horses.}$$

How many square feet of dry Pine can it saw per hour?

See page 150. $30 \times 39 = 1170$ square feet.

The saw is meant to be applied dire t on the wheel shaft.



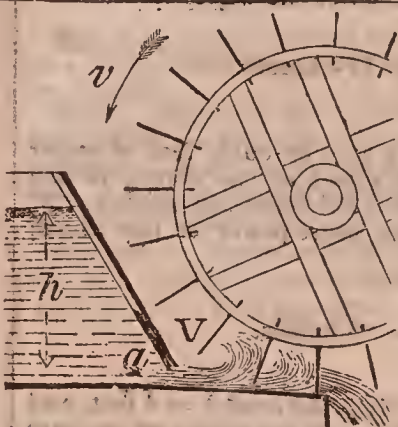
302.

Undershot wheel in a mid-stream.

$$H = \frac{a v}{200} (V - v)^2,$$

When $V = 2v$ about, the effect will be,

$$H = \frac{a V^2}{1600}, \quad a = \text{area of float.}$$



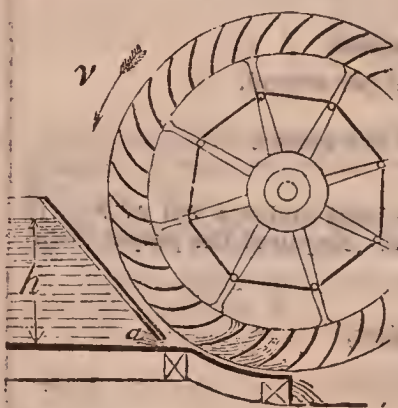
303.

Undershot-Wheel.

$$H = \frac{Q v}{454} (V - v),$$

$$H = \frac{m a v}{86.8} (V - v) \sqrt{h},$$

$$\text{When } V = 2v, \text{ about, } H = \frac{a h \sqrt{h}}{0.47}.$$



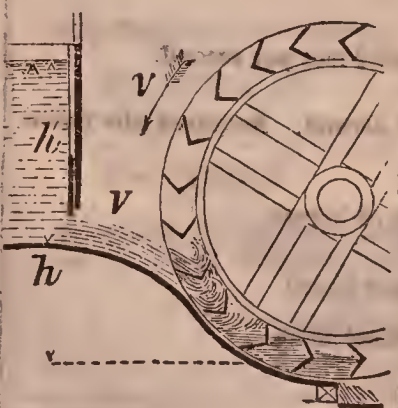
304.

Poncelet's Wheel.

$$H = \frac{Q v}{228} (V - v), \text{ when } h > 5 \text{ feet,}$$

$$H = \frac{Q v}{197} (V - v) \text{ when } h < 5 \text{ feet,}$$

$$Q = 8m a \sqrt{h}, \quad V = 6.91 \sqrt{h}.$$

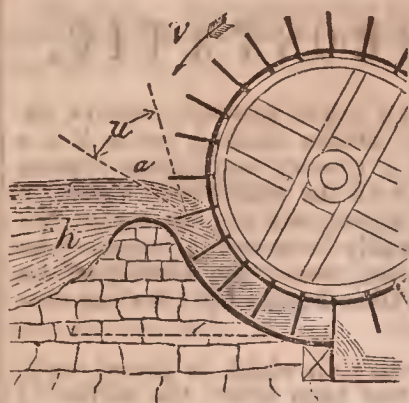


305.

Breast-Wheel with Parabolic drain.

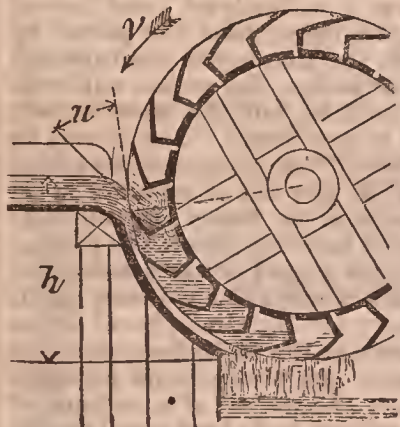
$$H = \frac{Q}{12} \left[h + \frac{v}{28} (V - v) \right],$$

$$Q = 6.5a \sqrt{h'}.$$

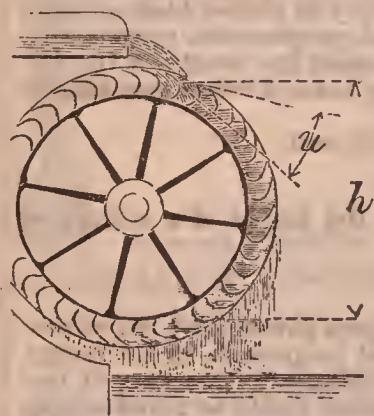
306. *Low-breast Wheel.*

$$H = \frac{Q}{11.2} \left[h + \frac{v}{32} (V \cos u - v) \right]$$

$$Q = kb. V = \frac{Q}{a}. \text{ See table for weirs.}$$

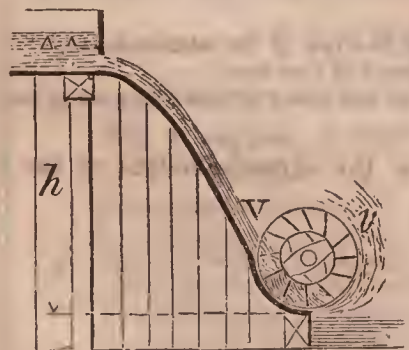
307. *Breast Wheel.*

$$H = \frac{Q}{11.4} \left[h + \frac{v}{25} (V \cos u - v) \right]$$

308. *Over-shot Wheel.*

$$H = \frac{Q}{13.7} \left[h + \frac{v}{21.5} (V \cos u - v) \right]$$

$$\text{Proper velocity about } n = \frac{35 D + 100}{D} \text{ revolutions per minute.}$$

309. *Saw-Mill Wheel.*

$$H = \frac{Q v}{200} (V - v)$$

Proper diameter of the Wheel,

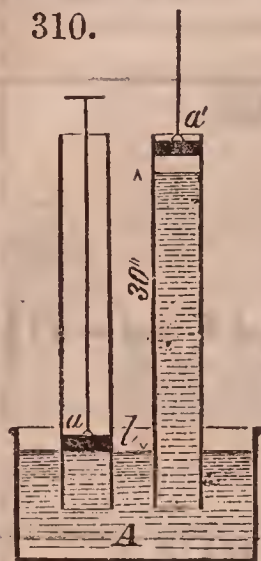
$$D = \frac{106}{n} \sqrt{h}, \text{ in feet,}$$

n = revolutions per min.

ATMOSPHERE. AEROSTATIC.

THE atmosphere round our earth, as well as all other gaseous matters, endeavours to occupy a larger space to infinity, (no known limits,) but as it is a material substance, it is under the action of force of gravity, and cannot expand farther than when its density is in equilibrium with the said force. Conceive the atmosphere to consist of a great number of layers, one on the top of the other; the density of the under layers will evidently be greatest, because the upper ones press on them, and they are all elastic; hence the density of the atmosphere is greater at the surface of the earth than higher up. We can now find out the weight and density of all these layers.

310.



A is a vessel full of mercury, in which is placed vertically a glass tube about 3 feet high above the surface *l*; in the glass tube is fitted, air-tight, piston *a*, just one square inch area, which can be moved by the piston-rod *c*; now the piston stand is at *a* on the level *l*, and in contact with the mercury in the tube; raise the piston by the piston rod and handle *c*, the mercury in the tube will follow until the height of 30 inches, the piston still continues to move higher in the tube, but the mercury will maintain its position at 30 inches from *l*. Now it may be supposed that it is some force of the piston that draws the mercury up in the tube; if so why did it separate at 30 inches? If the column becomes too heavy it could separate at *l*, and the 30 inches of mercury follow the piston; as this is not the case, but the weight of the atmosphere pressing on the surface *l* and forcing the mercury up in the tube until it (the mercury and the atmosphere) comes in equilibrium, which occurs at the 30 inches; and the piston only served to remove the atmospheric pressure in the tube; hence we have the weight of a column of atmospheric air with one square inch base equal to the weight of a column of mercury 30 inches high and one

sq. in. base. One cubic inch of mercury at 60° Fahr. weighs 0.491 pounds, this multiplied by the height, 30 inches, gives 14.73 pounds, the weight of the columns of mercury or atmosphere; this is generally termed "the atmospheric pressure per square inch."

The specific gravity of mercury at 60° Fahr. is 13.58, and

$$\frac{13.58 \times 30}{12} = 33.95 \text{ feet, the height of a column of}$$

water required to balance the atmosphere.

If the temperature and force of gravity were uniform throughout the atmosphere, the density would decrease in an arithmetical progression by the height from the ground, and by observing the altitude of columns of mercury at two different heights, the extreme height of the atmosphere would be found simply by the formula 7, page 64, in which

a = 0 the altitude of the column of mercury at the top of the atmosphere.

b = 30 inches, the column of mercury at the level of the sea.

δ = the difference of the columns of mercury at the level of the sea, and at a height *h* above the sea.

Then, *n* multiplied by the height *h*, should be the extreme height of the atmosphere, or

$$H = h \left(\frac{b}{\delta} + 1 \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad 1,$$

Example. The mountain Chimborazo, Ecuador, (South America,) is $h = 3.87$ miles high above the level of the sea; at its top the column of mercury is observed to be only 27.63 inches, and $\delta = 30 - 27.63 = 2.37$.

$$H = 3.87 \left(\frac{30}{2.37} + 1 \right) = 52.825 \text{ miles, the extreme height}$$

of the atmosphere.

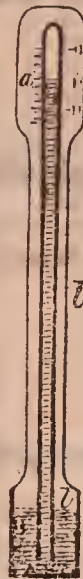
This is about the true height, but the calculation is incomplete by lack of many circumstances accompanied with higher calculus in mathematics, which can not be allowed to occupy room in this work.

The column is 30 inches when the temperature of the atmosphere is 32° Fah.
(See Barometer.)

BAROMETER.

311.

THE Barometer is based upon the same principle as the preceding experiment. It consists of a glass tube *b*, about 35 inches high, open at one end, *c*, and filled with distilled mercury; inverted in a small vessel *A*, also containing mercury. About the top of the column of mercury is placed a scale to indicate the height from *l*. When disturbances take place in the atmosphere by heat, condensation, &c., its weight and density will differ, and the column of mercury will fall and rise accordingly; hence by connection of the scale at *a*, it indicates the disturbance.



To Find the Density of the Atmosphere about a Barometer.

Letters denote.


h = altitude of the column of mercury in inches.

t = temperature of the atmosphere, Fahr.

$S =$ specific gravity of the air around the Barometer at the time h and t are observed.

$S = 1$ when $t = 32^\circ$.

$$S = \frac{h}{0.0624(448.77 + t)} \quad . \quad . \quad . \quad . \quad . \quad 2,$$

 *Example.* The Barometer has fallen to 26.31 inches and the temperature is $t = 60^{\circ}$. Required the specific gravity of the air?

$$S = \frac{26.31}{0.0624(448.77 + 60)} = 0.83.$$

This formula does not include the expansion of mercury from 32° to 60°, which must be separately reduced by the following formula.

$$h = H(0.9967962 - 0.0001001t) \quad , \quad - \quad - \quad - \quad - \quad 3,$$

in which H = the observed column at the temperature t , and h = the true column to be inserted in the formula 1.

To Measure Vertical Heights by the Barometer.

Letters Denote,

H = column of mercury } at the lower station.
 T = temperature of the air }
 h = column of mercury } at the higher station.
 t = temperature of the air }
 l = latitude of the place.
 f = vertical height, in feet, between the higher and lower stations.

$$x = \log. \left(\frac{H}{h} \times \frac{1}{1+0.0001001(T-t)} \right), \quad - \quad - \quad - \quad - \quad 4,$$

$$f = 60345.51x(1+0.002551 \cos. 2l)(1+0.00208(T+t-64^\circ)), \quad - \quad - \quad 5.$$

If the atmosphere is very calm the observations may be made one after the other by one Barometer and detached Thermometer; but the least disturbance of wind requires the observations at the upper and lower stations to be made at the same time. The reduction of the columns of mercury is included in the formula 5.

WIND. AREODYNAMIC.

THE motions and effects of gases by the force of gravity, are precisely the same as that of liquids. (See Hydraulics.)

The altitude or head of the atmosphere at uniform density will be the altitude of a column of water 33.95 feet, divided by the specific gravity of the air, 0.0012046, or,

$$\frac{33.95}{0.0012046} = 28183 \text{ feet,}$$

the velocity due at the foot of this head is (Formula 1, page 183.)

$V = 8.02 \sqrt{28183} = 1346.4$ feet per second, the velocity at which the air will pass into a vacuum.

Velocity of Wind.

When air passes into an air of less density, the velocity of its passage is measured by the difference of their density.

$$\left. \begin{array}{l} H = \\ h = \end{array} \right\} \text{density of the air in inches of mercury.}$$

t = temperature at the time of passage.

V = velocity of the wind in feet per second.

$$V = 1346.4 \sqrt{\frac{H-h}{h} (1+0.00208t)}, \quad - \quad - \quad - \quad - \quad 6.$$

The force of wind increases as the square of its velocity.

a = area exposed at right-angles to the wind, in square feet.

F = force of the wind in pounds.

H = horse-power.

v = velocity of the plane a in direction of the wind, + when it moves opposite, and - when it moves with the wind.

$$F = 0.002288a V^2, \quad \text{when } v = 0, \quad - \quad - \quad 7,$$

$$F = 0.002288a(V \mp v)^2, \quad - \quad - \quad - \quad - \quad 8,$$

$$H = \frac{av(V \mp v)^2}{241400}, \quad - \quad - \quad - \quad - \quad 9.$$

Example. A Rail-train running *ENE* 25 miles per hour, exposes a surface of 1000 square feet to a pleasant brisk gale *NE* by *E*. Required the resistance to the train in the direction it moves, and the horse-power lost?

ENE — *NE* by *N* = 3 points = $33^\circ 45'$.

$V = 14$ feet per second, a brisk gale.

$v = 25 \times 1.467 = 36.6$ feet per second.

$F = 0.002288 \sin. 33^\circ 46' \times 1000(14 + \cos. 33^\circ 45' \times 36.6)^2 = 305.1$ pounds.

$$H = \frac{305.1 \times 36.6}{550} = 20 \text{ horses.}$$

<i>M</i> les per hour.	feet per second.	Force per sq. ft pound.	Common Appellation of the Force of Wind.
1	1.47	0.005	Hardly perceptible.
2	2.93	0.020	Just perceptible.
3	4.4	0.044	
4	5.87	0.079	
5	7.33	0.123	Gentle pleasant wind.
6	8.8	0.177	
7	10.25	0.241	
8	11.75	0.315	Pleasant brisk gale.
9	13.2	0.400	
10	14.67	0.492	
12	17.6	0.708	Very brisk.
14	20.5	0.964	
15	22.00	1.107	
16	23.45	1.25	High wind.
18	26.4	1.55	
20	29.34	1.968	
25	36.67	3.075	Very high.
30	44.01	4.429	
35	51.34	6.027	
40	58.68	7.873	Storm or tempest.
45	66.01	9.963	
50	73.35	12.30	
55	80.7	14.9	Great storm.
60	88.02	17.71	
65	95.4	20.85	
70	102.5	24.1	Hurricane.
75	110	27.7	
80	117.36	31.49	
100	146.66	50	Tornado, tearing up trees, &c.

BALLOON.

To Find what Weight, and to what Height a Balloon can raise.

Letters denote.

C = cubic contents of the balloon, in feet.

s = specific gravity of the gas used to inflate the balloon, air = 1 at 32°.

W = the weight in pounds, it can raise from the ground.

w = the weight with which it is loaded, including the weight of the materials of which it is made.

f = height in feet to which it will raise.

T = temperature at the ground.

t = temperature at the height *f*.

H = Barometer column in inches, at the ground.

l = latitude of the place.

$$W = 0.07529C(1 - s), \quad 10,$$

$$x = \log. \left(\frac{Hw}{W} \times \frac{1}{1 + 0.0001001(T - t)} \right) \quad 11,$$

$$f = 60345.512(1 + 0.002551 \cos. 2l)(1 + 0.00208(V + t - 64)) \quad 12.$$

Balloons are commonly filled with Hydrogen gas whose specific gravity is *s* = 0.07, when pure, or about 14 times lighter than air, of which say 10 times to be relied upon, as some foreign heavier gases may accompany it.

WIND-MILLS.

The sail-shaft of vertical wind-mills should have an inclination from 12° to 15° with the level when built on low flat ground; on high ground, elevated from 1000 to 1500 feet within a circle of about two miles, the sail-shaft should incline from 3° to 6° with the level

Effect of Wind-Mills.

Letters denote.

A = projecting area of sails exposed to the wind, in square feet.

V = velocity of the wind in feet per second.

H = horse-power of the mill.

R = extreme } radii of sails in feet.
 r = inner }

$l = \sqrt{\frac{R^2 + r^2}{2}}$, radius of centre of percussion in feet.

n = number of revolutions of sails per minute.

v = mean angle of sails to the plane of motion.

The angle of the sails should be from 20° to 30° at the inner radius r , at the extreme radius R from 7° to 12° , and the mean angle $v = 15^\circ$ to 17° .

$$H = \frac{A l n \sin v \cos v}{1,540,000} \left(V - \frac{b n \sin v}{9.5} \right)^2$$

assume the mean angle $v = 16^\circ$, we have the horse power.

$$H = \frac{A l n}{5,800,000} \left(V - \frac{l n}{34.5} \right)^2$$

In order to utilise the maximum effect of wind, it is necessary to load the mill that the number of revolutions of the sails are proportional to the velocity of the wind.

Proper revolutions will be found by $n = \frac{3.16 V}{l \sin v}$.

$$\text{If } v = 16^\circ, n = \frac{11.5 V}{l}, \quad H = \frac{A V^3}{1,135,000}, \quad \text{and } A = \frac{1,135,000 H}{V^3}.$$

Example 1. A wind-mill is to be built of six horse power in brisk wind, $V = 20$ feet per second. Required the area of sails $A = ?$

$$A = \frac{1,135,000 \times 6}{20^3} = 851 \text{ sq. feet.}$$

Example 2. Four sails $\frac{851}{4} = 212.75$ sq. feet each. $212.75 = 6$ feet wide by 35.5 long, dimensions of the sails. Inner radius $r = 5$ feet and $R = 5 + 35.5 = 40.5$ feet. Required the radius of centre of percussion $l = ?$

$$l = \sqrt{\frac{40.5^2 + 5^2}{2}} = \sqrt{832.6} = 28.85 \text{ feet.}$$

Example 3. The mean angle of sails to be $v = 16^\circ$. Required the proper number of revolutions of the sails per minute in brisk wind of $V = 20$ feet per second $n, = ?$

$$\text{Revolutions } n = \frac{11.5 \times 20}{28.85} = 8 \text{ per minute.}$$

Example 4. A wind-mill has an area of $A = 750$ sq. feet exposed to high wind of $V = 50$ feet per second, and makes $n = 26$ revolutions per minute,—centre of percussion $l = 25$. Required the horse power of the mill $H = ?$ and proper number of revolutions per minute $n = ?$

$$H = \frac{750 \times 25 \times 26}{5,800,000} \left(50 - \frac{25 \times 26}{34.5} \right)^2 = 7.8 \text{ horses.}$$

$$n = \frac{11.5 \times 50}{25} = 23 \text{ rev. per minute.}$$

$$H = \frac{750 \times 25 \times 23}{5,800,000} \left(50 - \frac{25 \times 23}{34.5} \right)^2 = 8.26 \text{ horses.}$$

Composition of air by weight,	{ 75·55 Nitrogen. 22·32 Oxygen. 1·32 Carbonic acid and water vapor.
Composition of water by weight,	{ 88·9 Oxygen. 11·1 Hydrogen.
Composition of water by bulk,	{ 1 Oxygen. 2 Hydrogen.
Height of columns that press one pound per square inch.	{ 2·3 feet of water, fresh. 2·23 " sea-water. 2·05 inches of mercury at 60°.

LIGHT.

Light is the sensation transmitted by the eye and produces the sense of seeing.

Heat and Electricity produce light by making bodies luminous.

Intensity of Light is inverse as the distance from the luminous body.

Velocity of Light is 192500 miles per second.

Light passes from the sun 95000000 miles in 8 minutes.

Light can pass around the whole earth in $\frac{1}{8}$ of a second.

Solids must be heated to at least 600° to produce light in the dark; and to 1000° in day-light.

SOUND.

Velocity of Sound through Air,

v = velocity in feet per second.

t = temperature of the air, Fah. scale.

D = distance in feet the sound travels in the time T .

$$v = 1089 \cdot 42 \sqrt{1 + 0 \cdot 00208(t - 32)},$$

Velocity of sound in water is about 4 times that in air, and 8 times that through solids.

Intensity of sound is inversely as the square of the distance.

$$D = 1089 \cdot 42 T \sqrt{1 + 0 \cdot 00208(t - 32)},$$

$$T = \frac{D}{v}.$$

Example. A ship at sea was seen to fire a cannon, and 6·5 seconds afterwards the report was heard, the temperature in the air was 60°. Required the distance to the ship?

$$D = 1089 \cdot 42 \times 6 \cdot 5 \sqrt{1 + 0 \cdot 0028(60^\circ - 32)} = 7300 \text{ feet, or } 1 \cdot 38 \text{ miles.}$$

Descriptions of Sound.	Audible at a distance of	
	feet.	miles.
A powerful human voice in the open air, no wind,	460	0·087
Report of a musket, - - - - -	16000	3·02
Drum, - - - - -	10560	2
Music, strong brass band, - - - - -	15840	3
Cannonading, very strong, - - - - -	575000	90
In a barely observable breeze a strong human voice with the wind can be heard, - - - - -	15840	3

RINGING BELLS.

Letters denote.

D = diameter of the bell at the mouth, in inches.

d = diameter of the bell at the crown, in inches.

h = height of the bell from the mouth to the crown in inches.

S = thickness of sound bow in inches.

W = weight of the bell in pounds avoirdupois.

n = number of vibrations per second, corresponding with the key note of the bell, and to be found in the accompanying table I.

k = from 0.07 to 0.08, or a coefficient expressing the relative thickness of the sound bow to the diameter of the bell. In peals of bells, the sound bow is generally $S = 0.08D$ for the triple, and $S = 0.07D$ for the tenor; the intermediate bells in the peal having the intermediate proportions of sound bow.

Example 1. Required the weight of a bell $D = 62$ inches in diameter, and $S = 4\frac{1}{2}$ in. thickness of the sound bow, $W = ?$

$$\text{Formulae 1. } W = 0.25 \times 62^2 \times 4.5 = 4324.5 \text{ pounds.}$$

Example 2. A bell of 2,500 pounds is to be constructed with a sharp note, taking the sound bow $k = 0.075$. Required the diameter of the bell $D = ?$

$$\text{Formulae 10. } D = \sqrt[3]{\frac{4 \times 2500}{0.075}} = 51.084 \text{ inches}$$

Example 3. It is required to construct a bell with the key note D^{\sharp} in the first octave above zero, $n = 152.22$. To be of light weight with a full good note, for which latter case take $k = 0.07$. Required the diameter of the bell, $D = ?$

$$\text{Formulae 11. } D = \frac{58000 \times 0.07}{152.22} = 26.665 \text{ inches.}$$

Example 4. Required the key note of a bell with $D = 36.5$ in. diameter and $S = 2.75$ in., $n = ?$

$$\text{Formulae 4. } n = 58000 \times \frac{2.75}{36.5^2} = 119.7 \text{ vibrations.}$$

In the table the nearest number 120.82, in the first octave below zero, answers to the key note B , which will be the note of the bell.

Example 5. A bell of 6860 pounds is to be constructed with the key note C in the first octave below zero $n = 64$, see table I. Required the diameter of the bell $D = ?$

$$\text{Formulae 9. } D = 21.947 \sqrt[4]{\frac{6860}{64}} = 70.6175 \text{ inches}$$

Example 6. Required the thickness of sound bow for the bell in the preceding example? $D = 70.6175$ inches and $n = 64$. $S = ?$

$$\text{Formulae 12. } S = \frac{64 \times 70.6175^2}{58000} = 5.5027 \text{ inches.}$$

Example 7. Required the weight of a bell $D = 48$ inches diameter at the mouth, $d = 25$ inches at the crown, and $h = 34$ inches height from the mouth to the crown, $S = 3.5$ in., $W = ?$

$$\text{Formulae 17.}$$

$$W = 48 \times 25 \times 3.5 (0.5 - 0.002816 \times 25) + 0.00375 \times 34 \times 25^2 \times 3.5 = 2126.226 \text{ pounds.}$$

Formulas for Ringing Bells.

$$\begin{array}{llll}
 W = 0.25 D^2 S & \dots & 1 & D = 2 \sqrt{\frac{W}{S}} & \dots & 7 & S = \frac{\pi D^2}{58000} & \dots & 12 \\
 W = \frac{D^4 n}{232000} & \dots & 2 & D = 240.83 \sqrt{\frac{S}{n}} & \dots & 8 & S = \frac{4W}{D^2} & \dots & 13 \\
 W = 0.25 D^3 k & \dots & 3 & D = 21.947 \sqrt[4]{\frac{W}{n}} & \dots & 9 & S = k D & \dots & 14 \\
 n = 58000 \frac{S}{D^2} & \dots & 4 & D = \sqrt[3]{\frac{4W}{k}} & \dots & 10 & k = \frac{S}{D} & \dots & 15 \\
 n = 232000 \frac{W}{D^4} & \dots & 5 & D = 58000 \frac{k}{n} & \dots & 11 & k = \frac{4W}{D^3} & \dots & 16 \\
 n = 58000 \frac{k}{D} & \dots & 6 & & & & & & \\
 W = D d S (0.5 - 0.0002816 d) + 0.00375 h d^2 S & \dots & 17
 \end{array}$$

Table I. Vibrations per Second = n .

Key note.	Bass.			Zero.	Descant.	
	3rd Oct.	2nd Oct.	1st Oct.		1st Oct.	2nd Oct.
C	16.000	32.000	64.000		128.00	256.00
C \sharp	16.947	33.885	67.790		135.58	271.00
D	17.960	35.920	71.840		143.68	287.36
D \sharp	19.027	38.055	76.110		152.22	304.44
E	20.159	40.318	80.636		161.27	322.54
F	21.357	42.715	85.430		170.86	341.72
F \sharp	22.627	45.255	90.510		181.02	362.04
G	23.972	47.945	95.890		191.78	383.56
G \sharp	25.398	50.797	101.59		203.19	406.37
A	26.908	53.817	107.63		215.27	430.53
A \sharp	28.508	57.017	114.03		228.07	456.13
B	30.204	60.409	120.82		241.63	483.27
C	32.000	64.000	128.00		256.00	512.00

Table V.

Abscissa x	Ordinate y	Thickness of Metal.			
		$S = 1$	$S = 0.07D$	$S = 0.75D$	$S = 0.08D$
1	0.4142	1	0.700	0.750	0.800
1 $\frac{1}{2}$	0.686	0.800	0.560	0.600	0.640
2	0.867	0.653	0.459	0.490	0.522
2 $\frac{1}{2}$	0.974	0.547	0.382	0.410	0.437
3	1.025	0.474	0.331	0.355	0.379
3 $\frac{1}{2}$	1.030	0.423	0.295	0.317	0.338
4	1.000	0.380	0.266	0.285	0.304
4 $\frac{1}{2}$	0.955	0.351	0.245	0.263	0.281
5	0.875	0.327	0.228	0.245	0.261
5 $\frac{1}{2}$	0.775	0.301	0.211	0.226	0.241
6	0.665	0.291	0.203	0.218	0.233
6 $\frac{1}{2}$	0.530	0.286	0.200	0.214	0.228
7	0.390	0.279	0.195	0.209	0.223
7 $\frac{1}{2}$	0.235	0.272	0.190	0.204	0.217
8	0.075	0.267	0.186	0.200	0.213
8.74	0.78	0.333	0.233	0.250	0.266

To Construct a Bell.

When a bell is to be constructed, we generally have the weight or key note given by contract, the diameter and sound bow are calculated by the preceding formulae and examples, and then ready to proceed with the construction.

The diameter of the bell at the mouth, is divided into 10 equal parts, called strokes, which then is the scale and measurement for the construction. Make a decimal scale, as shown on plate VII.

The section of a bell is generally laid out on a piece of board represented by the dotted lines *a, b, c, d*, which then is cut out and used for turning up the mould for the bell. The board should be about 11 strokes long, and 2.5 strokes wide. Through the centre of the board draw the line *p, q*, parallel to *b, c*,—bisect the line *p, q*, and set four (4) strokes from the bisecting point towards each end, divide the strokes into halves, and number them as shown on the accompanying drawing. Through each division draw lines at right angles to *p, q*, set off the corresponding ordinates *y* expressed in strokes, table II, and join them by a curve-line, which then will be the centre of thickness of metal in the bell.

In the end of the first ordinate, as a centre, draw a circle with a diameter equal to the desired thickness of the sound bow, which should be from 0.7 to 0.8 strokes. At every succeeding ordinate draw a circle with the diameter noted in table II; for instance, if the thickness of the sound bow is $4\frac{1}{2}$ inches, then the thickness of metal or diameter is the circle at the third ordinate will be $4.5 \times 0.474 = 2.133$ inches; but if the sound bow is 0.7, 0.75 or 0.8 strokes, the thickness of metal at the third ordinate will be 0.331, 0.355, or 0.379 strokes. When all the thicknesses are thus drawn, draw the two lines tangent to the circles on each side of the centre line of the metal. From the origin *o*, draw the tangents to the sound bow; the line *o1* should not quite tangent, but leave a moulding about 0.02 strokes, as represented by the drawing. Prolong the $6\frac{1}{2}$ ordinate, and set off 1.79 strokes to *e*, which then is the centre for the curve on the top, draw the arc through the centre of the small circle at the 8th ordinate; join *e, 8*, set off from *e*, 0.46 strokes to the centre for the inside curve at the top.

Thickness of metal of the top should be 0.3 the sound bow at 8, and 0.333 at *r*.

Draw the ordinate at 8.74, set off 0.78 to *r*, join *r* and the abscissa 8.48, and prolong the line through *r*; then finish the drawing as shown on the plate.

When the board is cut out and ready for turning the mould, it must be carefully set, so that the outside diameter of the crown will be half the diameter of the mouth of the bell.

Should the constructor desire to make the bell lower in proportion to the diameter, he can do so by shortening the radius for the top of the crown, or lowering the crown; though not lower than to the seventh ordinate, which only affects the *continuation* of the *sound*. But whatever height chosen, it is necessary to maintain the diameter of the abscissa at 7.9 strokes, equal to half the diameter of the mouth of the bell, or to maintain the same position of the board to the axis of the bell, in order to attain the *maximum quality of tone*.

Clapper.

The weight of the clapper should be from one-fortieth to one-fiftieth the weight of the bell, the small bells take the largest clapper, and vice versa.

Bell Metal.

Thirty pounds of Tin to one hundred pounds of Copper is a good proportion.



HEAT. CALORIC.

THE *Physical constitution* of heat is yet under investigation by operative minds, its well known character and effect upon matter is the base for the investigation.

Heat resembles *light, electricity and magnetism*, and is thus far assumed to be a material substance.

Heat is contained in all matters, with no known exception. Two bodies containing different quantities of heat per unit, being placed in contact,—the heat will pass from one to the other, until it comes in equilibrium, that is when the two bodies contain equal quantities of heat per unit. It is this passing of heat that first comes under our notice. The body from which the heat passes will feel the other to be cold, and *vice versa*,—the one that receives the heat will feel the other to be warm, until there is no further passage, namely, when the bodies will feel neither warm nor cold to each other; hence the measure of the *emplex sensible heat*, is the difference between the heat per unit in the two bodies. Cold is only a want of heat.

Caloric is only another word of expression for heat.

Caloric is of two kinds, *sensible and latent*.

Sensible Caloric is that which is sensible to the touch, felt as temperature, and can pass freely from one body to another.

Latent Caloric is that which is insensible to the touch. It is contained in bodies without being felt as temperature, but can by chemical action become sensible; for instance, a piece of burned limestone put into water will get warm and heat it, although both were cold before; the latent caloric in the water was set free to sensible.

Influence of Heat on Matter's Coherence.

All bodies in nature expand when heated, and contract when cooled. Solid bodies vary but little by the difference in temperature. Liquids vary more, but gases are extremely susceptible to the impression of heat and cold.

Table of Linear Expansion of Solids.

Difference in temperatures.	Length = 1 at 32°. Names of bodies.	Length at T.°	k difference length per degree.
32° to 212°	Glass, - - -	1·00086183	0·00000478
32 to 392		1·00184520	0·00000546
32 to 572		1·00303252	0·00000583
32 to 212	Wrought iron, - - -	1·00118210	0·00000656
32 to 572		1·00440528	0·00000894
32 to 212		1·00122045	0·00000680
32 to 212	Soft good iron, - - -	1·00123504	0·00000687
32 to 212	Iron wire, - - -	1·00111120	0·00000618
32 to 212	Cast iron, - - -	1·00107915	0·00000600
32 to 212	Soft steel, - - -	1·00123956	0·00000689
32 to 212	Steel hardened and tem. 150°,	1·00171820	0·00000955
32 to 572	Copper, - - -	1·00564972	0·00001092
32 to 212		1·00284836	0·00001580
32 to 212		1·00146606	0·00000815
32 to 212	Gold, pure, - - -	1·00149530	0·00000830
32 to 212	Gold, hammered, - - -	1·00190868	0·00001060
32 to 212	Silver, pure, - - -	1·00201000	0·00001116
32 to 212	Silver, hammered, - - -	1·00187821	0·00001043
32 to 212	Brass, common cast, - - -	1·00193333	0·00001075
32 to 212	Brass wire or sheet, - - -	1·00088420	0·00000491
32 to 572	Platinum, pure, - - -	1·00275482	0·00000520
32 to 572		1·00095420	0·00000530
32 to 212		1·00294167	0·00001633
32 to 212	Zinc, pure or cast, - - -	1·00310833	0·00001722
32 to 212	Zinc, hammered, - - -	1·00270000	0·00001500
32 to 212	Tin, hammered, - - -	1·00271298	0·00001207
32 to 212	Tin, cast, - - -	1·00042280	0·00000235
32 to 212	Fire brick, - - -	1·00110410	0·00000613
33 to 212	Marble, - - -	1·00078940	0·00000438
32 to 212	Granite, - - -		

Table of Volume Expansion of Liquids.

<i>Difference in temperatures.</i>	<i>Names of Liquids.</i>	<i>Volume at T.^o</i>	<i>k difference in vol. per degree.</i>
32° to 212°	Mercury, - - -	1·00018018	0·000001000
212 to 392	" - - -	1·00018433	0·000001025
392 to 572	" - - -	1·00018868	0·000001048
32 to 212	Water, - - -	1·00046600	0·000002595
32 to 212	Salt, dissolved, - -	1·00050000	0·000002778
32 to 212	Sulphuric acid, - -	1·00060000	0·000003333
32 to 212	Oil of Turpentine and Ether,	1·00070000	0·000003890
32 to 212	Oil, common, - -	1·00080000	0·000004444
32 to 212	Alcohol and Nitric acid,	1·00100000	0·000005555

All gases expand and contract equally and uniformly; 0·0020825 its volume per degree of Fah. thermometer. The accompanying Table is the result of Mr. Dalton's experiments with air. The volume at 32° is equal to 1 or the unit.

Table for Volume Expansion of Air.

<i>Degrees.</i>	<i>Volume.</i>	<i>Degrees.</i>	<i>Volume.</i>	<i>Volume</i>
32°	1·000	80	1·1110	1 $\frac{7}{64}$
33	1·002	85	1·121	1 $\frac{1}{8}$
34	1·004	90	1·132	1 $\frac{1}{7 \cdot 56}$
35	1·007	100	1·152	1 $\frac{1}{6 \cdot 57}$
40	1·021	200	1·354	1 $\frac{1}{3}$
45	1·032	212	1·376	1 $\frac{1}{2 \cdot 66}$
50	1·043	302	1·558	1 $\frac{9}{16}$
55	1·055	392	1·739	1 $\frac{3}{4}$
60	1·066	482	1·919	1 $\frac{15}{16}$
65	1·077	572	2·098	2 $\frac{3}{2}$
70	1·089	680	2·312	2 $\frac{5}{16}$
75	1·099			

Letters Denote,

L = length or any linear measure of the body of the temperature T .

l = length or linear measure at the temperature t .

V = volume of liquids at the temperature T :

v = volume at the temperature t .

k = coefficient for the linear measure or volume as noted in the Tables.

The volume of solids is as $L^3 : l^3$.

The linear measure of liquids is a $\sqrt[3]{v} : \sqrt[3]{V}$.

Formulas of Linear Expansion of Solids.

$$L = l \left(1 + k(T - t) \right), \quad l = \frac{L}{1 + k(T - t)},$$

$$T = \frac{L - l}{kl} + t, \quad t = T - \frac{L - l}{kl},$$

Example 1. A copper rod of $L = 22\cdot55$ feet long is 140° warm. To what temperature must it be cooled to fit in a space of $l = 22\cdot52$ feet?

$$t = 140 - \frac{22\cdot55 - 22\cdot52}{22\cdot52 \times 0\cdot0000158} = 55\cdot7^\circ \text{ the answer.}$$

Formulas of Volume Expansion of Liquids.

$$V = v \left(1 + k(T - t) \right), \quad v = \frac{V}{1 + k(T - t)}$$

$$T = \frac{V - v}{k v} + t, \quad t = T - \frac{V - v}{k v}$$

Example 2. A vessel containing 5.68 cubic feet of water at $t = 42^\circ$, is closed up round the water, but a cylindrical pipe of 0.008 square feet, inside section, is raised up vertically from it; now let the temperature of the water be raised to $T = 130^\circ$. How high will the water rise in the pipe?

$$V = 5.68 [1 + 0.000002595(130 - 42)] = 5.681297 \text{ cubic feet,}$$

$$\text{and} \quad \frac{5.681297 - 5.68}{0.008} = 0.162 \text{ feet} = 1.945 \text{ inches,}$$

the height to which the water will raise in the pipe.

This is the principle upon which Thermometers are constructed, but the scale can only be approximated by this formula. The substances adopted for thermometers are spirits of wine and mercury; oil and ether has also been proposed, but the two former are best, and mercury is most generally used.

THERMOMETERS.

THERE are three different graduated Thermometers in use, namely *Fahrenheit's Celcius's*, and *Reamur's*.

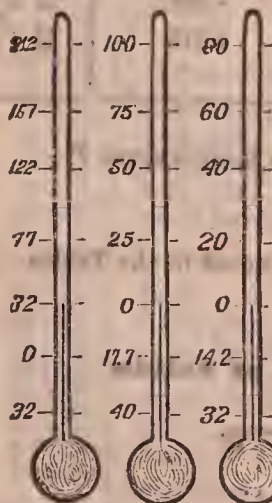
The first one, or *Fahrenheit's* is used in North America, England, and Holland.

The second or *Celcius's* in France, Sweden and Germany.

The third one, or *Reamur's*, was formerly used in France and some parts of Germany, but now only in Spain.

The Figures exhibit their difference.

Fahr. Celci. Ream.



Proportional Formulas for the Thermometrical Scales.

$$\text{Celci.} = \frac{5}{4} \text{ Ream.} = \frac{5}{9} (\text{Fahr.} - 32.)$$

$$\text{Ream.} = \frac{4}{5} \text{ Celci.} = \frac{4}{9} (\text{Fahr.} - 32.)$$

$$\text{Fahr.} = \frac{9}{5} \text{ Celci.} + 32 = \frac{9}{4} \text{ Ream.} + 32.$$

Example. How much is 68° Celcius on Fahrenheit's scale.

$$\text{Fahrenheit's} = \frac{9}{5} \times 68 + 32 = 154.4^\circ, \text{ the answer.}$$

Fluid boils when its vapour has the same density as the atmosphere where it boils, hence, fluid will boil sooner high up in the atmosphere than at the ground.

In vacuum water boils at 88° .

The mean temperature of the earth is about 50° ; at the torrid zone 75° . temperature one 50° ; and in the polar regions 36° .

Water can be kept in liquid to 20° .

Table of Temperatures when Bodies change Form.**SMELTING POINTS.**

Cast iron, fully sm.,	-	-	2754°
Gold, fine,	-	-	1983°
Silver, fine,	-	-	1850°
Copper,	-	-	2160°
Brass, common,	-	-	1900°
Zinc,	-	-	740°
Lead,	-	-	594°
Bismuth,	-	-	476°
Tin,	-	-	421°
1 Tin, 1 Bismuth,	-	-	283°
3 Tin, 2 Lead, 5 Bismuth,	-	-	212°
1 Tin, 1 Lead, 4 Bismuth,	-	-	201°
Antimony,	-	-	790°
Sulphur,	-	-	228°
Phosphorus,	-	-	109°
Beeswax, white,	-	-	155°
“ yellow,	-	-	142°
Tallow,	-	-	92°
Ice,	-	-	32°
Oil of Turpentine,	-	-	14°
Ice of strong Brandy,	-	-	7°
1 Snow and 1 Salt,	-	-	0°
Mercury,	-	-	- 39°

BOILING POINTS.

Mercury,	-	-	630°
Oil of Linseed,	-	-	600°
Sweet Oil,	-	-	412°
Sulphuric acid,	-	-	410°
Sulphur,	-	-	390°
Phosphorus,	-	-	374°
Oil of turpentine,	-	-	315°
Sea-water, salt,	-	-	217°
Water, distilled,	-	-	212°
Alcohol,	-	-	174°

MISCELLANEOUS.

Metals, red, daylight,	-	-	1077°
Iron red, daylight,	-	-	884°
Common fire,	-	-	790°
Iron bright red, in dark,	-	-	752°
Human blood is	-	-	98°
Cold greatest ever produced,	-	-	— 90°
Vinous fermentation,	-	-	— 60 to 70°
Acetous fermentation begins,	-	-	— 78°
Acetification ends,	-	-	— 88°
Phosphorous burns,	-	-	— 43°
A comfortable room about	-	-	60° to 70°

Table of Power for Transmission of Heat.**CONDUCTING POWER.**

Gold,	-	-	1000
Silver,	-	-	973
Iron,	-	-	347
Tin,	-	-	304
Copper,	-	-	898
Zinc,	-	-	363
Lead,	-	-	180
Platinum,	-	-	981
Marble,	-	-	24
Fire-brick,	-	-	11
Fire-clay,	-	-	11.4
Porcelain,	-	-	12.2

Water as the Standard.

Water,	-	-	10
Pine,	-	-	39
Lime,	-	-	39
Oak,	-	-	33
Elm,	-	-	32
Ash,	-	-	31
Apple,	-	-	28
Ebony,	-	-	22

RELATIVE CONDUCTING POWERS OF SOLIDS.

Hare's fur,	-	-	1.315
Eider-down,	-	-	1.305
Beaver's fur,	-	-	1.296
Raw silk,	-	-	1.284
Wool,	-	-	1.118
Lamp-black,	-	-	1.117
Cotton,	-	-	1.046
Lint,	-	-	1.032
Charcoal,	-	-	0.936
Ashes of wood,	-	-	0.927

Sewing-silk,	-	-	0.917
Air,	-	-	0.577

RELATIVE CONDUCTING POWER OF FLUIDS.

Mercury,	-	-	1000
Water,	-	-	357
Proof spirit,	-	-	312
Alcohol pure,	-	-	332

RADIATING POWER.

Water,	-	-	100
Lampblack,	-	-	100
Paper, writing,	-	-	98
Rosin,	-	-	96
Sealing wax,	-	-	95
Glass, common,	-	-	90
India ink,	-	-	88
Ice,	-	-	85
Red Lead,	-	-	80
Graphit,	-	-	75
Lead, tempered,	-	-	45
Mercury,	-	-	20
Lead, polished,	-	-	19
Iron, polished,	-	-	15
Tin and Silver,	-	-	12
Copper and Gold,	-	-	12

REFLECTING POWERS.

Brass,	-	-	100
Silv r,	-	-	90
Tinfoilium	-	-	85
Tin,	-	-	80
Steel,	-	-	70
Lead,	-	-	60
Glass,	-	-	10
Glass, oiled or waxed,	-	-	5
Lampblack,	-	-	0

Mixtures of.	Cold produce.	Degrees Fahr.
Nitrate of Ammonia, 1 } Water, 1 }	46°	From +50° to +4°.
Sulphate of Soda, 8 } Muriatic Acid, 5 }	50°	From +50° to +0°.
Dilute Sulphuric Acid, 5 } Snow, 4 }	23°	From -68° to -91°.

SPECIFIC CALORIC.

Specific Caloric is the relative quantity of heat contained in bodies of equal weight or volume, and of the same temperature.

Let two different substances of known weight or volume and temperature, be mixed together; the temperature of the mixture will dissolve the relative quantity of caloric in the ingredients.

Mixture of the same Substances.

Letters denote.

W = weight or volume of a substance of temperature T .

w = weight or volume of a similar substance but temperature t .

t' = temperature of the mixture $W+w$. We shall have,

$$t'(W+w) = WT + wt, \quad t' = \frac{WT + wt}{W+w},$$

$$W = \frac{w(t' - t)}{T - t'}, \quad T = \frac{w(t' - t)}{W} + t'.$$

Example 1. Let $W = 4.62$ cubic feet of water at $T = 150^\circ$ be mixed with $w = 5.43$ cubic feet at $t = 46$. Required the temperature of the mixture $t' = ?$

$$t' = \frac{4.62 \times 150 + 5.43 \times 46}{4.62 + 5.43} = 97.6^\circ \text{ the answer.}$$

Example 2. How much water of $T = 107^\circ$ must be mixed to $w = 27.3$ gallons of $t = 58^\circ$, the mixture of the water to be 75° ?

$$W = \frac{27.3(75 - 58)}{107 - 75} = 14.5 \text{ gallons.}$$

Mixture of different Substances.

W and w expressed by *weights* only. S and s = *Specific caloric* as given in the accompanying Table. We shall have,

$$WS(T - t') = ws(t' - t), \quad t' = \frac{WS T + ws t}{WS + ws}.$$

$$W = \frac{ws(t' - t)}{S(T - t)}, \quad T = \frac{t'(WS + ws) - ws t}{WS}.$$

Example 3. To what temperature must $W = 20$ pounds of iron be heated to raise $w = 131$ pounds of water of $t = 54^\circ$ to a temperature $t' = 64^\circ$. $T = ?$
From the Table we have $s = 1$, and $S = 0.1218$.

$$T = \frac{64(20 \times 0.1218 + 131) - 131 \times 1 \times 54}{20 \times 0.1218} = 602^\circ.$$

the required temperature, supposing no vapour escapes from the water.

If any chemical action takes place in the mixture, these formulas will not answer, because part of the *sensible caloric* may become *latent*, or *latent caloric* may be set free.

Table of Specific Caloric, Water as Unit.

Names of Substances.	Specific Caloric.		
	32° to 212°.		32°, 572°.
Water, - - - - -	1.0000		
Iron, - - - - -	0.1105		
Glass-crystal, - - - - -	0.1929		
Mercury, - - - - -	0.029	to 0.033	0.035
Lead, - - - - -	0.02819	to 0.0293	
Tin, - - - - -	0.04755	to 0.0514	
Sulphur, - - - - -	0.2085	to 0.188	
Lime, burned, - - - - -	0.2169		
9 Water, 10 Lime, - - - - -	0.43912		
Sulphuric acid, sp. g. = 1.87058, - - - - -	0.3346		
Nitric acid, sp. g. = 1.29895, - - - - -	0.66139		
Alcohol, sp. g. = 0.81, - - - - -	0.7		
Platinum, - - - - -	0.0344	to 0.0335	0.0355
Antimony, - - - - -	0.0507		0.0547
Zinc, - - - - -	0.0927		0.1015
Copper, - - - - -	0.094	to 0.0949	0.1013
Iron, - - - - -	0.1098	to 0.1105	0.1218
Glass, - - - - -	0.1770		0.19
Gold, - - - - -	0.0288		
Bismuth, - - - - -	0.0298		
Woods in average, - - - - -	0.48	to 0.6	
Sweet Oil, - - - - -	0.30961		
Nickel, - - - - -	0.1035		
Cobalt, - - - - -	0.1498		
Tellurium, - - - - -	0.0912		

SPECIFIC CALORIC OF GASES AT EQUAL DENSITY.

	Volume. air = 1.	Weight. air = 1.	Weight. water = 1.
Air, atmospheric, - - - - -	1.000	1.000	0.2669
Hydrogen, - - - - -	0.9033	12.34	3.2936
Oxygen, - - - - -	0.9764	0.8848	0.2361
Nitrogen, - - - - -	1.0000	1.0318	0.2754
Carbonic-oxid gas, - - - - -	1.034	1.0805	0.2884
Carbonic acid, - - - - -	1.2583	0.828	0.221
Nitro-oxid gas, - - - - -	1.3505	0.8878	0.2369
Gas of oils, - - - - -	1.553	1.5763	0.4207
Steam, - - - - -	1.96	3.136	0.847

Capacity for Caloric is the relative ability of bodies to retain the *specific caloric*. Capacity for caloric is inverse as the density of the substances. The *specific caloric* multiplied by the atom weight of a substance, gives the constant number 0.375 (average) which proves that the atoms have equal capacity for caloric in all substances. This is a fact with no known reason, but by it valuable results may be opened.

Table of Relative Capacity for Caloric.

Names.	Equal Weights.	Equal Volume.	Names.	Equal Weights.	Equal Volume.
Water	1.000	1.000	Zinc	0.102	
Copper	0.114	1.027	Tin	0.060	
Iron	0.126	0.993	Lead	0.043	0.487
Brass	0.116	0.971	Glass	0.187	0.443
Gold	0.050	0.966			
Silver	0.082	0.833			

When the volume diminishes the capacity for caloric will also be diminished and thus part of the caloric will profuse the body. A volume of air compressed to $\frac{1}{5}$ its bulk will fire tinder, which requires a temperature of about 550°.

STEAM.

Steam is the vapour into which water is converted by the application of heat.

Let AB be a cylindrical glass tube in which is fitted a piston a of one square inch area; consider this piston to have no friction or weight, and can be moved steam-tight from A to B . Let the tube be 1728 inches from A to B , the space under the piston a just one inch from the bottom being filled with water of 32° Fahr., which will be one cubic inch; weigh the whole apparatus. Now, place a lamp under the tube in a position as represented by the Figure, and notice the time, (say $10h, 5m.$) The temperature of the water will gradually increase, and the piston a maintain a contact with it until the water begins to boil, which time is to be carefully noticed; now ($10h, 15m.$) It will be found that temperature of the water has raised from 32° to 212° , which took $10h, 15m - 10h, 5m = 10$ minutes.

Let the lamp still remain and the boiling be continued. The piston a will now leave the water, and gradually ascend towards B , apparently leaving a space between itself and the water, the latter will gradually diminish as the piston ascends, which indicates that steam is gradually formed, and occupies the space between the water and the piston; and as the piston has no weight or friction it is evident that the density of the steam must be the same as the surrounding atmosphere.

But another important faculty of steam and water will now be manifest, namely, that the temperature of both will remain the same, 212° , as at the boiling point, ($10h, 15m.$) consequently the heat from the lamp which goes into the water and steam is not *sensible* but becomes *latent*. The water is now getting very low; observe carefully the moment when it apparently disappears on the bottom of the tube. - - "now" ($11h, 10m.$) The piston a will be found at B , 1700 inches from A , and the time from the boiling-point is ($11h, 10m - 10h, 15m$) = 55 minutes = $5\frac{1}{2}$ times that occupied to raise the water from 32° to $212^{\circ} = 180^{\circ}$; hence the quantity of heat from the lamp now contained in the steam is $180 \times 5\frac{1}{2} + 180 = 1170^{\circ}$ of which 180° is sensible and 990 latent. If the water had been enclosed in a vessel to prevent evaporation, and the same quantity of heat 1170° imparted to it, it would have a temperature of 1202° , which is about that of metals when red hot in daylight.

Again to the tube AB , at the time before noticed, viz., $11h, 10m$. Take the lamp away, weigh the apparatus, and it will be found the same weight as before; hence, the same quantity of water is still in the tube, but in the form of steam. The heat will now radiate from the tube, and it will be observed that the piston a gradually descends towards A , and the inner surface of the tube will be covered with a dew which will soon fall to the bottom as water, but still maintain the heat of 212° , until the piston a has fully reached its former position at A , when the same quantity of water (one cubic inch) will occupy the same space as before the lamp was put under it, but with a temperature of 212° .

The heat required to make steam of one cubic inch of water is able to raise $5\frac{1}{2} + 1$ cubic inches from 32° to 212° ; or steam at 212° , formed of one cubic inch of water, can raise $5\frac{1}{2}$ cubic inches of ice cool water from 32° to 212° , when mixed together, making $6\frac{1}{2}$ cubic inches.

Effect of Steam.

By the preceding experiment we find that one cubic inch of water will be 1700 cubic inches converted into steam; or one cubic inch of water makes one cubic foot of steam 212° , the same density as the surrounding atmosphere which is $14\frac{1}{2}$ pounds per square inch; the effect of steam in the experiment was consequently a weight of $F = 14\frac{1}{2}$ pounds raised 1700 inches = 142 feet in 55 minutes, or

a B

C

7700

930

γ δ ϵ



$$P = \frac{Fs}{t} = \frac{14.75 \times 142}{55 \times 60} = 0.635 \text{ Effects.}$$

See Formula 5, page 148.

Advantage of Using High Steam.

Let us now make the same experiment with the tube AB , and load the piston with $14\frac{1}{2}$ pounds, which will be a weight $F = 29.5$ pounds including the atmosphere. Set the lamp under as before, and the experiment is in operation. The temperature of the water will now not cease to increase when it has attained 212° ; nor will the piston a begin to raise after 10 minutes as in the former experiment; but, when the water has attained 250° , it will cease to increase, the piston commence to ascend, and steam to generate. The piston will now only raise 930 inches from A , which will occupy the same time as before. The mechanical effect of the steam is therefore 29.5 pounds raised 930 = 77.5 feet in 55 minutes, or,

$$P = \frac{29.5 \times 77.5}{55 \times 60} = 0.693 \text{ Effects,}$$

which exceeds the former experiment about 9 per cent.

$$\frac{0.64363}{0.70469} = 0.913 \text{ or } 100 - 91.3 = 8.6 \text{ per cent.}$$

an advantage of using higher steam.

This per centage will increase as the steam is used higher.

Advantage of Using Steam Expansively.

We now continue the latter experiment. The piston a stands at 930 inches from A ; take the lamp away, and remove the $14\frac{1}{2}$ pounds on the piston. The steam of 250° in the tube will now raise the piston a to B , at 1700 inches from A and the temperature of the steam will decrease from 250° to 212° ; consequently the same steam has produced an additional effect by raising $14\frac{1}{2}$ pounds (the pressure of the atmosphere,) $1700 - 930 = 770$ inches high = 64.165 feet, which for comparison, will here assume to be accomplished in the same time 55 minutes, we shall then have

$$P = \frac{14.75 \times 64.165}{55 \times 60} = 0.29127 \text{ Effects}$$

and $0.70469 + 0.29127 = 0.99596$ Effects produced by the same quantity of steam,

$$\text{and } \frac{0.64363}{0.99596} = 0.646. \quad 100 - 64.6 = 35.3 \text{ per cent.,}$$

gained by using the steam high and expansively.

Advantage taken of the Pressure of the Atmosphere by Vacuum,

Again to the latter experiment, the piston a stands at B , and the tube is full of steam at 212° ; let there now be introduced among the steam $5\frac{1}{2}$ cubic inches of ice cool water, (32°), the steam will immediately condense to water, and the piston a begin to descend; finally, between the piston and the bottom of the tube will be found $6\frac{1}{2}$ cubic inches of water at 212° , hence the atmospheric pressure has reproduced an effect equal to that the steam before expended on it, or 0.64363 effects.

The principal features of the application of steam to produce mechanical effects are now illustrated, and we will proceed to give the principal *Rules, Formulas, and Tables*, respecting its property.

If steam is reduced in volume, its density and temperature will increase; and when additional heat is applied to steam its density or volume will increase the same as if it was produced direct from water.

Tempera- ture of Steam	Atmosph. included.		Specific gravity, air = 1.	k Volume compared with water.	Number of atmos- pheres.	Atmosphere excluded.	
	Inches of Mercury.	Pounds persquare inch.				Inches of Mercury,	Pounds per square inch.
32°	0.200	0.098	0.0041	187407		-29.79	-14.60
40	0.263	0.129	0.0053	144529	0.01	-29.73	-14.57
50	0.375	0.184	0.0074	103350	0.01	-29.62	-14.52
60	0.524	0.257	0.0102	75421	0.02	-29.47	-14.44
70	0.721	0.353	0.0136	55862	0.02	-29.27	-14.35
80	1.000	0.490	0.0186	41031	0.03	-29.00	-14.21
90	1.36	0.666	0.0250	30425	0.05	-28.63	-14.03
100	1.86	0.911	0.0333	22873	0.06	-28.13	-13.79
103	2.04	1.000	0.0364	20958	0.07	-27.95	-13.70
110	2.53	1.240	0.0458	16667	0.08	-27.46	-13.46
120	3.33	1.632	0.0576	13215	0.11	-26.66	-13.07
130	4.34	2.129	0.0538	10328	0.14	-25.65	-12.57
140	5.74	2.813	0.0960	7938	0.19	-24.25	-11.89
145	6.53	3.100	0.108	7040	0.22	-23.46	-11.60
150	7.42	3.636	0.122	6243	0.25	-22.57	-11.06
155	8.40	4.166	0.137	5559	0.28	-21.59	-10.54
160	9.46	4.635	0.153	4976	0.31	-20.53	-10.07
165	10.68	5.23	0.171	4443	0.35	-19.31	-9.47
170	12.13	5.94	0.193	3943	0.4	-17.86	-8.76
175	13.62	6.67	0.215	3538	0.45	-16.37	-8.03
180	15.15	7.42	0.238	3208	0.50	-14.84	-7.28
185	17.00	8.33	0.265	2879	0.56	-12.99	-6.37
190	19.00	9.310	0.294	2595	0.63	-10.99	-5.39
195	21.22	10.40	0.325	2342	0.71	-8.77	-4.30
200	23.64	11.58	0.36	2118	0.79	-6.35	-3.12
205	26.13	12.80	0.394	1932	0.87	-3.86	-1.90
210	28.84	14.13	0.431	1763	0.96	-1.15	-0.57
211	29.41	14.41	0.440	1730	0.98	+ 0.58	- 0.29
212	30.00	14.70	0.448	1700	1.00	+ 0.00	+ 0.00
212.8	30.60	15.	0.457	1669	1.02	+ 0.60	+ 0.30
214.5	31.62	15.5	0.471	1618	1.05	+ 1.62	+ 0.80
216.3	32.64	16.	0.484	1573	1.09	+ 2.64	+ 1.30
218.	33.66	16.5	0.497	1530	1.12	+ 3.66	+ 1.80
219.6	34.68	17.	0.512	1488	1.15	+ 4.68	+ 2.30
221.2	35.70	17.5	0.529	1440	1.19	+ 5.70	+ 2.8
222.7	36.72	18.	0.540	1411	1.22	+ 6.72	+ 3.3
221.2	37.74	18.5	0.554	1377	1.25	+ 7.74	+ 3.8
225.6	38.76	19.	0.567	1343	1.29	+ 8.76	+ 4.3
227.1	39.78	19.5	0.581	1312	1.33	+ 9.78	+ 4.8
228.5	40.80	20.	0.595	1281	1.36	+10.80	+ 5.3
229.9	41.82	20.5	0.608	1253	1.40	+11.82	+ 5.8
231.2	42.84	21.	0.612	1225	1.43	+12.84	+ 6.3
232.5	43.86	21.5	0.636	1199	1.46	+13.86	+ 6.8
233.8	44.88	22.	0.65	1174	1.50	+14.88	+ 7.3
235.1	45.90	22.5	0.663	1150	1.53	+15.90	+ 7.8
236.3	46.92	23.	0.677	1127	1.56	+16.92	+ 8.3
237.5	47.94	23.5	0.690	1105	1.60	+17.94	+ 8.8
238.7	48.96	24.	0.704	1084	1.63	+18.96	+ 9.3
239.9	49.98	24.5	0.717	1064	1.67	+19.98	+ 9.8
241.	51.00	25.	0.730	1044	1.70	+21.00	+10.3
243.3	53.04	26.	0.756	1007	1.77	+23.04	+11.3

Temperature of Steam.	Atmosph. included.		Specific gravity, air = 1.	Volume compared with water.	Number of atmos- pheres.	Atmosphere excluded.	
	Inches of Mercury.	Pounds per square Inch.				Inches of Mercury.	Pounds per square inch.
245.5°	55.08	27	0.784	973	1.83	+25.08	+12.3
247.6	57.12	28	0.810	941	1.90	+27.12	+13.3
249.6	59.16	29	0.836	911	1.97	+29.16	+14.3
251.6	61.20	30	0.863	883	2.04	+31.20	+15.3
253.6	63.24	31	0.889	857	2.11	+33.24	+16.3
255.5	65.28	32	0.915	833	2.18	+35.28	+17.3
257.3	67.32	33	0.941	810	2.24	+37.32	+18.3
259.1	69.36	34	0.968	788	2.31	+39.36	+19.3
260.9	71.40	35	0.993	767	2.38	+41.40	+20.3
262.6	73.44	36	1.020	748	2.45	+43.44	+21.3
264.3	75.48	37	1.045	729	2.52	+45.48	+22.3
265.9	77.52	38	1.071	712	2.59	+47.52	+23.3
267.5	79.56	39	1.097	695	2.65	+49.56	+24.3
269.1	81.60	40	1.122	679	2.72	+51.60	+25.3
270.6	83.64	41	1.148	664	2.79	+53.64	+26.3
272.1	85.68	42	1.175	649	2.86	+55.68	+27.3
273.6	87.72	43	1.200	635	2.92	+57.72	+28.3
275.	89.76	44	1.225	622	3.00	+59.76	+29.3
276.4	91.80	45	1.249	610	3.06	+61.80	+30.3
277.8	93.84	46	1.275	598	3.13	+63.84	+31.3
279.2	95.88	47	1.567	586	3.20	+65.88	+32.3
280.5	97.92	48	1.325	575	3.26	+67.92	+33.3
281.9	99.96	49	1.351	564	3.32	+69.96	+34.3
283.2	102.0	50	1.376	554	3.40	+72.00	+35.3
284.4	104.0	51	1.400	544	3.47	+74.00	+36.3
285.7	106.1	52	1.426	534	3.53	+76.1	+37.3
286.9	108.1	53	1.450	525	3.60	+78.1	+38.3
288.1	110.2	54	1.477	516	3.67	+80.2	+39.3
289.3	112.2	55	1.500	508	3.74	+82.2	+40.3
290.5	114.2	56	1.523	500	3.81	+84.2	+41.3
291.7	116.3	57	1.548	492	3.88	+86.3	+42.3
292.9	118.3	58	1.575	484	3.94	+88.3	+43.3
294.2	120.4	59	1.598	477	4.01	+90.4	+44.3
295.6	122.4	60	1.621	470	4.08	+92.4	+45.3
296.9	124.4	61	1.646	463	4.15	+94.4	+46.3
298.1	126.5	62	1.671	456	4.22	+96.5	+47.3
299.2	128.5	63	1.698	449	4.28	+98.5	+48.3
300.3	130.5	64	1.719	443	4.35	+100.5	+49.3
301.3	132.6	65	1.743	437	4.42	+102.6	+50.3
302.4	134.6	66	1.755	434	4.49	+104.6	+51.3
303.4	136.7	67	1.794	425	4.55	+106.7	+52.3
304.4	138.7	68	1.818	419	4.62	+108.7	+53.3
305.4	140.8	69	1.839	414	4.69	+110.8	+54.3
306.4	142.8	70	1.868	408	4.76	+112.8	+55.3
307.4	144.8	71	1.891	403	4.82	+114.8	+56.3
308.4	146.9	72	1.915	398	4.89	+116.9	+57.3
309.3	148.9	73	1.938	393	4.96	+118.9	+58.3
310.3	151.0	74	1.963	388	5.03	+121.0	+59.3
311.2	153.0	75	1.991	383	5.09	+123	+60.3
312.2	155.1	76	2.011	379	5.17	+125.1	+61.3
313.1	157.1	77	2.036	374	5.23	+127.1	+62.3

Temperature of Steam.	Atmosph. included		Specific gravity, air = 1.	h Volume compared with water.	Number of atmospheres.	Atmosphere excluded.	
	Inches of Mercury.	Pounds per square Inch.				Inches of Mercury.	Pounds per square inch.
314.0°	159.1	78	2.060	370	5.30	+129.1	+ 63.3
314.9	161.2	79	2.081	366	5.37	+131.2	+ 64.3
315.8	163.2	80	2.105	362	5.44	+133.2	+ 65.3
316.7	165.3	81	2.128	358	5.51	+135.3	+ 66.3
317.6	167.3	82	2.152	354	5.57	+137.3	+ 67.3
318.4	169.3	83	2.178	350	5.64	+139.3	+ 68.3
319.3	171.4	84	2.203	346	5.71	+141.4	+ 69.3
320.1	173.4	85	2.228	342	5.78	+143.4	+ 70.3
321.0	175.5	86	2.248	339	5.85	+145.5	+ 71.3
321.8	177.5	87	2.275	335	5.91	+147.5	+ 72.3
322.6	179.6	88	2.295	332	5.98	+149.6	+ 73.3
323.5	181.6	89	2.322	328	6.05	+151.6	+ 74.3
324.3	183.6	90	2.343	325	6.12	+153.6	+ 75.3
325.1	185.8	91	2.365	322	6.19	+155.6	+ 76.3
325.9	187.8	92	2.389	319	6.26	+157.8	+ 77.3
326.7	189.8	93	2.411	316	6.32	+159.8	+ 78.3
327.5	191.9	94	2.435	313	6.39	+161.9	+ 79.3
328.2	193.9	95	2.459	310	6.46	+163.9	+ 80.3
329.0	196.0	96	2.483	307	6.53	+166.0	+ 81.3
329.8	198.0	97	2.505	304	6.60	+168.0	+ 82.3
330.5	200.0	98	2.530	301	6.66	+170.0	+ 83.3
331.3	202.0	99	2.558	298	6.73	+172.0	+ 84.3
332.0	204.0	100	2.583	295	6.80	+174.0	+ 85.3
335.8	214.2	105	2.703	282	7.13	+194.2	+ 90.3
339.2	224.4	110	2.815	271	7.47	+194.4	+ 95.3
342.7	234.6	115	2.947	259	7.82	+204.6	+100.3
345.8	244.8	120	3.036	251	8.15	+214.8	+105.3
349.1	255.0	125	3.178	240	8.5	+225.0	+110.3
352.1	265.2	130	3.270	233	8.83	+235.2	+115.3
355.0	275.4	135	3.405	224	9.16	+245.4	+120.3
357.9	285.6	140	3.497	218	9.51	+255.6	+125.3
360.6	295.8	145	3.626	210	9.83	+265.8	+130.3
363.4	306.0	150	3.712	205	10.2	+276.0	+135.3
368.7	326.4	160	3.941	193	10.9	+296.4	+145.3
373.6	346.8	170	4.028	183	11.5	+316.8	+155.3
378.4	367.2	180	4.375	174	12.2	+337.2	+165.3
382.9	387.6	190	4.585	166	12.9	+357.6	+175.3
387.3	408.0	200	4.82	158	13.6	+378.0	+185.3
403.8	509.	250	5.90	129	17.0	+479.	+235.3
420.3	612.	300	7.00	109	20.4	+582.	+285.3
435.0	714.	350	8.00	953	23.8	+684.	+345.3
446.5	816.	400	8.95	851	27.2	+786.	+385.3
471.3	1019.	500	10.9	700	34.0	+989.	+485.3
487.0	1223.	600	12.8	597	40.8	+1193.	+585.3
519.	1631.	800	15.7	486	54.4	+1601.	+785.3
548.	2038.	1000	19.7	387	68.3	+2008.	+985.3

Letters denote.

F = force of the steam, or pressure per square inch in pounds.

I = inches of Mercury that balances the steam.

T = temperature of the steam in degrees of Fahrenheit's Thermometer.

Formulas for Steam above 212°.

$$F = \left(\frac{T}{202} + 0.52 \right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad 1,$$

$$I = \left(\frac{T}{180} + 0.584 \right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad 2,$$

$$T = (\sqrt{F} - 0.52)202, \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.$$

Example 1. The temperature of a quantity of steam is found to be 275°. Required the density in pounds per square inch?

$$F = \left(\frac{275}{202} + 0.52 \right)^2 = 44.3 \text{ pounds per square inch.}$$

By logarithms, $\frac{275}{202} + 0.52 = 1.881.$

$$\log 1.881 = 0.274389$$

$$\log 44.3 = \frac{0.646334}{6} \text{ or } 44.3 \text{ pound per square inch.}$$

The properties of steam are calculated and contained in the accompanying Table, as noted on the top. The two last columns contain the inches of mercury and pressure per square inch commonly expressed in practice; it is 0 at the temperature 212°, and below that temperature it is negative, which denotes so much vacuum. If the temperature in a condenser is 120, the vacuum is 13.07 pounds.

To Find the Weight of Steam,

RULE. Multiply the specific gravity of the steam by the weight of one cubic foot of air = 0.07529, and the product is the weight per cubic foot of the steam in pounds.

To Find the Quantity of Water of which a given quantity of Steam has been, or can be produced.

RULE. Divide the cubic contents of the steam by the volume k in the Table, and the quotient is the cubic content of the water.

Force or Feed Pumps.

Letters denote.

d = diameter } of the force pump, single acting.

s = stroke,

D = diameter } of the steam cylinder piston, in inches, double acting.

S = stroke

k = volume given in the Table at the given pressure of steam.

The stroke of the steam piston is only that under which steam is admitted to the cylinder.

$$d = 2D \sqrt{\frac{S}{k s}}, \quad s = 4 \frac{D^2 S}{k d^2}, \quad . \quad . \quad 4, 5,$$

Slip water included.

Example. Required the diameter of a force-pump having the same stroke as the cylinder piston $s = 38$ inches, diameter of cylinder $D = 30$ inches, the steam is cut off at $\frac{1}{4}$ the stroke, and the steam pressure $+ 50$ pounds per square inch. Here $k = 437$, and $S = 19$ inches, because steam is cut off at $\frac{1}{4}$ the stroke.

$$d = 2 \times 30 \sqrt{\frac{19}{437 \times 38}} = 2.03 \text{ inches.}$$

To find the Quantity of Condensing Water.

Letters denote.

q = condensing water of temperature t , in cubic feet.

Q = steam of temperature T , in cubic feet.

k = volume in the Table.

t' = temperature in the condenser when the steam and water are mixed.

$$q = \frac{1.4 Q (990 + T - t')}{k(t' - t)}, \quad - \quad - \quad - \quad - \quad 6,$$

Dimensions of the Air Pump.

d = diameter } of the air pump, single acting.

s = stroke

D = diameter } of the steam cylinder, double acting.

S = stroke

$$d = 2.3 D \sqrt{\frac{S(990 + T - t')}{k s (t' - t)}}, \quad - \quad - \quad - \quad - \quad 7,$$

Assume $t' = 100^\circ$, and $t = 50^\circ$, we shall have,

$$d = 0.326 D \sqrt{\frac{S(940 + T)}{k s}}, \quad 8,$$

$$s = 0.106 D^2 \frac{S(940 + T)}{k d^2}, \quad 9,$$

$$d = 0.23 D \sqrt{\frac{S(940 + T)}{k s}}, \quad 10,$$

$$s = 0.053 D^2 \frac{S(940 + T)}{k d^2}, \quad 11.$$

Example. A single acting air-pump is to be constructed for an engine $D = 38$ inches, $S = 45$ inches stroke of the cylinder; the stroke of the air-pump can be 32 inches, and the exhaust steam is 261° . Required the diameter of the air-pump? $k = 767$.

$$d = 0.326 \times 38 \sqrt{\frac{45(940 + 261)}{767 \times 32}} = 18.25 \text{ inches.}$$

Slip water included. T and k must be taken for the exhaust steam, as the steam may have had worked expansively; the area of the foot valve must be calculated from the following formulas.

Foot Valve in the Air Pump.

To render an air-pump to work well, and with the greatest advantage, it is necessary to pay particular attention to the following formulas. The force by which the water is driven from the condenser through the foot valve into the air-pump is limited by the pressure in the condenser; this pressure is the vacuum subtracted from 14.7 pounds; it is noted in the third column where the temperature in the condenser is opposite, in the first column. Every pound of this pressure per square inch balances a column of water 27 inches high, which is the head that presses the water from the condenser.

Letters denote.

A = area of the air-pump piston.

a = area of the foot-valve, or bucket-valve.

D = diameter of the air-pump-piston.

d = diameter of the foot-valve, when round.

S = stroke of air-pump piston, in feet.

P = pressure in the condenser at the temperature T .

n = number of strokes of the air-pump piston per minute.

$$a = \frac{A S n}{100 \sqrt{P}}, \quad 12,$$

$$d = \frac{D \sqrt{S n}}{10 \sqrt[4]{P}}, \quad 15,$$

$$S = \frac{100 a \sqrt{P}}{n A}, \quad 13,$$

$$S = \frac{100 d^2 \sqrt{P}}{n D^2}, \quad 16,$$

$$n = \frac{100 a \sqrt{P}}{A S}, \quad 14,$$

$$n = \frac{100 d^2 \sqrt{P}}{D^2 S}, \quad 17.$$

Example. The area A of an air-pump-piston is 2.35 square feet, stroke of piston $S = 3.6$ feet, to make $n = 40$ strokes per minute, and the pressure to be $P = 3.2$ pounds. Required the area of the foot-valve.

$$a = \frac{2.35 \times 3.6 \times 40}{100 \sqrt{3.2}} = 1.85 \text{ square feet.}$$

To Find the Velocity and Quantity of the Injection Water through the Adjustage into the Condenser.

Letters denote.

v = velocity in feet per second.

h = head of the press water; + when above, and - below the adjustage.

F = vacuum, noted - or negative in the last column, but is positive in the formulas.

q = quantity of water discharged in cubic feet, per second.

a = area of all the holes in the adjustage in square feet.

d = diameter } of the injection pipe, in feet.

L = length }

n = double strokes of cylinder-piston, or revolutions per minute.

A , D , and S , dimensions of the steam cylinder, in feet.

T = temperature, and k = volume coefficient of the exhaust steam.

$$a = \frac{q}{5 \sqrt{2F+h}}, \quad 18,$$

$$q = 5a \sqrt{2F+h}, \quad 21,$$

$$v = 8 \sqrt{2F+h} \quad 19,$$

$$d = 0.35 \sqrt[5]{\frac{L q^2}{2F+h}}, \quad 22,$$

$$q = \frac{n S D^2 (940+T)}{55k}, \quad 20,$$

$$a = \frac{n S D^2 (940+T)}{275k \sqrt{2F+h}}, \quad 23,$$

Example. Required the diameter of an injection pipe $L = 10$ feet long, which shall supply $q = 1.3$ cubic feet of water per second into a vacuum of 12 pounds per square inch, the head of press water $h = 2$ feet?

$$d = 0.35 \sqrt[5]{\frac{10 \times 1.3}{2 \times 12 + 2}} = 0.3055 \text{ feet} = 3\frac{1}{16} \text{ inches.}$$

Area of Steam Passages.

a = area of the steam pipe, sq. in.

A = area of the cylinder piston, sq. in.

d = diameter of the pipe, in inches.

D = diameter, S = stroke of cylinder, in inches.

$$a = \frac{A S n}{35000}, \quad d = \frac{D \sqrt{S n}}{186}, \quad 24, 25.$$

Example. Required the diameter of a steam-pipe for a cylinder $D = 40$ inches. Stroke of piston $S = 48$ inches, and $n = 38$ revolutions per minute?

$$d = \frac{40 \sqrt{48 \times 38}}{186} = 9.2 \text{ inches, nearly.}$$

Steam Ports to the Cylinder.

$$a = \frac{A S n}{30600}, \quad 26,$$

Safety Valve.

Three-fourths of the fire grate in square feet is a good proportion for the safety valve in square inches.

Notation of Letters corresponds with Figure 3, Plate VIII.

a = area of safety valve in square inches.

P = pressure per square inch in the boiler

W = weight on the safety valve lever

Q = weight of the safety valve and lever

l = lever for W

e = " aP

x = " Q

} in pounds.

} in inches.

Balance the lever over a sharp edge, and the centre of gravity Q is found; measure the distance x from the fulcrum C .

$$a P e = W l + Q x \quad 27,$$

$$W = \frac{a P e - Q x}{l}, \quad 29,$$

$$P = \frac{W l + Q x}{a e}, \quad 28,$$

$$l = \frac{a P e - Q x}{W}, \quad 30,$$

Example. Area of the safety valve $a = 9$ square inches, $e = 4\frac{1}{2}$ inches, $W = 50$ pounds, weight of the lever and safety valve $Q = 15$ pounds, and $x = 17$ inches. Required at what distances l , l' and l'' will the weight W indicate pressures of $P = 30$, $P' = 40$, and $P'' = 50$ pounds?

$$l = \frac{9 \times 30 \times 4.5 + 15 \times 17}{50} = 29.2 \text{ inches,}$$

from the fulcrum C the weight W will indicate $P = 30$ pounds.

$l' = 37.9$ inches, when $P' = 40$ pounds.

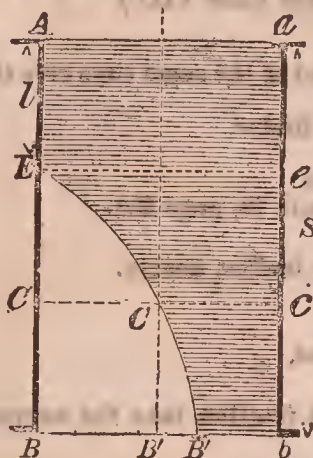
$l'' = 45.8$ " " $P'' = 50$ "

and thus the lever can be graduated.

Cut-off Steam,

IN order to save steam, or more correctly to employ its effect to a higher degree, the admittance of steam to the cylinder is shut off when the piston has moved a part of the stroke; from the cut-off point the steam acts expansively with a decreased pressure on the piston, as represented by the accompanying Figure.

306.



Let the steam be cut off at $\frac{1}{3}$ of the stroke, and Aa represent the total pressure, say 20 pounds per square inch which will continue to the point E where the admittance of steam is shut off at $\frac{1}{3}$ the stroke S . The steam $Aa e E$, is now acting expansively on the piston, and the pressure decreases as the volume increases, when the piston has attained Cc or $\frac{2}{3}$ of S , the pressure $Cc = 10$ pounds, only half the pressure $Aa = 20$ because the volume $Aa e E$ is only half of $Aa c C$, and so on until the piston has attained Bb the pressure $Bb = \frac{1}{3} \times 20 = 6.66$ pounds.

The mean pressure, or the effectual pressure, throughout the stroke, will be about 13.33 pounds per square inch, or 66 per cent., but the quantity of steam used is only 33 per cent., hence 33 per cent. is gained by using the steam expansively.

Letters denote.

l = the part of the stroke S with full steam.

P = pressure per square inch under full admittance of steam.

F = the effectual force per square inch throughout the stroke

p = pressure per square inch at the end of the stroke

$$p = \frac{l}{S}, \text{ and } F = P \left(2.3(\log.S - \log.l) + 1 \right) \frac{l}{S}.$$

Example. Stroke of piston is $S = 6.4$ feet, and the steam is cut off at $l = 2.8$ feet, the pressure under full steam is $P = 50$ pounds. Required the effectual pressure $F = ?$

From
Subtract

$$\log.6.4 = 0.806180$$

$$\log.2.8 = 0.447158$$

$$2.3 \times 0.359022 + 1 = 1.69 \text{ nearly.}$$

$$F = \frac{50 \times 1.69 \times 2.8}{6.4} = 37 \text{ pounds.}$$

Table of Expanded Steam.

	F .	p .
Cut off steam at.	0.3845P	0.125P
	0.597P	0.25P
	0.698P	0.33P
	0.741P	0.375P
	0.844P	0.5P
	0.918P	0.625P
	0.935P	0.666P
	0.965P	0.75P
	0.991P	0.875P

Example. The pressure in the boiler is $P = 68$ pounds; cut off the steam at $\frac{2}{3}$ of the stroke. Required the effectual pressure throughout the stroke, and density of the exhaust steam?

$$F = 0.741 \times 68 = 50.4 \text{ pounds effectual pressure.}$$

$$p = 0.375 \times 68 = 25.5 \text{ pounds density of exhaust.}$$

SLIDE VALVES.

THE slide valve motion is one of the most important features in causing a steam engine to work well, and to employ the effect of steam economically. The author of this book being well acquainted with disarrangements on this point, has here endeavoured to give a good working-drawing of the proper proportions and arrangements of slide-valve motion. (See Plate VIII.)

Main Valve.

It will be best to assume a certain size cylinder, and at the same time give the proportions for any size.

$D = 34$ inches, diameter of the cylinder.

$S = 18$ inches stroke of piston.*

$n = 56$ double strokes per minute.

We have the area of the steamports m , from Formula 26, page 238.

$$a = \frac{34^2 \times 0.785 \times 18 \times 56}{30600} = 30 \text{ square inches, nearly.}$$

$$m = \frac{D+S}{26} = \frac{34+18}{26} = 2 \text{ inches,}$$

the width of the steamport; if the quotient gives a fraction take the nearest quarter or eighth.

$$\frac{a}{m} = \frac{30}{2} = 15 \text{ inches, breadth of steamport.}$$

$r = \frac{1}{2} m$ about = 1 inch, the exhaust port $o = 2m - \frac{1}{2}r = 3\frac{1}{2}$ inches, and $f = o + 2r = 5\frac{1}{2}$ inches, $h = f - \frac{1}{2}r = 5\frac{1}{4}$ inches, $k = 1\frac{1}{2}m = 3$ inches, and $i = h + 2k = 11\frac{1}{4}$ inches, $e = m = 2$ inches.

* The stroke and diameter is here rather out of proportion, but we will maintain them in the calculations as they suit the drawing, which is purposely made to show the *slide valves* on a large scale. The rules will however suit any proportions of diameter and stroke.

To Find the Stroke of the Eccentric.

s = stroke of the eccentric in inches.

$s = i - f - \frac{1}{2}r = 5\frac{1}{4}$ inches.

The lap $L = \frac{1}{2}(i - f - 2m) = \frac{7}{8}$ inches.

The *lead* of the valve, or opening of the steamport when the crank pin stands o. the centre should be about

$$l = \frac{m\sqrt{n}}{80} = \frac{2\sqrt{56}}{80} = \frac{1}{4} \text{ inches, nearly.}$$

Having finished the main valve and ascertained the stroke of the eccentric, it is now required to find the position of the centre b , (Plate VI.,) of the eccentric, to the crank-pin. Suppose the crank pin of the engine stands at a on the centre nearest to the cylinder, and the eccentric rods are attached direct to the valve rods; draw the line dd , at right-angle to the centre-line aa'' of the engine. then

$$\text{the angle, sin. } W = \frac{2(L+l)}{s} = \frac{2(\frac{7}{8} + \frac{1}{4})}{5\frac{1}{4}} = 0.409, \text{ or } W = 24^\circ 10'.$$

See Plates VIII and IX.

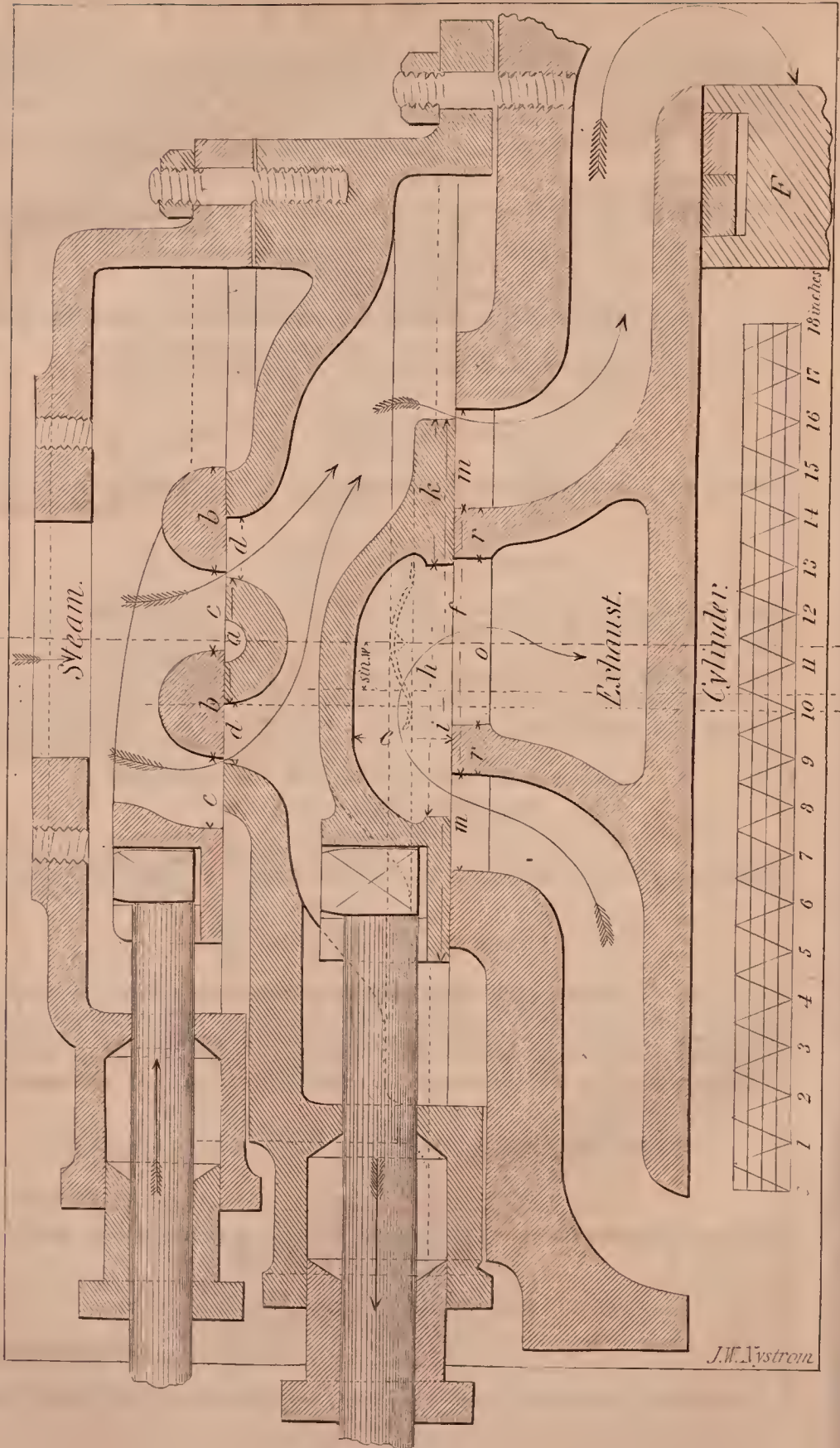
To Find the position of the Crank-Pin at the moment the Main Valve opens.

$$y = \frac{Sl}{s \cos. W} = \frac{18 \times 0.25}{5.5 \times 0.9123} = 0.9 \text{ inches, nearly,}$$

from the centre line.

Slide Valves.

Plate VIII.



J.W. Nyström

Eccentrics.

Plate IX.

Fig. 1.



Fig. 2.

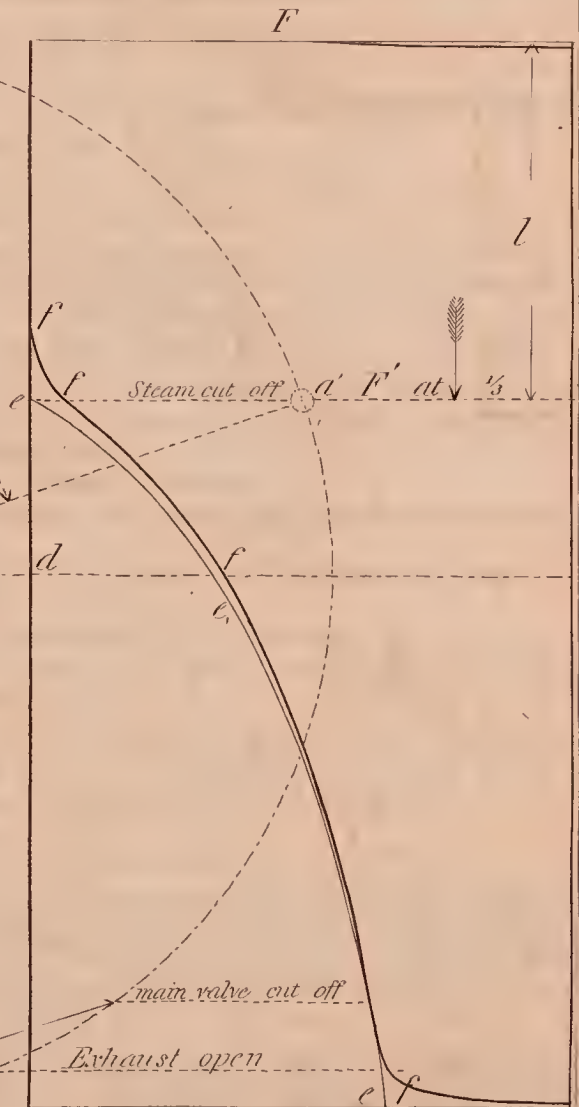
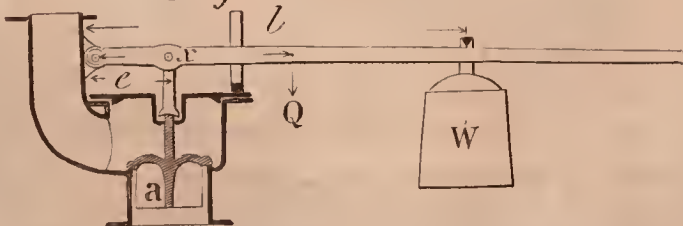


Fig. 3.



J.W. Nyström

To Find the position of the Crank at the moment the Exhaust opens.

$$x = \frac{S}{2} \left(\sin. W - \frac{1}{s} (f - h) \right) = \frac{18}{2} \left(0.409 - \frac{1}{5.5} (5.5 - 5.25) \right) = 3.27 \text{ inches}$$

from the centre line.

To Find the position of the Crank Pin when the Main Valve cuts off the Steam.

$$X = \frac{2SL}{s} = \frac{2 \times 18 \times \frac{7}{8}}{5.5} = 5.727 \text{ inches.}$$

To Find at what part of the Stroke the Main Valve Cuts off the Steam,

$$\text{Will cut off at} = 1 - \frac{4L^2}{s^2} = 1 - \left(\frac{2 \times \frac{7}{8}}{5.5} \right)^2 = 0.899 \text{ of the stroke.}$$

The greater the lap is, the sooner will the main-valve cut off, but if the lap is increased the stroke of the eccentric must also be equally increased. It does not work well to cut off much by the main-valve, especially when the engine works fast; for very slow motion it may answer to cut off at $\frac{1}{2}$ the stroke.

It will be noticed that the centre of the eccentric is always ahead of the crank pin with an angle $90^\circ + w$. Hence when the engine is to be reversed, the centre b must have the same position on the opposite side of the centre-line, or the eccentric must be moved forwards an angle of $90^\circ - 2w$.

Cut-off Valve.

The width of the cut off ports should be about $d = \frac{1}{4}m = 1\frac{1}{4}$ inch, and their

$$\text{breadth } \frac{a}{2d} = \frac{30}{2 \times 1\frac{1}{4}} = 12 \text{ inches, when two ports are used.}$$

Proportions of the Valve.

$a - b = c - d$, $a + d = b + c$, and $a = 2d$, and the stroke of the cut-off valve eccentric $s = 2b$, we shall have $a = 2\frac{1}{2}$, $b = 2\frac{1}{4}$, $c = 1\frac{1}{2}$, $d = 1\frac{1}{4}$, and $s = 4\frac{1}{2}$ inches.

Let us assume the steam to be cut off at $\frac{1}{2} = l$ of the stroke S , the position of the crank-pin a' will then be $\sin. u = 2l = 0.666$, or $u = 70^\circ 30'$; at the same time the position of the centre c' of the cut off eccentric will be

$$\sin. z = \frac{d+c}{s} = \frac{1\frac{1}{4} + 1\frac{1}{2}}{4\frac{1}{2}} = 0.612, \text{ or } z = 37^\circ 50',$$

and $V = u - z = 70^\circ 30' - 37^\circ 50' = 32^\circ 40'$, the position of the centre c when the crank-pin a is on the centre. This Table will show the positions of the centre a and c , at different cut offs. Letters correspond with Figure 1, Plate VI.

Cut off at l .	v	$\sin. v$	stroke of eccen. s .	z	u	F .	p .
$\frac{1}{4}$	$22^\circ 10'$	0.377	$2b$	$37^\circ 50'$	60°	0.5880	0.250
$\frac{1}{3}$	$32^\circ 40'$	0.539	$2b$	$37^\circ 50'$	$70^\circ 30'$	0.6914	0.333
$\frac{2}{5}$	$31^\circ 55'$	0.527	$c+a$	$43^\circ 35'$	$75^\circ 30'$	0.7332	0.375
$\frac{1}{2}$	$42^\circ 35'$	0.675	$b+c$	$47^\circ 25'$	90°	0.8350	0.500
$\frac{3}{5}$	$46^\circ 30'$	0.7193	$a+b-c$	58°	$104^\circ 30'$	0.910	0.625
$\frac{4}{5}$	$51^\circ 30'$	0.7933	$a+b-c$	$58^\circ 30'$	$109^\circ 30'$	0.985	0.666

It will now be observed that the effectual pressure F in this Table is less than in the Table on page 239, owing to the valve not cutting off the steam instantly, but gradually, so that the density of the steam in the cylinder is already diminished at the cut off point. The valve will cut off quicker the less the angle z is.

See Figure 2, Plate VIII. The actual pressure will not form a sharp corner at e , or follow the line e, e, e , as would be due when cut off at $\frac{1}{2}$ the stroke, but the line f, f', f, f' will be the true diagram. Including the steam in the ports and steam-chest, the density at the end of the stroke will correspond nearly with the Table.

BLOWING-OFF. SALT WATER. SATURATION.

SEA water contains about 0.03 its weight of salt. When salt water boils, fresh water evaporates, and the salt remains in the boiler, consequently the proportion of salt increases as the water evaporates, until it has reached 0.36 weight to the water; the salt will then commence to saturate in the boiler, and the water in solution will hold 0.36 weight of salt to 1 of water.

To prevent this deposit in the boiler, it is necessary to keep the salt below this proportion, which is overcome by withdrawing (blow off) part of the super-salted water, while less salted (feed water,) water is replaced. It is found in practice that when the proportions are kept 0.12 of salt to 1 weight of water, the deposit will be very slight. To obtain this it will be necessary to blow off

$$\frac{0.03}{0.12} = 0.25 \text{ parts of the feed water, or}$$

if a brine-pump is used, it should be at least 0.25 of the feed pump.

W = cubic feet of super-salted water to be blown off per minute.

D , S , n , and k , as before, we shall have,

$$W = \frac{D^2 S n}{3000k},$$

Example. $D = 30$ inches, stroke of piston 36 inches, cut off at half stroke $S = 18$, making 14 revolutions per minute, with a pressure of 30 pounds per square inch, $k = 610$. How much water must be blown off per minute?

$$W = \frac{30^2 \times 18 \times 14}{3000 \times 610} = 0.124 \text{ cubic feet.}$$

Heat Wasted by Blowing Off.

Letters denote.

w = water evaporated } in cubic feet per unit of time.
 W = water blown off }
 t = temperature of the feed water.
 T = " " blowing off.
 H = heat wasted, per cent.

$$H = \frac{W(T - t)}{w(990 + T - t)}.$$

Example. Let the quantity of water blown off be $\frac{1}{2}$ of the feed water, we have $W = 1$, and $w = 2$, the boiling point of the water will then be $T = 215.5^\circ$, let the feed water taken from the hot-well be $t = 100^\circ$. Required the quantity of heat lost?

$$H = \frac{1(215.5^\circ - 100)}{2(990 + 215.5 - 100)} = 0.052 \text{ or } 5.2 \text{ per cent.}$$

This is a very trifling quantity of heat lost.

Proportion of Salt.	Boiling point T .	Water blown off. per cent. W .	Heat lost. per cent.	Specific gravity.
0.03	213.2°	100	100	1.03
0.06	214.4	50	10.35	1.06
0.09	215.5	33.3	5.2	1.09
0.12	216.7	25	3.5	1.12
0.15	217.9	20	2.66	1.15
0.18	219.	16.6	2.14	1.18
0.21	220.2	14.3	1.80	1.21
0.24	221.4	12.5	1.56	1.24
0.27	222.5	11.1	1.38	1.27
0.30	223.7	10.0	1.23	1.30
0.33	224.9	9.07	1.12	1.33
0.36	226	Water saturates.		1.36

Heat wasted by Incrustation.

The conducting power of iron for heat, is about 30 times that of saturated scales, hence a considerable portion of heat is lost when the scales become thick in a boiler.

t = thickness of the scale in 16th of an inch.
 H = heat wasted, in per cent.

$$H = \frac{t^2}{32+t^2}.$$

Example. The scale in a boiler is 5 sixteenths of an inch thick. How much heat is lost by it?





















$$H = \frac{5^2}{32+5^2} = 0.438 \text{ or } 44 \text{ per cent, nearly,}$$

which goes out through the chimney.

This is merely to show that the heat lost by blowing off is but trifling, compared with the heat lost by saturation of scales, which additionally injures the boiler by softening and fracturing the iron, and final explosions.

When boilers are taken good care of by cleaning and blowing off at short intervals, the scales need not exceed 1 sixteenth of an inch.

To Command the Engineer how to Manœuvre the Engine in a Steamboat;

Go ahead	-	-		-	-	-	one stroke.
Back	-	-			-	-	two strokes.
Stop	-	-		-	-	-	one stroke.
Slowly	-	-		-	-	-	two short.
Full speed	-	-		-	-	-	three short.
Go ahead slowly	-	-			-	-	one long, two short.
Back slowly	-	-				-	two long, two short.
Go ahead, full speed	-	-				-	one long, three short.
Back fast	-	-					two long, three short.
Hurry	-	-			-	-	three short repeated.

SCREW PROPELLERS,

PLATE X., is a drawing of a Screw-propeller with proportions thus far known to be the most effective, particularly when the steam-engine is applied direct to the propeller shaft; its pitch is twice the diameter at the periphery, but contracts towards the centre; at the hub the pitch is lessened by the amount of slip assumed. When the propeller is geared from the engine, the pitch is generally less in proportion to the diameter.

$p = \frac{1}{4}$ of the pitch at the periphery.

$p'' =$ " " " " hub.

s = the assumed slip in a fraction of p .

Then $p = p'' + s$.

By these two pitches p and p'' , the helices, $ac b$ at the periphery and $d c f$ at the hub are constructed as for common screws.

The actual pitch of the propeller at the centre of effort of the blades o is represented by p' at $r = 0.725R$ from the centre; or the actual pitch $= 4p'$.

Letters Denote,

P = pitch of the propeller } at the periphery.
 W = angle of the blades }

D = diameter, R = radius, extreme.

L = length parallel with the centre-line.

m = number of blades.

b = extreme breadth of the propeller blades over the edge, between the corners e, e .

e = circle arc in the angle v .

v = the projected angle of the blades.

a = the projected area of the blades.

A = the true inclined surface of the blades.

\bigcirc = acting area of the propeller.

H = horse-power required to drive the propeller n revolutions per minute.

Formulas for Screw Propellers,

$$\cot. W = \frac{P}{\pi D}, \quad - \quad - \quad - \quad 1,$$

$$b = \sqrt{\frac{v^2 \pi^2 D^2}{129600} - L^2}, \quad 7,$$

$$P = \cot. W \pi D, \quad - \quad - \quad 2,$$

$$a = \frac{0.785 D^2 v m}{360}, \quad - \quad - \quad 8,$$

$$P = \frac{360 L}{v}, \quad - \quad - \quad - \quad 3,$$

$$A = \frac{R m}{2.25} (b + L), \quad - \quad - \quad - \quad 9,$$

$$P = \frac{\pi D L}{e}, \quad - \quad - \quad - \quad 4,$$

$$\bigcirc = \frac{2.5 D^2}{\sqrt{P^2 + \pi^2 D^2}}, \quad - \quad - \quad 10,$$

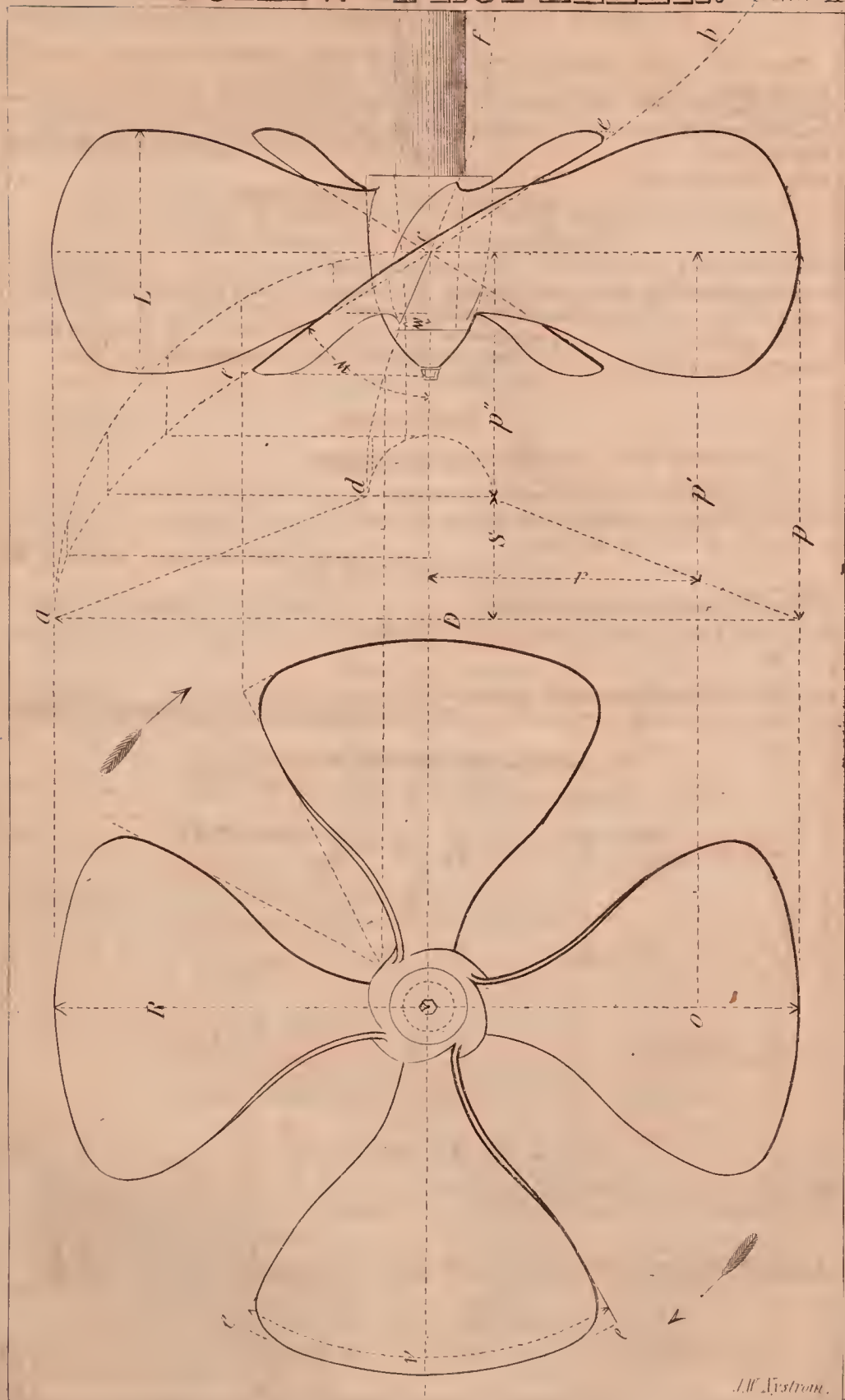
$$P = \frac{\pi D L}{\sqrt{b^2 - L^2}}, \quad - \quad - \quad 5,$$

$$H = \frac{D^3 n^2}{480000} \left(L S \cos. W + 0.11 \right) \quad 11,$$

$$v = \frac{360 L}{P}, \quad - \quad - \quad - \quad 6,$$

$$n = \frac{78}{D} \sqrt[3]{\frac{H}{(L S \cos. W + 0.11)}} \quad 12.$$

SCREW PROPELLER. *Plate X.*





Example 1. The diameter of a propeller is 10 feet 6 inches, and the angle $W = 58^\circ$ at the periphery. Required the pitch $P =$ in feet?

$$P = \cot.58^\circ \times 3.14 \times 10.5 = 20.6 \text{ feet.}$$

Example 2. The propeller on Plate VII. is of dimensions $D = 15$ feet, $L = 5$ feet, $W = 57^\circ 30'$, the slip is 38 per cent: or $S = 0.38$. What power is required to drive it 40 revolutions per minute, $H =$?

$$H = \frac{15^3 \times 40^3}{480000} \left(5 \times 0.38 \times \cos.57^\circ 30' + 0.11 \right) = 509 \text{ horses, nearly.}$$

Example 3. A Propeller of diameter $D = 12$ feet, angle $W = 64^\circ$, and length $L = 3$ feet 6 inches, is to be driven by a steam engine of 450 horses, the slip $S = 0.28$. How many revolutions will it make per minute, $n =$?

$$n = \frac{78}{12} \sqrt[3]{\frac{450}{(3.5 \times 0.28 \times \cos.64^\circ + 0.11)}} = 61 \text{ revolutions}$$

per minute.

Explanation of Tables.

Table I. is for finding the pitch and acting area of propellers; the column marked W contains the angle of the propeller blades, as marked on the drawing.

To Find the Pitch.

RULE. Multiply the diameter of the propeller by the tabular coefficient in the pitch column opposite the given angle, and the product is the pitch of the propeller.

Example. The diameter of a propeller is 12 feet, the angle $W = 60^\circ$; diameter of the hub 1.5 feet, and the angle $w = 16^\circ$.

Required the $\begin{cases} \text{pitch at the periphery?} \\ \text{pitch at the hub?} \\ \text{pitch at the centre of effort?} \end{cases}$

$$\begin{aligned} \text{Pitch at the periphery} &= 12 \times 1.814 = 21.768 \text{ feet.} \\ \text{" " hub} &= 1.5 \times 10.97 = 16.455 \text{ feet.} \end{aligned}$$

Let d and p be diameter and pitch of the hub.

$P =$ pitch at the centre of pressure. We shall have,

$$(P - p) : (P - p) = (D - d) : (0.725D - d).$$

and

$$P = p + \frac{(P - p)(0.725D - d)}{D - d},$$

$$P = 16.455 + \frac{(21.768 - 16.455)(0.725 \times 12 - 1.5)}{12 - 1.5} = 20.097 \text{ feet.}$$

To Find the Acting Area.

RULE. Multiply the square of the diameter of the propeller by the tabular coefficient in the column O opposite the given angle, and the product is the acting area of the propeller.

Example. The diameter of a propeller $D = 13$ feet, 3 inches, and the angle $W = 60^\circ$. Required the acting area?

$$O = 13.25^2 \times 0.679 = 119.2 \text{ square feet.}$$

TABLE I.

Table for finding the Pitch and Acting Area of
Propellers. $D = 1$.

Angle. W	Pitch. P	Act. Area O	Angle. W	Pitch. P	Act. Area. O
5°	36·	0·068	47°	2·930	0·573
6	30·	0·082	48	2·828	0·583
7	25·65	0·095	49	2·730	0·582
8	22·4	0·109	50	2·635	0·601
9	19·85	0·123	51	2·545	0·610
10	17·82	0·136	52	2·455	0·618
11	16·16	0·150	53	2·370	0·625
12	14·79	0·163	54	2·283	0·634
13	13·60	0·176	55	2·200	0·642
14	12·60	0·190	56	2·120	0·650
15	11·04	0·203	57	2·040	0·657
16	10·97	0·217	58	1·963	0·665
17	10·27	0·229	59	1·888	0·672
18	9·67	0·242	60	1·814	0·679
19	9·12	0·255	61	1·740	0·686
20	8·64	0·268	62	1·670	0·692
21	8·19	0·281	63	1·600	0·699
22	7·77	0·294	64	1·530	0·705
23	7·40	0·306	65	1·465	0·711
24	7·06	0·319	66	1·400	0·716
25	6·75	0·331	67	1·333	0·722
26	6·45	0·344	68	1·270	0·728
27	6·17	0·356	69	1·205	0·731
28	5·91	0·368	70	1·142	0·736
29	5·67	0·380	71	1·114	0·741
30	5·45	0·392	72	1·021	0·745
31	5·23	0·404	73	0·960	0·750
32	5·03	0·415	74	0·900	0·754
33	4·85	0·427	75	0·842	0·757
34	4·66	0·439	76	0·783	0·761
35	4·50	0·450	77	0·725	0·764
36	4·33	0·461	78	0·668	0·767
37	4·175	0·472	79	0·611	0·770
38	4·025	0·483	80	0·555	0·772
39	3·885	0·494	81	0·498	0·775
40	3·745	0·504	82	0·442	0·777
41	3·620	0·515	83	0·386	0·779
42	3·500	0·523	84	0·331	0·780
43	3·370	0·535	85	0·275	0·781
44	3·260	0·545	86	0·220	0·782
45	3·141	0·555	87	0·165	0·783
46	3·035	0·564	88	0·110	0·784

TABLE II.

Table for finding the Exponent and Coefficient of Vessels.

<i>Full Lines.</i>		<i>Hollow Lines.</i>	
Exponent x .	Coefficient k .	Exponent x .	Coefficient k .
1	0.000	0.68	1.71
0.95	0.024	0.67	1.77
0.90	0.228	0.66	1.84
0.88	0.326	0.65	1.90
0.86	0.432	0.64	1.96
0.84	0.558	0.63	2.00
0.82	0.692	0.62	1.97
0.80	0.836	0.61	1.93
0.79	0.902	0.60	1.88
0.78	0.978	0.59	1.82
0.77	1.050	0.58	1.77
0.76	1.12	0.57	1.72
0.75	1.20	0.56	1.67
0.74	1.28	0.55	1.61
0.73	1.35	0.54	1.55
0.72	1.43	0.53	1.50
0.71	1.51	0.52	1.44
0.70	1.59	0.51	1.38
0.69	1.64	0.50	1.32

TABLE III.

Table for finding the Slip and Acting Area. $\bigcirc = 1$.

Slip.	Act. Area.	Slip.	Act. Area.	Slip.	Act. Area.	Slip.	Act. Area.
S.	\bigcirc	S.	\bigcirc	S.	\bigcirc	S.	\bigcirc
per cent.		per cent.		per cent.		per cent.	
5	84.85	28	4.150	51	0.927	74	0.208
6	60.35	29	3.820	52	0.888	75	0.192
7	46.35	30	3.555	53	0.840	76	0.177
8	39.00	31	3.333	54	0.784	77	0.163
9	32.20	32	3.090	55	0.737	78	0.149
10	27.00	33	2.880	56	0.697	79	0.137
11	22.07	34	2.710	57	0.655	80	0.125
12	19.80	35	2.535	58	0.611	81	0.113
13	17.32	36	2.366	59	0.581	82	0.103
14	15.20	37	2.222	60	0.546	83	0.092
15	13.52	38	2.085	61	0.511	84	0.083
16	12.00	39	1.952	62	0.479	85	0.074
17	10.82	40	1.827	63	0.455	86	0.066
18	9.715	41	1.727	64	0.422	87	0.058
19	8.820	42	1.634	65	0.394	88	0.050
20	8.000	43	1.523	66	0.369	89	0.045
21	7.282	44	1.430	67	0.347	90	0.037
22	6.700	45	1.354	68	0.323	91	0.031
23	6.111	46	1.275	69	0.300	92	0.026
24	5.635	47	1.195	70	0.281	93	0.021
25	5.200	48	1.127	71	0.263	94	0.017
26	4.785	49	1.068	72	0.241	95	0.012
27	4.444	50	1.000	73	0.225		

PADDLE-WHEELS.

Letters denote.

a = area of all the floats immersed, square feet.

W = the immersed angle of the paddle-wheels, at the centre of pressure.

R = radius from the centre of motion to the centre of pressure of floats.

O = acting area, square feet.

$$O = a \cos. \frac{1}{2} W.$$

Example. Let the area of all the immersed paddles be $a = 120$ square feet, the angle $W = 60^\circ$, and $\frac{1}{2} \times 60 = 20^\circ$. Required the acting area,

$$O = 120 \times \cos. 20^\circ = 112.76 \text{ square feet.}$$

Vessels. Resistance to,

Letters denote.

l = length in the loadline of the vessel, in feet.

\mathcal{X} = greatest immersed section, in square feet.

\mathcal{O} = area of resistance, square feet.

Q = displacement of the vessel in tons.

H = horse-power required to propel the vessel M statute miles per hour.

S = slip of the propeller or paddle-wheels.

k = coefficient of the vessel.

F = resistance of the vessel, in pounds.

$$x = \frac{35Q}{\mathcal{X}l}, \quad - \quad - \quad 1,$$

$$\log. F = 0.63346 + \log. \mathcal{O} + 175 \log. M, \quad 5,$$

$$\mathcal{O} = \mathcal{X} \sqrt{\frac{\mathcal{X}}{\mathcal{X} + k l^2}}, \quad 2,$$

$$\log. H = \log. \mathcal{O} + 2.75 \log. M - 1.94448, \quad 6,$$

$$F = 4.3 \mathcal{O} M^2, \quad - \quad 3,$$

$$S = \frac{\sqrt{\mathcal{O}^3}}{\sqrt{\mathcal{O}^3} + \sqrt{O^3}} \quad 7,$$

$$H = \frac{\mathcal{O} M^3}{88}, \quad - \quad 4,$$

$$\log. M = \frac{\log. H + 1.94448 - \log. \mathcal{O}}{2.75}, \quad 8,$$

The formulas 3 and 4, give the resistance and horse power too large, owing to the square and cube of M ; but the formulas 5 and 6 give it as correct as may be desired in practice.

Example. Find the area of resistance of a vessel with dimensions $l = 230$ feet, $\mathcal{X} = 490$ square feet, $Q = 1900$ tons?

$$\text{Exponent } x = \frac{35 \times 1900}{490 \times 230} = 0.59. \quad \text{Find this number}$$

in the column x , Table II., in the next column $k = 1.82$, which inserted in the formula 2 will give the area of resistance.

$$\mathcal{O} = 490 \sqrt{\frac{490}{490 + 1.82 \times 230^2}} = 35 \text{ square feet.}$$

Speed of Steamboats.

Screw Propellers.	Paddle-Wheels.
$M = \frac{P n}{88}(1 - S), \quad - \quad 1,$	$M = \frac{R n \cos. \frac{1}{2} W}{14}(1 - S), \quad 4,$
$n = \frac{88 M}{P(1 - S)}, \quad - \quad 2,$	$n = \frac{14 M}{R \cos. \frac{1}{2} W(1 - S)}, \quad 5,$
$S = 1 - \frac{88 M}{P n}, \quad - \quad 3,$	$S = 1 - \frac{14 M}{n R \cos. \frac{1}{2} W}, \quad 6,$

Example. The pitch of a propeller is $P = 30$ feet, and makes 40 revolutions per minute; the slip is $S = 0.4$. Required the speed of the vessel?

$$M = \frac{30 \times 40}{88}(1 - 0.4) = 8.18 \text{ miles per hour.}$$

Explanation of Table III.

To find the Slip.

RULE. Divide the acting area of the propeller, or paddle-wheels by the area of resistance of the vessel; find the quotient in the columns of *acting area*, and opposite, in the slip column, is the *slip per cent*.



TONNAGE OF VESSELS.

THE United States Custom House measurement of tonnage of vessels is expressed by the formula.

$$T = \frac{b d}{95} \left(l - \frac{3}{5} b \right),$$

Letters Denote,

T = *tonnage* of the vessel.

b = extreme *beam* in feet, taken above the main-wales.

d = *depth* of the vessel in feet. In double-decked vessels half the beam b is taken as the depth. For single decked vessels, the depth is taken from the under side of the deck plank to the ceiling of the hold.

l = *length* of the vessel in feet, from the fore-part of the steam to the after part of the stern-post, measured on the upper deck.

Example. The dimensions of a vessel are $l = 186$ feet, $b = 30$, and $d = 15$ feet, for a double-decked vessel. Required her tonnage?

$$T = \frac{30 \times 15}{95} \left(186 - \frac{3}{5} 30 \right) = 795.77 \text{ tons.}$$

To Approximate the Tonnage of the Displacement.

$$Q = \mathcal{W} l \times \begin{cases} 0.015 \text{ very sharp vessels.} \\ 0.022 \text{ very full vessels.} \end{cases}$$

SAN JACINTO,

United States Propeller Steamer of War.

Dimensions.

$l = 210$ feet long in the load line.

$\mathcal{O} = 477$ square feet, greatest immersed section when loaded to 16 feet. 9 inches draft of water, and $Q = 2150$ tons, displacement.

Required her exponent $x = ?$ coefficient $k = ?$ and area of resistance $\mathcal{O} = ?$

$$\text{Formula 1. page 248, Exponent } x = \frac{35 \times 2150}{477 \times 210} = 0.75.$$

Find this exponent in table II. page 247, and opposite the coefficient $k = 1.2$.

$$\text{Formula 2. } \mathcal{O} = 477 \sqrt{\frac{477}{477 + 1.2 \times 210^2}} = 45 \text{ square feet.}$$

Steam Engine of the San Jacinto.

I am indebted to B. H. Bartol, Esqr. for the following dimensions of the Engine and Propeller for the San Jacinto, which was in progress at the Southwark Foundry (Merrick & Sons,) Philadelphia, when this book went to press.

Two horizontal Engines to gear the propeller 2.25 turns to 1.

$d = 70$ inches, diameter } of Cylinders. $D = 15$ feet, diameter } of the
 $s = 4$ feet, stroke of piston } $P = 22.5$ feet, pitch } Propeller.
 $L = 3$ feet, length }

$||| = 195.5$ square feet, fire-grate in three boilers.

Cut off the steam at about $\frac{1}{4}$ the stroke.

$p = 15$ pounds pressure per square inch in the boilers, excluding the atmosphere.

$\dagger = 2.25$ proportion of gearing.

From these dimensions I will make a calculation, anticipating the performance of the San Jacinto when finished.

Calculation.

Required the acting area of the propeller $\mathcal{O} = ?$

$$\frac{P}{D} = \frac{22.5}{15} = 1.5, \text{ find this number in the Pitch column, Table I., page 246;}$$

in the next columns $W = 64^\circ$ nearly, and $\mathcal{O} = 0.7 \times 15^2 = 157.5$ square feet.

Required the slip $S = ?$

$$\frac{\mathcal{O}}{\mathcal{O}} = \frac{157.5}{45} = 3.5, \text{ find this number in the column of acting area Table III.}$$

page 247, opposite, in the slip column $S = 30$ per cent nearly.

$$H = \frac{D^3 n^3}{480000} (L S \cos W + 0.11) \quad \text{--- form. 11, page 244.}$$

$$H = \frac{2 a s p n}{24250 \dagger} \quad \text{--- form. 22, page 149.}$$

From these two formulas we obtain the number of revolutions of the propeller per minute,

$$n = \frac{5.5 d}{D} \sqrt{\frac{p s}{\dagger D (L S \cos W + 0.11)}}$$

Assume the effectual pressure per square inch on the piston, including the vacuum to be $p = 25$ pounds, we shall have

$$n = \frac{5.5 \times 70}{15} \sqrt{\frac{25 \times 4}{2.25 \times 15 (3 \times 0.3 \times \cos 64^\circ + 0.11)}} = 62 \text{ revolutions per minute, or } n = 12.2 \sqrt{p}$$

lutions per minute, or $n = 12.2 \sqrt{p}$

$$\text{Formula 1. page 249. } M = \frac{22.5 \times 62}{88} (1 - 0.30) = 11 \text{ miles per hour,}$$

or about 9.6 sea miles.

WABASH.

U. S. Propeller Steam Frigate.

Wabash is one of the six steam frigates authorized by Congress, 1854, to be constructed for the U. S. Navy.

Dimensions.

l = 271 feet, length in the load line. Beam 54 feet 4 inches.

\mathcal{A} = 860 square feet, greatest immersed section.

Q = 4700 tons tonnage of displacement.

Propeller, true screw with two blades.

D = 17 feet 4 inches, diameter
 P = 23 feet, pitch
 L = 3 feet 6 inches, length } of the propeller.

Two engines, applied direct on the propellershaft.

d = 72 inches diameter
 s = 3 feet stroke of piston
 p = 20 pounds effectual pressure } steam cylinders.

Four boilers, with vertical tubes.

\square = 330 square feet total fire grate in the boilers, with 11500 square feet of heating surface.

Steam pressure about 15 pounds per square inch in the boilers, cut off the steam at $\frac{1}{2}$ the stroke.

From these dimensions, for which I am indebted to B. H. Bartol, Esq., I will make a calculation of the frigate's performance under steam.

The calculations for the San Jacinto on the preceding page, agrees with her actual performance in smooth water, as near as can be observed.

Calculation,

Example 1. Required the area of resistance, \emptyset = ?

Formula 1, page 248. Exponent $x = \frac{35 \times 4700}{860 \times 271} = 0.705$.

Coefficient k = 1.59 table II. page 247.

For. 2, p. 248. Area of resistance $\emptyset = 860 \sqrt{\frac{860}{860 + 1.59 \times 271^2}} = 73.6$ sq. ft.

Example 2. Required the acting area of the propeller, \bigcirc = ?

$$\frac{P}{D} = \frac{23}{17.33} = 1.327,$$

Find this number in the pitch column, table I., page 246, in the next column $W = 67^\circ$ angle of the propeller blades at the periphery, and the acting area $\bigcirc = 0.722 \times 17.33^2 = 217$ square feet.

Example 3. Required the slip of the propeller, S = ?

$$\frac{\bigcirc}{\emptyset} = \frac{217}{73.6} = 2.94$$

Find this number in the column \bigcirc table III. page 247, opposite in the slip column $S = 32$ per cent slip of the propeller.

Example 4. Required the number of revolutions of the propeller with 20 pounds effectual pressure per square inch on the cylinder pistons, $n = ?$

$$\text{Revolu's } n = \frac{5.5 \times 72}{17.33} \sqrt{\frac{20 \times 3}{17.33(3.5 \times 0.32 \times \cos 67^\circ + 0.11)}} = 57\frac{1}{2} \text{ pr. mt.}$$

Or number of revolutions $n = 12.85 \sqrt{p}$.

Example 5. Required the speed of the frigate under $57\frac{1}{2}$ revolutions and 32 per cent slip of the propeller, $M = ?$

$$\text{Speed } M = \frac{23 \times 57.5}{88} (1 - 0.32) = 10\frac{1}{4} \text{ statute miles.}$$

Or 8.85 sea miles (knot) per hour

Example 6. Required the horse-power of the engines under the above circumstances, $H = ?$

$$\text{Form. 22, p. 149. Power } H = \frac{2 \times 4071.5 \times 3 \times 20 \times 57.5}{24250} = 1158 \text{ horses}$$

Example 7. Required the horse-power of the fire-grate $\square = ?$ with 15 pounds steam-pressure in the boilers, cut off at one-third the stroke and vacuum in the condensor 12 pounds.

$$\text{Form. 4, page 254. } H = \frac{330 \times 883 \times 7(15 + 0.8 \times 12)}{31400} = 1600 \text{ horses.}$$

By this calculation the power of the fire-grate exceeds the power of the engine in the preceding example by $1600 - 1158 = 442$ horses; consequently the steam can be kept higher than 15 pounds per square inch in the boilers, and the propeller accordingly making more revolutions per minute.

Example 8. Assume the revolutions to $n = 65$ per minute, and find to what pressure the steam can be kept? Cut off at 12 inches of the stroke in two cylinders makes $S = 24$ inches.

$$k = \frac{D^2 S n}{4.66 w \square} = \frac{72^2 \times 24 \times 65}{4.66 \times 7 \times 330} = 756,$$

Find this number (nearest 748) in the k column, page 233, opposite in the last column is the required pressure, 21 pounds per square inch.

65 revolutions = 10 knots per hour.

Example 9. Required the consumption of coal per hour in the four boilers with 21 pounds of steam cut off at $\frac{1}{3}$ the stroke and 65 revolutions per minute $C = ?$ The coal to evaporate $w = 7$ pounds of water and $k = 748$ at 21 pounds.

$$\text{Form. 5, p. 255. Coal } C = \frac{2 \times 3 \times 72^2 \times 12 \times 65}{7 \times 748} = 4633 \text{ pounds,}$$

or 2 tons pr. hour.

The Wabash is now under building at the Philadelphia Navy Yard, machinery and propeller constructed by Merrick and Sons, Philadelphia. She will be ready under steam early in the spring, 1856.

FRESH WATER CONDENSOR.

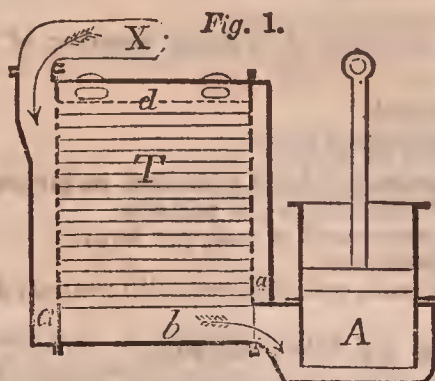


Fig. 1.

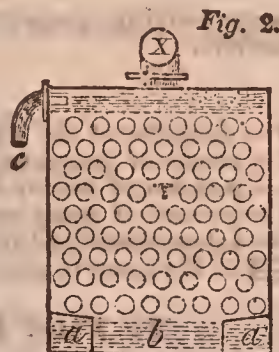


Fig. 2.

Fig. 1, is a longitudinal, and Fig. 2, a transverse section of a fresh water condenser with horizontal tubes.

A, air-pump. *a*, fresh water. *X*, exhaust-pipe. *b*, hot well. *T*, tubes. *c*, injection pipe. *d*, strainer.

The tubes are of copper one inch outside diameter, thickness of metal, No. 22 or 24 wire gauge, weighs $5\frac{1}{2}$ ounces per foot. The space occupied by the tubes should be about cubical, that is, the sides of the tube-plate should be about the length of the tubes.

Between the injection and the tubes is a horizontal strainer, to spread the cold water uniformly over the tubes. Steam inside the tubes.

Letters Denote.

A = condensing area of all the tubes in square inches.

l = length of tubes, height and breadth of tube-plate in inches.

N = number of tubes in the condensor.

D = diameter of steam cylinder. *S* = stroke of piston, in inches.

n = number of revolutions of engine per minute.

T = temperature of exhaust steam

k = volumu coefficient of steam } see steam table, page 232.

$$A = \frac{D^2 S \sqrt{n}}{5.5 k} (940 + T), \quad l = 0.853 \sqrt[3]{A}, \quad N = 0.5128 l^2, \quad A = 1.61 l^3.$$

Example. A fresh water condensor is to be constructed for an engine of *D* = 62 in. *S* = 76 in. making *n* = 34 revolutions per minute; *T* = 230°, *k* = 1225. Required the condensing area of tubes, *A* = ?

$$A = \frac{62^2 \times 76 \times \sqrt{34}}{5.5 \times 1225} (940 + 230) = 295,769 \text{ square inches.}$$

Required the length of tubes, and sides of tube plate, *l* = ?

$$l = 0.853 \sqrt[3]{295,769} = 57 \text{ inches nearly.}$$

Required the number of tubes in the condensor, *N* = ?

$$N = 0.5128 \times 57^2 = 1666 \text{ tubes.}$$

Number of tubes in the top row $\frac{l}{1.5} = \frac{57}{1.5} = 38$, side row $\frac{57}{1.3} = 44$ tubes.

The tubes to be placed zigzag, as shown in Fig. 2. Should the location for the condensor not permit the cubical form, we have,

$$\text{Length tubes } l = \frac{A}{1.61 l^2}, \quad \text{Breadth of tube plate } b = \frac{A}{1.61 l^2}, \quad \text{Height } h = \frac{A}{1.61 l^2}$$

Fresh water produced $G = \frac{D^2 S n}{164 k}$, gallons per minute, or about 75 per cent of the feed water.

Temperature in the hot well about 116° to 115°.

“ of fresh water 120° to 130°.

For fresh water condensers the capacity of the air pump should be about 10 per cent larger than by the rules on page 236.

STEAM BOILERS.

The accompanying proportions are averages of a great number of good marine boilers.

Letters denote.

D = diameter of the steam-cylinder in inches.

S = stroke of piston under which steam is fully admitted, in inches.

n = number of double strokes, or revolutions per minute.

w = pounds of water evaporated per pound of coal, per hour.

k = volum coefficient from the steam table.

\square = fire grate in square feet, for each cylinder, and with natural draft.

To Find the Area of Fire Grate.

$$\square = \frac{D^2 S n}{4.66 w k} \quad n = \frac{4.66 w k \square}{D^2 S} \quad 1, 2.$$

Example 1. A steam engine of $D = 54$ inches diameter of the cylinder, and stroke of piston 96 inches, cut off at $\frac{1}{2}$, $S = 48$ inches; is to make 22 revolutions per minute. Anthracite coal to be used, that evaporates $w = 7$ pounds of water per pound of coal, and to carry 27 pounds of steam per square inch, $k = 649$. Required the area of fire grate $\square = ?$ in square feet.

$$\square = \frac{54^2 \times 48 \times 22}{4.66 \times 7 \times 649} = 145.34 \text{ square feet.}$$

Example 2. A steamboiler of $\square = 128$ square feet, is to be used for an engine of $D = 36$ inches diameter, and 64 inches stroke, — cut off the steam at $\frac{2}{3}$ then $S = 42.66$ inches. Steam pressure to be kept at 25 pounds per square inch $k = 679$. $w = 6.5$. Required for how many revolutions per minutes can the steam be kept at 25 pounds?

$$n = \frac{4.66 \times 6.5 \times 679 \times 128}{36^2 \times 42.66} = 47.6 \text{ revolutions.}$$

Horse Power of the Fire Grate.

H = horse power of the fire grate.

P = pressure in the boiler in pounds per square inch, excluding the atmosphere.

p = vacuum in the condensor in pounds per square inch.

$$\square = \frac{H x}{k w (P + 0.8 p)}, \quad H = \frac{\square k w (P + 0.8 p)}{x} \quad 3 \ 4.$$

Cut off the steam at	$\frac{1}{4}$ the stroke.	$x = 27700$.	saves 55	} per cent. of fuel.
	$\frac{1}{2}$ " "	$x = 31400$.	" 49	
	$\frac{3}{4}$ " "	$x = 38400$.	" 38	
	$\frac{1}{2}$ " "	$x = 45500$.	" 26	
	$\frac{3}{4}$ " "	$x = 49100$.	" 20	

Steam admitted throughout the stroke $x = 61700$, " 0 per cent.

Example 3. Steamboilers are to be constructed for an engine of 650 horses, the steam to be cut off at $\frac{1}{2}$ the stroke, and $P = 36$ pounds per square inch. $k = 544$, $w = 7.5$ pounds of water evaporated per pound of coal. Required the fire grate in the boilers $\square = ?$ in square feet.

$$\square = \frac{650 \times 38400}{544 \times 7.5 (36 + 0.8 \times 11)} = 136 \text{ square feet.}$$

Example 4. Required the horse-power of a fire grate $\text{■} = 112$ square feet, to carry 18 pounds steam, and cut off at $\frac{2}{3}$ the stroke? $k = 810$, $w = 7$ pounds.

$$H = \frac{112 \times 18 \times 810 \times 7}{45500} = 251.2 \text{ horses.}$$

Consumption of Coal.

C = coal consumed in pounds per hour.

$$C = \frac{3 D^2 S n}{w k} \quad C = \frac{14 H x}{k w (P + 0.8p)} \quad 5, 6.$$

Example 5 A steam engine of $D = 42$ inches diameter, and 48 inches stroke, cut off the steam at $\frac{1}{3} S = 16$ inches, is to make $n = 65$ revolutions per minute with a pressure of 34 pounds per square inch, $k = 564$, and $w = 6$ pounds. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{3 \times 42^2 \times 16 \times 65}{6 \times 564} = 1625 \text{ pounds per hour.}$$

Example 6. A pair of steam engines of $H = 260$ horses are to be worked with $P = 28$ pounds per square inch, cut off at $\frac{2}{3}$ the stroke, $k = 635$, the coal to evaporate $w = 6.5$ pounds of water per pound of coal. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{14 \times 260 \times 31400}{630 \times 6.5 (28 + 0.8 \times 10)} = 775 \text{ pounds per hour.}$$

It will be observed in the formulas 4 and 6, that the higher steam used, the less fuel and fire grate is required for the same power,—the proportion of fuel will be nearly as the square root of the steam pressure; and still more fuel is saved by cutting off the steam at an early part of the stroke.

Fire Surface.

1. In the common single returning flue boilers, the whole amount of fire surface from the grate to the water line should be about 25 ■ .
2. In flue and returning tubular boilers the whole amount of fire surface 30 ■ .
3. Boilers with vertical flues, (2 inches diameter,) fire outside and water inside, fire surface 35 ■ .

Area of Flues. (Calorimeter.)

In the common single returning flue boilers, the area of the first row should be about 0.18 ■ .

Returning row, (flue or tube) 0.13 ■ .

Area of chimneys 0.16 ■ .

Evaporative Power, Weight and Bulk of Fuel.

Names of Fuel	Water evap'rat'd per pound of coal. W.	Weight per cubic foot.	Cubic feet to stow a ton.
Bitumenous or Anthracite coal	6 to 9	50 to 54	40 to 44
Coke, Natural Virginia	8 to 9	48	48
" Cumberland	8 to 10	32	70
Wood, Dry Pine	4 to 5	20 to 25	100

Lap Welded Iron Tubes.

Outside Diameter.	Standard Thickness W. G.	lbs. Weight per foot.	1856,		Fire Surface per foot.
			Price per foot.	Price of each safe end.	
1½	14	1·68	\$0·25	\$0·16	0·3273
1½	14	1·60	0·23	0·16	0·3926
1¾	13	1·80	0·31	0·18	0·4581
2	13	2·08	0·35	0·18	0·5236
2½	13	2·25	0·39	0·20	0·5890
2½	12	2·50	0·43	0·23	0·6545
2¾	12	3·12	0·48	0·26	0·7200
3	11	3·50	0·53	0·29	0·7853
3½	11	4·12	0·60	0·32	0·8508
3½	10	5·15	0·70	0·35	0·9163
3¾	10	5·28	0·85	0·38	0·9817
4	10	5·54	1·00	0·40	1·0472
5	9	7·	1·60	0·45	1·3680
6	8	10·	2·20	0·50	1·5708
7	8	12·	2·80	0·60	1·8326

The "Safe End," of extra quality and thicker iron, is welded to one or both ends of flues to order, which gives increased strength to the flue where it connects with the tube sheet.

Tube varying in thickness from standard gauge, made to order.

Manufactured and for sale by Morris, Tasker and Morris, Pascal iron works, Philadelphia.

WEIGHT OF STEAM BOILERS.

Thickness and Weight of Boiler Iron.

Parts of the Boiler.	Thickness by W Gauge.	Thickness in Inches.	Weight per Sq. Foot.
Boiler legs	1	0·3	12·1
Steam chimney	2	0·284	11·4
Furnes and crowns	3	0·259	10·4
Shell and steam drum	3	0·259	10·4
Flues from 13 to 18 in. diameter	4	0·238	9·6
" 10 to 13 "	5	0·22	8·85
" 8 to 10 "	6	0·203	8·17
" 6 to 8 "	7	0·18	7·27
Smoke pipe	11	0·12	4·83

Weight of Tubular Boilers.

Coefficient for Weight of Boilers.	Coefficient for Weight of Rivets and Braces.	Fire Surface in Square Feet.
16	3·6	above } 2000
18	4·	below }
Weight of Flue Boilers.		
27	5·5	above } 1000
32	6·	below }

Smoke pipe and grate bars excluded.

Example 1. Required the weight of a flue boiler with 709 square feet fire surface? also the weight of rivets and braces?

Weight of the boiler $709 \times 32 = 22,688$ pounds

of which rivets & braces $709 \times 6 = 4,254$ "

Example 2. Required the weight of a tubular boiler with 2,460 square feet fire surface? also the weight of rivets and braces?

Weight of the boiler $2460 \times 16 = 39,360$ pounds

of which rivets & braces $2460 \times 3·6 = 8,856$ "

OPTICS.

Optics is that branch of philosophy which treats of the property and motion of light.

Mirrors.

Example 1. Fig. 307. Before a *concave mirror* of $r = 6$ feet radius, is placed an object $O = 1$, at $d = 1.75$ feet from the vertex. Required the size of the image $I = ?$

$$\text{image } I = \frac{O r}{r - 2 d} = \frac{1 \times 6}{6 - 2 \times 1.75} = 2.4.$$

Example 2. Fig. 308. Before a *concave mirror* of $r = 5.25$ feet radius, is placed an object $O = 1$, at $D = 4.5$ feet from the vertex. Required the size of the inverted image $I = ?$

$$\text{image } I = \frac{O r}{2 D - r} = \frac{1 \times 5.25}{2 \times 4.5 - 5.25} = 1.4$$

Example 3. Fig. 309. Before a *convex mirror* of $r = 1.8$ feet radius, is placed an object $O = 1$, at $D = 3.15$ feet from the vertex. Required the size of the image $I = ?$, and the distance in the mirror $d = ?$

$$\text{image } I = \frac{1 \times 1.8}{2 \times 3.15 + 1.8} = 0.222 \quad \text{distance } d = \frac{3.15 \times 1.8}{2 \times 3.15 + 1.8} = 0.699 \text{ ft.}$$

Example 4. Fig. 310. A *parabolic mirror* is $h = 1.31$ feet high, and $d = 2.15$ feet in diameter. Required the focal distance $f = ?$ from the vertex.

$$\text{focal distance } f = \frac{d^2}{16 h} = \frac{2.15^2 \times 12}{16 \times 1.31} = 2.646 \text{ inches.}$$

Optical Lenses.

Example 5. Fig. 316. A double *convex lens*, of *crown glass*, having its radii $R = r = 6$ inches. Required its principal focal distance $f = ?$
For crown glass the index of refraction is $m = 1.52$. See table.

$$f = \frac{6}{2(1.52 - 1)} = 5.768 \text{ inches.}$$

Microscope.

Letters denote.

p = magnifying power of a lens.

D = limit of distinct vision.

α = limit of distinct sight, which for *long-sighted* eyes is about 10 or 12 inches, and *near-sighted* 6 to 8 inches. For common eyes take $\alpha = 10$ inches.

D = limit distance of the object from the optical centre at distinct vision.

Example 6. Fig. 322. Required the magnifying power of a single microscope with principal focal distance, $f = 4.3$ inches?

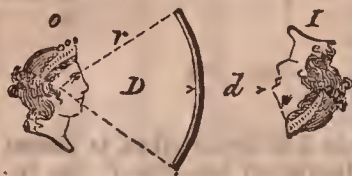
$$\text{Mag. power } p = \frac{\alpha + f}{f} = \frac{10 + 4.3}{4.3} = 3.325 \text{ times.}$$

307 *Spherical Concave Mirror.*

r = radius, and $f = \frac{1}{2} r$, focal distance of the mirror.

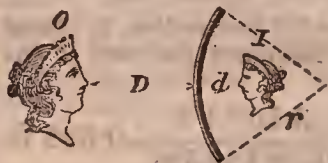
$$I = \frac{O r}{r - 2d} \cdot D = \frac{dr}{r - 2d}.$$

The image disappears when $d = f = \frac{1}{2} r$.

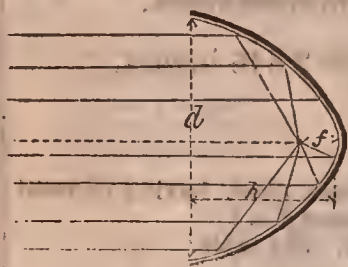
308 *Spherical Concave Mirror.*

$$I = \frac{O r}{2D - r} \cdot d = \frac{Dr}{2D - r}.$$

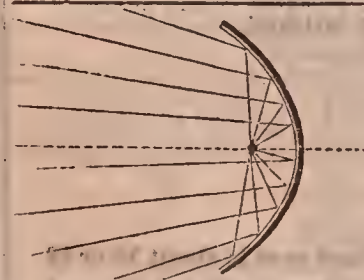
When the object is beyond the focal distance the image will be inverted.

309 *Spherical Convex Mirror.*

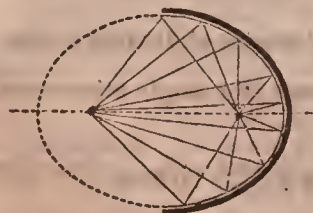
$$I = \frac{O r}{2D + r} \cdot d = \frac{Dr}{2D + r}.$$

310 *Parabolic Concave Mirror.*

$$f = \frac{d^2}{16h} \cdot h = \frac{d^2}{16f}.$$

311 *Hyperbolic Concave Mirror.*

Heat, Light, or Sound emanating from the foci of a hyperbola will be reflected diverged, from the concave surface.

312 *Elliptic Concave Mirror.*

Emanating rays from either of the two foci in an ellipse, will be refracted by the convex surface to the other foci.

Astronomical Telescopes and Opera Glasses.

Example 7. Fig. 325. The principal focal distance $f = 0.65$ inches of the ocular or eye-lens. $F = 58$ inches the principal focal distance of the objective-lens. Required the magnifying power of the telescope $I = ?$

$$\text{image } I = \frac{O F}{f} = \frac{1 \times 58}{0.65} = 89.23 \text{ times the object.}$$

The telescope is to be used at the limited distance $D = 1380$ feet and $D = \infty$. Required the proper lengths $l = ?$ and micrometrical motion of the ocular or eye-lens? when the limit of distinct sight $a = 10$ in. $F = 58 : 12 = 4.833$ feet. $f = 0.65 : 12 = 0.05416$ feet.

$$l = \frac{1380 \times 4.833}{1380 - 4.833} + \frac{10 \times 0.05416}{10 + 0.05416} = \frac{4.89035}{0.05386}$$

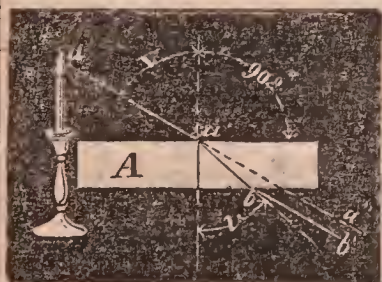
When $D = 1380$ feet, the length $l = 4.94421$ feet.

When $D = \infty$, $l = 4.8333 + 0.05386 = 4.88719$ "

Micrometrical motion of eye lens $\left\{ \begin{array}{l} 0.05702 \text{ "} \\ 0.68424 \text{ inches.} \\ \frac{11}{13} \text{ nearly.} \end{array} \right.$

Table of Refractive Indices.

Substances.	Index. m.	Substances.	Index. m.
Cromate of Lead -	$\left\{ \begin{array}{l} 2.97 \\ 2.50 \end{array} \right.$	Quartz - - -	1.54
Realgar - - -	2.55	Muriatic Acid - -	1.40
Diamond - - -	2.45	Water - - -	1.33
Glass, flint - -	1.57	Ice - - -	1.30
Glass, crown -	1.52	Hydrogen - - -	1.000138
Oil of Cassia -	1.63	Oxygen - - -	1.000272
Oil of Olives -	1.47	Atmospheric air -	1.000294



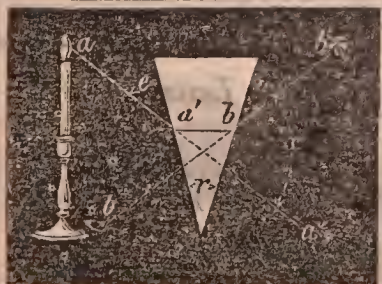
314

Prism.

An emergent rays of light $a a'$ falling upon a transparent medium A (say a glass prism) will be transmitted through in the direction $a b$, and delivered in the direction $b b'$, parallel to $a a'$.

$V =$ angle of incident, $v =$ angle of refraction.

$$\text{Index of refraction } m = \frac{\sin. V}{\sin. v}.$$



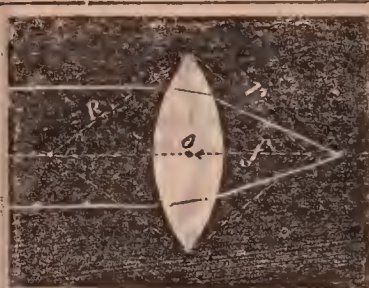
315 Given the direction of the emergent rays $a a'$, angles e and r .—to find the angles u and x ,—or the direction of the rays $b b'$.

$$\cos. z = \frac{\cos. e}{m}, \quad \cos. u = m \cos. (180 - z - r).$$

$$x = 180 - (e + r + u).$$

When $e = u$, the angle x is smallest.

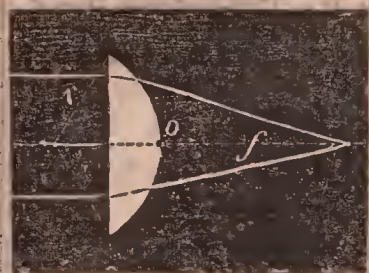
An eye in b' will see the candle in the direction $b' b''$.

316 *Double Convex Lens.*

$$f = + \frac{Rr}{(m-1)(R+r)}, \quad \text{the principal focal distance.}$$

$$f = \frac{r}{2(m-1)}, \quad \text{when } R = r$$

o = optical centre of the lens.



317

Plano Convex Lens.

$$f = + \frac{r}{m-1}.$$

The optical centre is in the convex surface.

318 *Convex-concave Lens (Meniscus.)*

$$f = + \frac{Rr}{(m-1)(R-r)}.$$

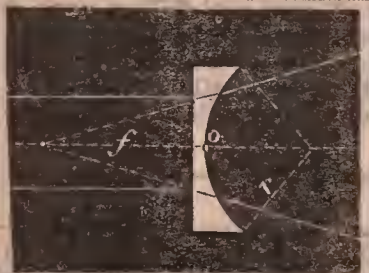
Draw the radii R' and r' parallel to one another.—Draw no , then o is the optical centre.



319

Double Concave Lens.

$$f = - \frac{Rr}{(m-1)(R+r)}.$$

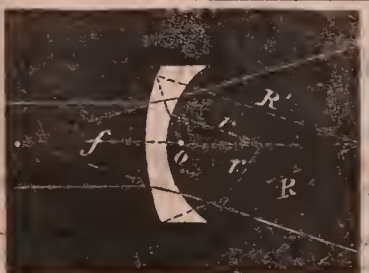


320

Plano Concave Lens.

$$f = - \frac{r}{m-1}.$$

The optical centre is in the concave surface.



321

Concavo-convex Lens.

$$f = - \frac{Rr}{(m-1)(R-r)}.$$

Draw R and r' parallel to one another. Draw no , then o is the optical centre.

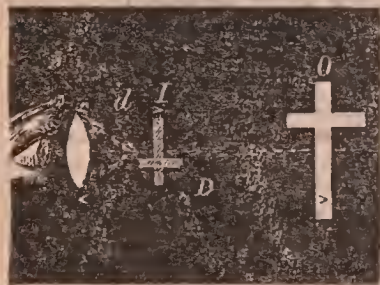


322

Single Microscope.

$$I:O = f:f-d, \quad I = \frac{O.f}{f-d}, \quad D = \frac{d.f}{f-d},$$

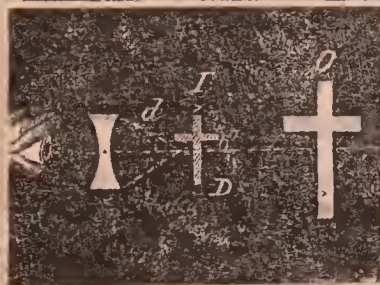
$$\mathfrak{P} = \frac{a+f}{f}, \quad \mathfrak{D} = \frac{d.f}{f-d}, \quad d = \frac{a.f}{a+f}.$$



323

When the object O is beyond the focal distance the image I will be inverted.

$$I:O = f:D-f, \quad I = \frac{O.f}{d-f}, \quad d = \frac{D.f}{D-f}.$$



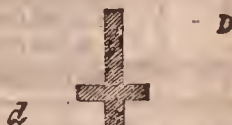
324

Diminishing Power of a Double Concave Lens.

$$I:O = f:f+D, \quad I = \frac{O.f}{f+D},$$

$$D = \frac{f(O-I)}{I}, \quad d = \frac{D.f}{D+f}.$$

325

Astronomical Telescope.

$$I:O = F:f,$$

$$I = \frac{O.F}{f},$$

$$d = \frac{D.f}{F}.$$

$$l = \frac{D.F}{D-F} \pm \frac{a.f}{a+f}.$$

$$\left(\text{if } D = \infty, \quad l = F + \frac{a.f}{a+f} \right)$$

* + for astronomical telescope,

— for opera-glasses.

326

Opera Glass.

Formulas are the same as for Astronomical Telescope.

GEOGRAPHY.



THE *Earth* on which we live, is a round ball or sphere, with a mean diameter of 7914 statute miles. The whole surface of the earth is 196800000 square miles, of which only one fourth or nearly 50,000000 square miles is land, and about 150,000000 square miles water.

Table of Area and Population of the whole Earth.

<i>Divisions of the Earth.</i>	<i>Area in Square Miles.</i>	<i>Population.</i>	<i>Proportion to Square Mile.</i>
America, - - - - -	14,491,000	54,677,000	4
Europe, - - - - -	3,760,000	258,354,000	69
Asia, - - - - -	16,313,000	455,562,000	28
Africa, - - - - -	10,936,000	61,604,000	5
Oceanica, - - - - -	4,500,000	23,261,000	5
Total, - - - - -	50,000,000	853,458,000	17

About $\frac{1}{30}$ th of the whole population are born every year, and nearly an equal number die in the same time; making about one born and one dead per second.

The Earth is not a perfect sphere, it is flatted at the Poles. The following are her true dimensions in statute miles or 5280 feet.

Dimensions of the Earth.

<i>Diameter</i>	{ 7898·8809 miles at the Poles. 7911·92 " mean, or in 45° lat. 7924·911 " at the Equator.
<i>Difference</i> - - -	26·0302 " Poles and Equator.
<i>Flatted</i> - - - -	13·015 " at each Pole.
<i>Circumference</i>	{ 24802·486 " round the Poles. 24851·640 " Mean, or in 45° lat. 24884·22 " round the Equator.

To Find the radius of the Earth in any given Latitude

$$R = 3955·96(1 + 0·00164 \cos.2l), \text{ statute miles.}$$

Definitions,

Axis of the Earth is an imaginary diameter around which the earth revolves.

Poles of the Earth are the two extremities of the axis, and are called North and South.

Equator of the Earth is the great circle at equal distances from the Poles; it divides the Earth into the Northern and Southern Hemispheres.

Meridian is any great circle of the Earth drawn through the Poles; hence the Meridian runs north and south, and are at right-angles to the Equator.

The *Equator* and *Meridians* are divided into degrees, minutes, and seconds.

Latitude is the degrees on a Meridian counted from the Equator.

Longitude is the degrees on the Equator or on circles parallel with the Equator, counted at right angles from a Meridian.

Parallels are circles drawn through equal Latitudes; they are parallel and concentric with the Equator, and at right-angles to the Meridians.

East and West is the direction of the Equator and Parallels, or at right-angles to north and south. Turn your face towards the south, the east is on the left hand, and the west on the right.

The time in which the earth makes one revolution, is divided into 24 hours,

$$\text{and } \frac{360^\circ}{24} = 15^\circ \text{ per hour.}$$

To Reduce Longitude into Time.

RULE. Divide the number of degrees, minutes, and seconds by 15, and the quotient will be the time.

Example 1. Longitude $74^\circ 48' 15''$, what is it in time?

$4h 59^m 13^s$ the answer.

To Find the Difference in Time between two Places.

RULE. Divide the difference in longitude by 15, and the quotient is the difference in time.

Example 2. Required the difference in time between New York and Cincinnati?

$$\begin{array}{rcl} \text{Longitude of Cincinnati} & - & 84^\circ 27' W \\ \text{" " New York} & - & 74^\circ 07' W \\ \hline \text{Difference in longitude} & - & 10^\circ 20' \end{array}$$

$$\frac{10 \times 60 + 20}{15} = 41 \text{ minutes } 20 \text{ seconds,}$$

the difference in time. When it is 12 o'clock in Cincinnati, it is $12h 41' 20''$ in New York.

Example 3. Required the difference in time between Philadelphia and Paris?

$$\begin{array}{rcl} \text{Longitude of Philadelphia} & 75^\circ 10' W \\ \text{" " Paris} & - & 2^\circ 20' E \end{array}$$

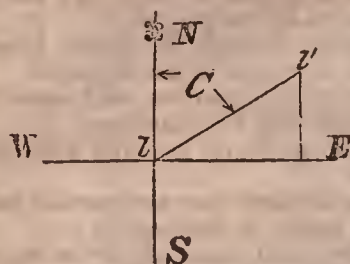
Difference in longitude $77^\circ 30'$ divided by 15 will be $5h 10^m$ the difference in time. When it is 12 o'clock in Philadelphia, it is $5h 10^m$ o'clock in Paris.

Example 4. A vessel sails from New York for Liverpool, after she has been at sea about one week, her difference in time with New York was found to be $2h 7^m 45^s$. Required her longitude from New York?

$$15(2h 7^m 45^s) = 31^\circ 58' 15'' \text{ from New York.}$$

NAVIGATION.

To navigate a vessel upon the supposition that the earth is a level plane, on which the meridians are drawn north and south, parallel with each other; and the parallels east and west, at right-angles to the former.



The line NS represents a meridian north and south; the line WE represents a parallel east and west.

A ship in l sailing in the direction of ll' , and having reached l' , it is required to know her position to the point l , which is measured by the line ll' , and the angle $Nl l'$; and imagined by the lines la and al' .

While the vessel is running from l to l' , the distance is measured by the log and time; and the course $Nl l'$ is measured by the compass commonly expressed in points.

These four quantities bear the following names.

$d = ll'$, distance from l to l' in miles.

$C = Nl l'$, course, or points from the meridian.

$\bar{d} = la$, departure or difference in longitudes, in miles.

$u = al'$, difference in latitudes, in miles.

l = latitude in degrees.

L = difference in longitude, in degrees or time.

Formulas for Plane Sailing.

$$\bar{d} = d \sin. C, \quad - \quad 1,$$

$$\bar{d} = u \tan. C, \quad - \quad 2,$$

$$\bar{d} = 60 \cos. l \, L, \quad - \quad 3,$$

$$\bar{d} = \sqrt{d^2 - u^2}, \quad - \quad 4,$$

$$u = d \cos. C, \quad - \quad 5,$$

$$u = \bar{d} \cot. C, \quad - \quad 6,$$

$$u = \frac{60 L \cos. l}{\tan. C}, \quad - \quad 7,$$

$$u = \sqrt{d^2 - \bar{d}^2}, \quad - \quad 8,$$

$$d = \frac{\bar{d}}{\sin. C}, \quad - \quad 9,$$

$$d = \frac{u}{\cos. C}, \quad - \quad 10,$$

$$d = \frac{60 L \cos. l}{\sin. C}, \quad - \quad 11,$$

$$d = \sqrt{\bar{d}^2 + u^2}, \quad - \quad 12,$$

$$\cos. l = \frac{\bar{d}}{60 L}, \quad - \quad 13,$$

$$\cos. l = \frac{d \sin. C}{60 L}, \quad - \quad 14,$$

$$\cos. l = \frac{u \tan. C}{60 L}, \quad - \quad 15,$$

$$L = \frac{\bar{d}}{60 \cos. l}, \quad - \quad 16,$$

$$L = \frac{d \sin. C}{60 \cos. l}, \quad - \quad 17,$$

$$L = \frac{u \tan. C}{60 \cos. l}, \quad - \quad 18,$$

$$\cos. C = \frac{u}{d}, \quad - \quad 19,$$

$$\sin. C = \frac{\bar{d}}{d}, \quad - \quad 20,$$

$$\tan. C = \frac{\bar{d}}{u}, \quad - \quad 21,$$

$$\sin. C = \frac{60 L \cos. l}{d}, \quad 22,$$

$$\tan. C = \frac{60 L \cos. l}{u}, \quad 23,$$

See Table of Formulas for Plane Sailing.

Example 1. A vessel sails *east north east* (6 points,) 236 miles. Required her departure Δ ? and difference in latitude u ?

Formula 1. $\Delta = d \sin.C = 236 \times \sin.6 \text{ points} = 218 \text{ miles departure, and } u = d \cos.c = 236 \times \cos.6 \text{ points} = 90.3 \text{ miles difference in latitude.}$

Example 2. A ship sails in north latitude in a course $C = ESE\frac{1}{2}E = 6\frac{3}{4}$ points, at a distance of 132 miles she made a difference in longitude of $L = 3^{\circ} 34'$. What latitude is she in?

$$\text{Formula 14.} \quad \cos.l = \frac{d \sin.C}{60L} = \frac{132 \times \sin.6\frac{3}{4}}{60 \times 3 + 34} = 0.59832,$$

or $l = 53^{\circ} 15'$ the latitude.

In high latitudes and very long distances, the preceding formulas will not give such correct results as may be desired, because they are set up with the supposition that the earth is a level plane; but by the aid of spherical trigonometry, we are enabled to ascertain courses and distances *correctly*, from and between any known points on the earth.

Spherical Distances.

For the spherical formulas, letters will denote.

l = lower latitude, in degrees from the equator.

l' = highest latitude, " " " "

C = course, from the latitude l to l' .

C' = course, from " l' to l .

d = shortest distance between l and l' in degrees of the great circle.

L = difference in longitude between l and l' , in degrees, or time.

$\tan.m = \cot.l' \cos.L.$

$n = 90 - l - m.$

$-l$, when l and l' are on one side of the equator.

$+l$, when l is on one side and l' on the other. Then

$$\cos.d = \frac{\sin.l' \cos.n}{\cos.m}, \quad 1,$$

$$\sin.C = \frac{\sin.L \cos.l'}{\sin.d}, \quad 2,$$

$$\sin.C' = \frac{\sin.L \cos.l}{\sin.d}, \quad 3,$$

Example. Required the shortest distance and course from New York to Liverpool?

$l = 40^{\circ} 42' \text{ N. latitude}$
 $74^{\circ} \text{ " W. longitude}$ } New York.

$l' = 53^{\circ} 22' \text{ N. latitude}$
 $2^{\circ} 52' \text{ W. longitude}$ } Liverpool.

$L = 71^{\circ} 8'$ difference in longitude.

$\tan.m = \cot 53^{\circ} 22' \times \cos.71^{\circ} 8' = 13^{\circ} 31'.$

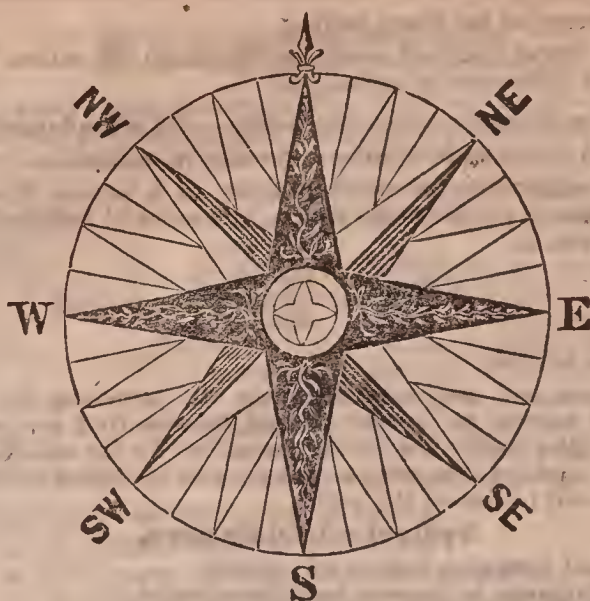
$n = 90^{\circ} - 13^{\circ} 31' - 40^{\circ} 42' = 35^{\circ} 47'.$

$$\text{For. 1.} \quad \cos.d = \frac{\sin.53^{\circ} 22' \times \cos.35^{\circ} 47'}{\cos.13^{\circ} 31'} = 47^{\circ} 58'.$$

Shortest distance $= 47^{\circ} \times 60 + 58 = 2878$ geographical miles.

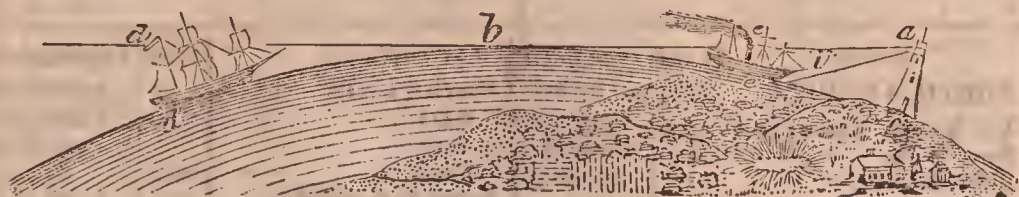
$$\sin.C = \frac{\sin.71^{\circ} 8' \times \cos.53^{\circ} 22'}{\sin.47^{\circ} 58'} = 49^{\circ} 23' = 4\frac{3}{4} \text{ points}$$

course from New York $NE\frac{3}{4}E$.



North.	South.	Points.	Degrees.	sine C.	Cos. C.	tan. C.
N.	S.	$\left\{ \begin{array}{l} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 2^{\circ} 45' \\ 5 \quad 37 \\ 8 \quad 26 \end{array} \right.$	$\left\{ \begin{array}{l} .0491 \\ .0879 \\ .1544 \end{array} \right.$	$\left\{ \begin{array}{l} .9988 \\ .9952 \\ .9880 \end{array} \right.$	$\left\{ \begin{array}{l} .0492 \\ .0883 \\ .1982 \end{array} \right.$
N. by E. and N. by W.	S. by E. and S. by W.	$\left\{ \begin{array}{l} 1 \\ 1\frac{1}{4} \\ 1\frac{1}{2} \\ 1\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 11 \quad 15 \\ 14 \quad 4 \\ 16 \quad 52 \\ 19 \quad 41 \end{array} \right.$	$\left\{ \begin{array}{l} .1936 \\ .2430 \\ .2 \quad 01 \\ .3368 \end{array} \right.$	$\left\{ \begin{array}{l} .9811 \\ .9700 \\ .9570 \\ .9416 \end{array} \right.$	$\left\{ \begin{array}{l} .1989 \\ .2505 \\ .3032 \\ .3577 \end{array} \right.$
N. N. E. and N. N. W.	S. S. E. and S. S. W.	$\left\{ \begin{array}{l} 2 \\ 2\frac{1}{4} \\ 2\frac{1}{2} \\ 2\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 22 \quad 30 \\ 25 \quad 19 \\ 28 \quad 7 \\ 30 \quad 56 \end{array} \right.$	$\left\{ \begin{array}{l} .3827 \\ .4276 \\ .4713 \\ .5140 \end{array} \right.$	$\left\{ \begin{array}{l} .9239 \\ .9039 \\ .8820 \\ .8577 \end{array} \right.$	$\left\{ \begin{array}{l} .4142 \\ .4730 \\ .5343 \\ .5993 \end{array} \right.$
N. E. by N. and N. W. by N.	S. E. by S. and S. W. by S.	$\left\{ \begin{array}{l} 3 \\ 3\frac{1}{4} \\ 3\frac{1}{2} \\ 3\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 33 \quad 45 \\ 36 \quad 44 \\ 39 \quad 22 \\ 42 \quad 11 \end{array} \right.$	$\left\{ \begin{array}{l} .5555 \\ .5981 \\ .6343 \\ .6715 \end{array} \right.$	$\left\{ \begin{array}{l} .8314 \\ .8014 \\ .7731 \\ .7410 \end{array} \right.$	$\left\{ \begin{array}{l} .6883 \\ .7463 \\ .8204 \\ .9062 \end{array} \right.$
N. E. and N. W.	S. E. and S. W.	$\left\{ \begin{array}{l} 4 \\ 4\frac{1}{4} \\ 4\frac{1}{2} \\ 4\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 45 \quad 0 \\ 47 \quad 49 \\ 50 \quad 37 \\ 53 \quad 26 \end{array} \right.$	$\left\{ \begin{array}{l} .7071 \\ .7410 \\ .7731 \\ .8014 \end{array} \right.$	$\left\{ \begin{array}{l} .7071 \\ .6715 \\ .6345 \\ .5981 \end{array} \right.$	$\left\{ \begin{array}{l} 1.000 \\ 1.103 \\ 1.218 \\ 1.348 \end{array} \right.$
N. E. by E. and N. W. by W.	S. E. by E. and S. W. by W.	$\left\{ \begin{array}{l} 5 \\ 5\frac{1}{4} \\ 5\frac{1}{2} \\ 5\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 56 \quad 15 \\ 59 \quad 4 \\ 61 \quad 52 \\ 64 \quad 41 \end{array} \right.$	$\left\{ \begin{array}{l} .8314 \\ .8577 \\ .8820 \\ .9039 \end{array} \right.$	$\left\{ \begin{array}{l} .5555 \\ .5140 \\ .4713 \\ .4276 \end{array} \right.$	$\left\{ \begin{array}{l} 1.496 \\ 1.668 \\ 1.870 \\ 2.114 \end{array} \right.$
E. N. E. and W. N. W.	E. S. E. and W. S. W.	$\left\{ \begin{array}{l} 6 \\ 6\frac{1}{4} \\ 6\frac{1}{2} \\ 6\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 67 \quad 30 \\ 70 \quad 19 \\ 73 \quad 7 \\ 75 \quad 56 \end{array} \right.$	$\left\{ \begin{array}{l} .9239 \\ .9416 \\ .9570 \\ .9700 \end{array} \right.$	$\left\{ \begin{array}{l} .3827 \\ .3368 \\ .2901 \\ .2430 \end{array} \right.$	$\left\{ \begin{array}{l} 2.414 \\ 2.795 \\ 3.285 \\ 3.991 \end{array} \right.$
E. by N. and W. by N.	E. by S. and W. by S.	$\left\{ \begin{array}{l} 7 \\ 7\frac{1}{4} \\ 7\frac{1}{2} \\ 7\frac{3}{4} \end{array} \right.$	$\left\{ \begin{array}{l} 78 \quad 45 \\ 81 \quad 34 \\ 84 \quad 22 \\ 87 \quad 11 \end{array} \right.$	$\left\{ \begin{array}{l} .9811 \\ .9880 \\ .9952 \\ .9988 \end{array} \right.$	$\left\{ \begin{array}{l} .1936 \\ .1544 \\ .0879 \\ .0491 \end{array} \right.$	$\left\{ \begin{array}{l} 5.027 \\ 6.744 \\ 11.14 \\ 20.32 \end{array} \right.$
East or West . . .		8	90°	1.000	0.000	∞

To Find the Distances of Objects at Sea.



Height in feet.	Distance in miles.	Height in feet.	Distance in miles.	Height in feet.	Distance in miles.	Height in feet.	Distance in miles.
0.582	1	11	4.39	30	7.25	200	18.72
1	1.31	12	4.58	35	7.83	300	22.91
2	1.87	13	4.77	40	8.37	400	26.46
3	2.29	14	4.95	45	8.87	500	29.58
4	2.63	15	5.12	50	9.35	1000	32.41
5	2.96	16	5.29	60	10.25	2000	59.20
6	3.24	17	5.45	70	11.07	3000	72.50
7	3.49	18	5.61	80	11.83	4000	83.7
8	3.73	19	5.77	90	12.55	5000	93.5
9	3.96	20	5.92	100	13.23	1 mile.	96.1
10	4.18	25	6.61	150	16.22		

The distance being the tangent $a b$ in statute miles, at the elevation $a c$, in feet.

Example 1. The light-house at a is 100 feet above the level of the sea. Required the distance $a b$.

Height 100 feet = 13.23 miles.

Example 2. The flag of a ship is seen from a in d . Required the distance a, d , when the flag is known to be 50 feet above the level d' of the sea?

Height of the light 100 = 13.23 miles a, b

Height of the flag 50 = 9.35 " b, d ,

Distance to the ship = 22.58 miles a, d .

Example 3. A steamer is seen at e , the horizon b seen in the masts is assumed to be 16 feet above the level e' . Required the distance to the ship?

Height of the light 100 = 13.23 miles a, b ,

The assumed height 16 = 5.29 " $e b$,

Distance to the ship = 7.94 miles $a e$,

To Find the Distance by an Observed Angle w .

Letters denote.

d = distance in statute miles ($a e'$) to the object observed.

t = the tangent ($a b$) in statute miles, or distance to the horizon.

v = the observed angle $e a e'$, of the horizon and the leadline of the object.

r = radius of the earth.

w = the angle $b a c$.

$$\cos. w = \frac{t}{\sqrt{t^2 + r^2}},$$

$$d = \cos. (w - v) \sqrt{t^2 - r^2} - \sqrt{\cos.^2 (w - v) (t^2 + r^2) - t^2}.$$

POPULATION OF COUNTRIES AND CITIES IN THE WORLD.

<i>Names.</i>	<i>Year</i>	<i>Populat'n</i>	<i>Names.</i>	<i>Year.</i>	<i>Population</i>
NORTH AMERICA	1854	40,000,000	Patagonia		1,200,000
U. S. of America	1850	23,191,876	Chili		1,200,000
New York - - - -	1853	850,000	Santiago - - - -		80,000
Philadelphia - - - -	"	575,000	Valparaiso - - - -	1848	60,000
Baltimore - - - -	"	195,000	New Grenada		2,363,054
Cincinnati - - - -	"	160,186	Bogota - - - -		40,000
New Orleans - - - -	"	145,449	Peru	1851	2,279,085
Boston - - - -	1850	136,880	Lima - - - -	1850	100,000
Pittsburg - - - -	1853	110,241	Venezuela	1854	1,419,289
St. Louis - - - -	"	100,000	Caraccas - - - -	1853	63,000
Chicago - - - -	"	60,625	Great Britain & Ireland		27,686,609
Buffalo - - - -	"	60,000	London - - - -	1856	2,500,000
Louisville - - - -	"	51,726	Manchester - - - -	1851	401,321
Albany - - - -	"	50,763	Liverpool - - - -	"	376,065
Providence - - - -	"	47,500	Glasgow - - - -	"	347,001
Newark, N. J. - - - -	"	45,500	Dublin - - - -	"	254,850
Charleston - - - -	1850	42,985	Edinburg - - - -	"	158,015
Washington - - - -	"	40,001	France		35,779,222
Rochester - - - -	1853	40,000	Paris - - - -	1851	1,053,262
Troy - - - -	1850	28,785	Marseilles - - - -	1852	192,527
Richmond - - - -	"	27,570	Lyons - - - -	"	156,169
Savannah - - - -	1853	23,458	Spain	1849	13,936,218
San Francisco - - - -	"	60,000	Madrid - - - -	1850	260,000
British America	"	3,634,850	Barcelona - - - -		121,815
Montreal - - - -	1851	57,715	Portugal	1850	3,471,203
Toronto - - - -	1855	50,000	Lisbon - - - -	"	455,217
Quebec - - - -	1852	42,052	Belgium	1849	4,359,090
Halifax - - - -	1851	33,582	Brussels - - - -	1846	123,874
St. John - - - -	1852	22,745	Holland		3,962,290
Cuba	1853	1,009,060	Amsterdam - - - -	1852	228,800
Havana - - - -	"	147,360	Denmark		2,412,926
Santiago - - - -	"	85,242	Copenhagen - - - -	1852	133,140
Matanzas - - - -	"	81,397	Hamburg Free City		200,690
Puerto Princip - - - -	"	46,532	Bremen ditto - - - -		53,156
Hayti	"	943,000	Sweden & Norway	1850	4,810,812
Portau Prince - - - -	"	20,000	Stockholm - - - -	1855	100,000
St. Domingo - - - -	"	10,000	Gottenburg - - - -	"	30,000
Jamaica		377,433	Christiania - - - -		26,500
Kingston - - - -		35,000	Prussia	1856	17,178,091
Mexico		7,853,394	Berlin - - - -	1852	441,931
Mexico City - - - -		200,000	Austria	1850	36,514,466
Guadalaxara - - - -		70,000	Vienna - - - -	1846	407,980
La Puebla - - - -		50,000	Italy		24,733,385
San Luis Potosi - - - -		40,000	Rome - - - -	1856	177,461
Central America		2,146,000	Naples - - - -	1851	416,475
New Guatimala - - - -		50,000	Palermo - - - -	1850	167,222
SOUTH AMERICA		16,000,000	Turkey		35,360,000
Brazil		6,065,000	Constantinople - - - -		786,990
Rio Janeiro - - - -	1853	400,000	Russia	1851	60,098,821
Bahia - - - -		120,000	St. Petersburg - - - -	1852	533,241
Bolivia		1,030,000	Moscow - - - -	1840	349,068
La Paz - - - -		20,000	Odessa - - - -	1850	71,392
Equador		500,000	Sevastopol - - - -	1855	40,000
Quito - - - -		60,000	China		387,632,907
Plata		820,000	Pekin - - - -		2,000,000
Buenos Ayres - - - -	1854	85,000	Canton - - - -		1,000,000
Cordova - - - -	"	13,000			
Paraguay		1,000,000			

<i>Names of Places.</i>	<i>Latitude.</i>	<i>Longitude.</i>		<i>Names of Places.</i>	<i>Latitude.</i>	<i>Longitude.</i>		
N. AMERICA AND WEST INDIES.				GERMANY.				
Quebec, - - -	46° 49' N	71 16	West	Berlin, - - -	52 31 N	13 24	East	
Halifax, - - -	44 38 "	63 35		Bern, - - -	46 57 "	7 25		
Portland light	43 36 "	70 12		Rotterdam, -	51 54 "	4 28		
Buffalo, - - -	42 53 "	78 55		Antwerp, - -	51 13 "	4 24		
Chicago, - - -	42 0 "	87 35		Amsterdam, -	52 22 "	4 51		
Newburyport light.	42 48 "	70 49		Bremen, - - -	53 5 "	8 49		
Boston State House,	42 21 "	71 4		Hague, - - -	52 4 "	4 16		
Nantucket light,	41 23 "	70 3		Hamburg, - -	53 33 "	9 56		
Newport Custom,	41 29 "	71 19		Lubeck, - - -	53 52 "	10 49		
New York, - - -	40 42 "	74 00.7		AUSTRIA.				
Philadelphia, -	39 57 "	75 10	from	Vienna, - - -	48 13 "	16 23	from	
Cape Henlopen,	38 46 "	75 4		Venice, - - -	45 26 "	12 21		
Cincinnati, - -	39 6 "	84 27		Trieste castle,-	45 39 "	13 46		
St. Louis, - - -	38 36 "	89 36		TURKEY.				
Richmond, - - -	37 32 "	77 27		Ragusa, mole,	42 38 "	18 7		
Washington City,	38 53 "	77 0.3		Athens Philopa.,	37 58 "	23 44		
Baltimore, - - -	39 18 "	76 37		Salonica, - -	40 39 "	22 57		
Cape Hatteras,	35 14 "	75 30		Constantinople,	41 1 "	28 59		
Charleston light,	32 42 "	79 54		SWEDEN AND NORWAY.				
Savannah, - - -	32 5 "	81 8		Greenwich,	Stockholm, - -	59 21 "		18 4
Cape Florida light,	25 41 "	80 5	Gothenburg, -		57 42 "	11 57		
Pensacola, - - -	30 24 "	87 10	Christiania, -		59 55 "	10 52		
Mobile, - - -	30 42 "	87 59	Bergen, - - -		60 24 "	5 20		
New Orleans, - -	29 57 "	90 0	Wisby Gotland,		57 39 "	18 17		
Porto Rico, - - -	18 29 "	66 7	DENMARK.					
Cape Hayti's City,	19 46 "	72 11	Copenhagen, -		55 41 "	12 34		
Havana, - - -	23 9 "	82 22	Elseneur, - -		56 2 "	12 37		
Vera Cruz, - - -	19 12 "	96 9	RUSSIA.					
Mexico, - - -	19 26 "	99 5	St. Petersburg,		59 56 "	30 19		
Porto Bello, - -	9 34 "	79 40	Greenwich,	Moscow, - - -	55 46 "	35 33		
Porto Cabello,	10 28 "	68 7		Revel, - - -	59 26 "	24 46		
Cape St. August'e,	8 21 S	34 57		Riga, - - -	56 51 "	23 57		
Rio Janeiro, - -	22 56 S	43 9		Cronstadt, - -	59 58 "	29 51		
Buenos Ayres, -	34 36 "	58 22		Abo, - - -	60 27 "	22 15		
Cape Horn, - - -	55 59 "	67 16		Odessa, - - -	46 27 "	30 42		
Valparaiso Fort,	33 2 N	71 41		SPAIN.				
Panama Ft. N.E.,	8 57 "	79 31		Madrid, - - -	40 25 "	3 42 W		
San Francisco,	37 47 "	122 21		Barcelona, - -	41 23 "	2 11 E		
ENGLAND.				Algiers light, -	36 49 "	3 1 "		
London, - - -	51 31 "	0 6	Greenwich,	Gibraltar, - -	36 6 "	5 20 W		
Liverpool, - - -	53 22 "	2 52		Carthage obser.,	37 36 "	1 1 "		
Greenwich, - - -	51 29 "	0 0		PORTUGAL.				
Glasgow, - - -	55 52 "	4 16 W		Oporto, - - -	41 11 "	8 38 "		
Dublin, - - -	53 23 "	6 20		Lisbon, - - -	38 42 "	9 9 "		
Edinburgh, - - -	55 57 "	3 12		Cape St. Vincent,	37 3 "	9 2 "		
Bristol, - - -	51 27 "	2 35		SICILY.				
Dover, - - -	51 8 "	1 19 E		Messina, - - -	38 12 "	15 35 E		
FRANCE.				Palermo, - - -	38 8 "	13 22 "		
Paris Observatory,	48 50 "	2 20	Greenwich,	Malta, - - -	35 54 "	14 13 "		
Havre de Grace,	49 29 "	0 6		CHINA.				
Cherbourg, - - -	49 38 "	1 37		Peking, - - -	39 54 "	116 28 "		
Marseilles Observ.	43 18 "	5 22		Canton, - - -	23 7 "	113 14 "		
Antwerp, - - -	51 13 "	4 24		CAPE OF GOOD HOPE.				
Calais, - - -	50 58 "	1 51		Sidney, Australia,	34 0 "	151 23 "		
ITALY.				Jerusalem. Pales.,	31 48 N	37 20 "		
Florence, - - -	43° 46 "	11 16						
Leghorn, - - -	43 32 "	10 18						
Rome, St. Peter's,	41 51 "	12 27						
Naples, light, -	40 50 "	14 16						
Ancona, light,-	43 33 "	13 30						

Distances by Sea

between the most important places on the earth : in
Geographical miles, 60 per degree.

* Round Cape of Good Hope.

+ Round Cape Horn.

From San Francisco 1980 to Sandwich Islands.
From Canton, 4500 "

		Canton.									
		San Francisco					5820				
		Cape Horn.					6100				
		New Orleans.					7620				
		New York.					2129				
		St. Petersburg.					4516				
		Stockholm.					377				
		Copenhagen					495				
		Gottenburg.					139				
		Hamburg.					360				
		Antwerp.					348				
		Paris.					518				
		London.					410				
		Liverpool.					710				
		Lisbon.					900				
		Gibraltar					330				
		Rome.					1018				
		Constantinople.					1150				
		Cape Good Hope.					7320				
		Batavia					5180				
		Port Jackson					3860				

Distances in Statute Miles.

Nearest travelling distances
between Cities in Europe.

Distances in Statute Miles.	Nearest travelling distances between Cities in Europe.										London												
	Warsaw		Rome		Munich		Vienna		Turin		Berne												
	Triest	407	342	338	456	584	796	942	855	1351													
	Constantinople		1200	1000	1050	1679	1409	1456	1584	1909	2084	2160	2350	2086	2412	2704	2492						
	Odessa	750	805	1400	1907	1369	1099	1856	2000	1336	1511	1836	2750	2485	1900	2450	2238						
	Moscow		910	1660	1953	2751	3158	2683	2413	3191	2964	2489	2664	3173	4004	3739	3114	3717	3505				
	St. Petersburg		395	1200	1950	708	1506	1774	1271	1162	1742	1544	969	1144	1478	2484	2219	1594	1953	1741			
	Stockholm		440	835	1953	2775	1148	1633	1928	1291	1370	1799	1606	866	691	1082	2251	1986	1334	1626	1413		
	Copenhagen		435	875	1270	1772	2340	967	1198	1493	856	935	1364	1171	431	256	647	1816	1551	899	1191	979	
	Lubeck		234	669	1109	1504	1514	2123	709	748	1146	639	718	849	643	214	39	352	1599	1334	710	896	684

Sailing distances

between the most important seaports in the United States,
in Geographical miles, 60 per Degree.

	Boston.		Halifax.	
	107	284	390	360
	Nantucket light.			
	193	211		
	Sandy Hook.			
	18	131		
	New York.			
	149	252		
	Cape Henlopen.			
	103	234		
	Philadelphia.			
	238	403		
	Cape Henry.			
	152	555		
	Baltimore.			
	174	661		
	Washington.			
	178	923		
	Norfolk.			
	90	795		
	Richmond.			
	214	865		
	Cape Hatteras.			
	281	786		
	Charleston.			
	95	1065		
	Savannah.			
	542	1162		
	Key West.			
	85	1930		
	Havana.			
	640	2685		
	New Orleans.			
	850	2800		
	Vera Cruz.			
	1407	2638		
	Porto Bello			
	1370	2738		

ASTRONOMY.

ASTRONOMY is that branch of Philosophy which treats of the properties of heavenly bodies.

The mean Solar day is divided into 24 hours.

* A Sidereal year is - - - - - 365d 6h 9m 9.6s.

† An Anomalistic year is - - - - - 365d 6h 13m 49.3s.

× A Tropical year is - - - - - 365d 6h 13m 49.3s.

* *Sidereal year* is the time in which the Earth makes one revolution round the Sun, in reference to a fixed star.

† *Anomalistic year* is the time which the Earth occupies between each perihelion to the sun.

× *Tropical year* is the periodical return of seasons.

Mean distance from the Earth to the Sun 95000000 miles or 11992 diameters of the Earth.

Inclination of the Ecliptic to the Equinoctial $23^{\circ} 28' 40''$.

The Sun subtends an angle from the Earth of $32' 3''$.

Horizontal parallax of the Sun $8.6''$ seconds.

Velocity of a point at the Equator by the rotation of the Earth 152.5 feet per second.

Cycle of the Sun is the period of 28 years at which the days of the week return to the same days at the month.

The Moon.

Distance from the Earth to the Moon is 237000 miles, = 30 diameters of the Earth, = about 0.25 the diameter of the Sun, or the diameter of the Sun is twice the diameter of the Moon's orbit around the Earth.

Diameter of the Moon is 2160 miles, or about 0.2729 of the Earth's diameter.

Volume of the Moon compared with the Earth is 0.02024.

Density of the Moon compared with the Earth is 0.5657.

Mass of the Moon compared with the Earth is 0.011399.

Inclination of the Moon's orbit to the Ecliptic $5^{\circ} 8' 48''$.

The Moon subtends an angle from the Earth of $31' 7''$.

Mean Sidereal revolution of the Moon 27.32166 days.

Mean Synodical revolution of the Moon 29.5305887 days.

The Moon passes the meridian in periods of 24.814 hours, or 48m 50s later every day.

Moon's Age is the number of days from the last new-moon.

Number of Months for the Moon's Age.

January, 0,	April, 2,	July, 5,	October, 8,
February, 2,	May, 3,	August, 6,	November, 10,
March, 1,	June, 4,	September, 8,	December, 10.

To Find the Moon's Age on any given day.

RULE. Add together the day of the month, Epact for the year, and the Tabular number for the month, the sum will be the moon's age. If it exceed 30, reject 30's, and the remainder will be the moon's age.

Example. Find out if it will be moonlight at Christmas, 1855?

Number of day = 25 in December

Epact for 1855 (see Table) = 12

Tabular number of December = 10

$47 - 30 = 17$ the moon's age

or in its third quarter, consequently moon-light at mid-night.

Golden Number or Lunar Cycle

is the period of 19 years at which the changes of the moon fall on the same days of the month.

To find the Golden number.

RULE. Add one to the given year, divide the sum by 19, and the remainder will be the Golden number.

<i>Yrs.</i>	<i>Dags.</i>	<i>Dom.</i> <i>let-</i> <i>ter.</i>	<i>Epoct.</i>	<i>Yrs.</i>	<i>Days.</i>	<i>Dom.</i> <i>let-</i> <i>ter.</i>	<i>Epoct.</i>	<i>Yrs.</i>	<i>Days.</i>	<i>Dom.</i> <i>let-</i> <i>ter.</i>	<i>Epoct.</i>
1800	Saturd.*	FE	4	1834	Saturd.	E	20	1868	Sund.*	ED	6
1801	Sunday.	D	15	1835	Sunday.	D	1	1869	Monday.	C	17
1802	Monday.	C	26	1836	Tuesd.*	CB	12	1870	Tuesd.	B	28
1803	Tuesday	B	7	1837	Wedsd.	A	23	1871	Wedsd.	A	9
1804	Thursd*	AG	18	1838	Thursd.	G	4	1872	Friday*	GF	20
1805	Friday.	F	29	1839	Friday.	F	15	1873	Saturd.	E	1
1806	Saturd.	E	11	1840	Sund.*	ED	26	1874	Sunday.	D	12
1807	Sunday.	D	22	1841	Monday.	C	7	1875	Monday.	C	23
1808	Tuesd.*	CB	3	1842	Tuesd.	B	18	1876	Wedsd.*	BA	4
1809	Wedns.	A	14	1843	Wedsd.	A	29	1877	Thursd.	G	15
1810	Thursd.	G	25	1844	Friday*	GF	11	1878	Friday.	F	26
1811	Friday.	F	6	1845	Saturd.	E	22	1879	Saturd.	E	7
1812	Sunday*	ED	17	1846	Sunday.	D	3	1880	Mond.*	DC	18
1813	Monday.	C	28	1847	Monday.	C	14	1881	Tuesd.	B	29
1814	Tuesd.	B	9	1848	Wedsd.*	BA	25	1882	Wedsd.	A	11
1815	Wedns.	A	20	1849	Thursd.	G	6	1883	Thurs.	G	22
1816	Friday*	GF	1	1850	Friday.	F	17	1884	Saturd.*	FE	3
1817	Saturd.	E	12	1851	Saturd.	E	28	1885	Sunday.	D	14
1818	Sunday.	D	23	1852	Mond.*	DC	9	1886	Monday.	C	25
1819	Monday.	C	4	1853	Tuesd.	B	20	1887	Tuesd.	B	6
1820	Weds.*	BA	15	1854	Wedsd.	A	1	1888	Thurs.*	AG	17
1821	Thursd.	G	26	1855	Thursd.	G	12	1889	Friday.	F	28
1822	Friday.	F	7	1856	Saturd.*	FE	23	1890	Saturd.	E	9
1823	Saturd.	E	18	1857	Sunday.	D	4	1891	Sunday.	D	20
1824	Monda*.	DC	29	1858	Monday.	C	15	1892	Tuesd.*	CB	1
1825	Tuesd.	B	11	1859	Tuesd.	B	26	1893	Wedsd.	A	12
1826	Wedsd.	A	22	1860	Thurs.*	AG	7	1894	Thursd.	G	23
1827	Thursd.	G	3	1861	Friday.	F	18	1895	Friday.	F	4
1828	Saturd.*	FE	14	1862	Saturd.	E	29	1896	Sund.*	ED	15
1829	Sunday.	D	25	1863	Sunday.	D	11	1897	Monday.	C	26
1830	Monday.	C	6	1864	Tuesd*.	CB	22	1898	Tuesd.	B	7
1831	Tuesd.	B	17	1865	Wedsd.	A	3	1899	Thurs.	A	18
1832	Thurs.*	AG	23	1866	Thursd.	G	14	1900	Friday*.	GF	29
1833	Friday.	F	9	1867	Friday.	F	25				

In Leap years take January, February.**

<i>February,</i> <i>March,</i> <i>*November.</i>	<i>February*</i> <i>August.</i>	<i>May.</i>	<i>January</i> <i>October.</i>	<i>January,*</i> <i>April,</i> <i>July.</i>	<i>September</i> <i>December.</i>	<i>June.</i>
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Example. On what day in the week will the fourth of July fall in the year 1868?

See Table 1868 = Sunday*. In the Table of months, July column, Sunday is on the 5th; consequently the fourth falls on Saturday.

To Find the Latitude of a place by the Meridian Altitude of the Sun,

Letters denote.

A = meridian altitude of the sun's centre above the horizon, in degrees and minutes. (At sea the sun's lower limb is generally observed, then for corrections of semi-diameter dip of horizon, parallax, add 12 minutes to the observed altitude, and the sun will be the centre altitude very nearly.)

D = declination of the sun at the time of observation, to be found on page 278.

l = latitude of the place of observation.

$$l = 90 - A \pm D, \quad - \quad - \quad - \quad 1,$$

$$A = 90 - l \pm D, \quad - \quad - \quad - \quad 2,$$

$$D = \mp 90 \pm A \pm l, \quad - \quad - \quad - \quad 3,$$

Where the quantities have double signs, *plus* and *minus*, use the upper one when the latitude and declination are of *equal names*; and the lower one when the latitude and declination are of *different names*.

Example. On the 25th day of October, 1853, in north latitude was observed the sun's meridian centre altitude to be $A = 37^\circ 53'$; the declination on that day was $D = 12^\circ 10'$ south. Required the latitude?

$$l = 90 - 37^\circ 53' - 12^\circ 10' = 39^\circ 57' \text{ the latitude of Philadelphia.}$$

To Find the Time when the Sun Rises or Sets.

Let v be the angle of time before or after six o'clock when the sun rises or sets; this angle divided by 15 and added to, or subtracted from six o'clock will be the true time when the sun rises or sets.

$$\sin.v = \tan.D \tan.l, \quad - \quad - \quad - \quad 4,$$

Example. What time does the sun rise and set, on the 27th day of July, 1854, in north latitude $l = 42^\circ 6'$?

$$\text{Sun's declination } \begin{cases} D = 19^\circ 12' \text{ in the morning.} \\ D = 19^\circ 5' \text{ in the evening.} \end{cases} \text{ North.}$$

$$\sin.v = \tan.19^\circ 12' \times \tan.42^\circ 6' = 0.31454 \text{ or } v = 18^\circ 20'.$$

$$18^\circ 20' : 15 = 1^h 13^m 20^s \text{ subtract from } 6^h$$

$$5^h 59^m 60^s$$

$$\text{Sun rises at } 4^h 46^m 40^s \text{ in the morning.}$$

$$\sin.v = \tan.19^\circ 5' \times \tan.42^\circ 6' = 0.312611 \text{ or } v = 18^\circ 13'.$$

$$6^h$$

$$18^\circ 13' : 15 = 1^h 12^m 56^s \text{ add to } 6^h$$

$$\text{Sun sets at } 7^h 12^m 56^s$$

To Find the Length of Day and Night.

RULE. Double the time when the sun sets is the length of the day.

RULE. Double the time when the sun rises is the length of the night.

To Find the apparent Time by an Altitude of the Sun.

Let L be the angle of time from 12 o'clock, (noon,) when the sun's altitude a is observed,

a = the observed altitude of the sun, (if the sun's lower limb is observed add 12' for corrections),

A and v , same as in the preceding examples.

$$\cos.L = \frac{\sin.a(1 \pm \sin.v)}{\sin.A} \mp \sin.v, \quad - \quad - \quad 5,$$

The sign + or - is to be used as before described.

Example. On the 11th of May, 1853, the sun's altitude in the afternoon was observed to be $a = 42^\circ 30'$, in the latitude $l = 33^\circ 40'$; the sun's declination at the time of observation was $D = 18^\circ N$. Required the apparent time.

$$\sin.v = \tan.18^\circ \times \tan.33^\circ 40' = 0.21642, \quad - \quad 4,$$

$$A = 90 - 33^\circ 40' + 18 = 74^\circ 20' \quad - \quad 2,$$

$$\cos.L = \frac{\sin.42^\circ 30' (1 + 0.21642)}{\sin.74^\circ 20'} - 0.21642 = 0.63709, \quad 5,$$

or $50^\circ 25'$ is the angle L , $\frac{50^\circ 25'}{15} = 3^h 21^m 40^s$, the apparent time of observation.

If the altitude is taken in the forenoon, subtract the obtained time from 12^h and the remainder is the apparent time.

<i>Planets.</i>	<i>Sigs.</i>	<i>Mean distance from the Sun.</i>	<i>Mean Sidereal period in mean Solar days.</i>	<i>Eccentricity in parts of the semi-axis.</i>	<i>Velocity in the orbit in miles per second.</i>	<i>Diameter in miles.</i>	<i>Density.</i>	<i>Mass.</i>	<i>Volume.</i>	<i>Time of rotations in hours.</i>
Sun,	☉	0.3870981	87.969258	0.2055149	30.4	882000	0.25	344000	1378000	607, 48 ^m
Mercury,	☿	0.3870981	87.969258	0.2055149	30.4	3140	1.12	0.06966	0.06218	24 5
Venus,	♀	0.7233316	224.70078	0.0068607	22.3	7806	0.92	0.877	0.9531	23 21
Earth,	♁	1.0000000	365.25636	0.0167836	18.9	7926	1.00	1.000	1.000	24 0
Mars,	♂	1.5236293	686.97964	0.0933070	15.33	4100	0.95	0.1313	0.1384	24 37
Flora,		2.2016870	1153.249	0.1565570	12.77	250?			0.000031	
Vesta,	♁	2.3610810	1325.147	0.0895694	12.30					
Iris,		2.3806240	1341.636	0.2299424	12.25					
Metis,		2.3856070	1345.85	0.1202532	12.25					
Juno,	♂	2.6708370	1594.296	0.2548847	11.57	79?				27 0?
Ceres,	♀	2.7680510	1682.125	0.0766523	11.8	163			0.000012	
Pallas,	♂	2.7728580	1686.51	0.2398150	11.35					
Jupiter,	♃	5.202776	4332.5848	0.0481621	8.31	87000	0.24	317.5	1332.5	9 56
Saturn,	♄	9.5387861	10759.2198	0.0561505	6.14	79160	0.14	139.5	996.2	10 29
Uranus,	♅	19.18239	30686.3208	0.0466794	4.33	34500	0.24	198.0	82.47	9 30
Neptune.	♆	30.0368	60126.71	0.0387195	3.45	41500	0.14	20.	143.5	

SUN'S DECLINATION.

Days.	JAN. Sou.	FEB. Sou.	MAR. Sou.	APR. Nor.	MAY Nor.	JUNE Nor.	JULY Nor.	AUG. Nor.	SEP. Nor.	OCT. Sou.	NOV. Sou.	DEC. Sou.	Days.
1	22 59	16 59	7 26	4 41	15 11	22 7	23 6	17 58	8 10	3 19	14 34	21 53	1
2	22 53	16 42	7 3	5 4	15 29	22 15	23 2	17 42	7 48	3 43	14 53	22 2	2
3	22 47	16 24	6 40	5 27	15 47	22 22	22 57	17 27	7 26	4 6	15 11	22 11	3
4	22 41	16 5	6 17	5 50	16 4	22 29	22 52	17 11	7 4	4 29	15 30	22 19	4
5	22 34	15 48	5 54	6 13	16 21	22 36	22 46	16 55	6 42	4 52	15 48	22 27	5
6	22 27	15 29	5 30	6 36	16 38	22 42	22 40	16 38	6 20	5 15	16 6	22 34	6
7	22 19	15 11	5 7	6 58	16 55	22 48	22 34	16 21	5 57	5 38	16 24	22 41	7
8	22 11	14 52	4 44	7 21	17 11	22 53	22 27	16 4	5 35	9 1	16 41	22 47	8
9	22 3	14 32	4 20	7 43	17 27	22 59	22 20	15 47	5 12	6 24	16 59	22 53	9
10	21 54	14 13	3 57	8 5	17 43	23 3	22 13	15 29	4 49	6 47	17 16	22 58	10
11	21 44	13 53	3 33	8 27	17 58	23 7	22 5	15 12	4 26	7 10	17 32	23 3	11
12	21 35	13 33	3 10	8 49	18 13	23 11	21 56	14 54	4 3	7 32	17 48	23 8	12
13	21 24	13 13	2 46	9 11	18 28	23 15	21 48	14 35	3 40	7 55	18 4	23 12	13
14	21 14	12 53	2 22	9 32	18 43	23 18	21 39	14 17	3 17	8 17	18 20	23 15	14
15	21 3	12 32	1 59	9 54	18 57	23 20	21 29	13 58	2 54	8 39	18 36	23 18	15
16	20 51	12 11	1 35	10 15	19 11	23 23	21 20	13 39	2 31	9 1	18 51	23 21	16
17	20 39	11 50	1 11	10 36	19 24	23 24	21 9	13 20	2 8	9 23	19 5	23 23	17
18	20 27	11 29	0 48	10 57	19 37	23 26	20 59	13 1	1 45	9 45	19 20	23 25	18
19	20 15	11 8	0 24	11 18	19 50	23 27	20 48	12 41	1 21	10 7	19 34	23 26	19
20	20 2	10 46	0 0	11 38	20 03	23 27	20 37	12 21	0 58	10 29	19 47	23 27	20
21	19 49	10 24	0 23	11 59	20 15	23 27	20 25	12 1	0 35	10 50	20 1	23 27	21
22	19 35	10 3	0 47	12 19	20 27	23 27	20 13	11 41	0 11	11 11	20 13	23 27	22
23	19 21	9 41	1 11	12 39	20 40	23 26	20 2	11 21	0 12	11 32	20 26	23 27	23
24	19 7	9 18	1 34	12 59	20 51	23 25	19 48	11 0	0 36	11 53	20 38	23 25	24
25	18 52	8 56	1 58	13 18	21 1	23 24	19 37	10 40	0 59	12 14	20 50	23 24	25
26	18 37	8 34	2 21	13 38	21 12	23 22	19 24	10 19	1 22	12 35	21 1	23 22	26
27	18 21	8 11	2 45	13 57	21 22	23 20	19 10	9 58	1 46	12 55	21 12	23 19	27
28	18 6	7 49	3 8	14 16	21 32	23 17	18 56	9 37	2 9	13 15	21 24	23 16	28
29	17 49		3 32	14 34	21 41	23 14	18 42	9 15	2 33	13 35	21 34	23 13	29
30	17 33		3 55	14 53	21 50	23 10	18 28	8 54	2 56	13 55	21 44	23 9	30
31	17 16		4 18		21 59		18 13	8 32		14 14		23 5	31

Declination of the Sun and equation of time for the years

Leap years 1852,	56,	60,	64,	in New York at 6h a.m.
1853,	57,	61,	1865,	" " apparent noon.
1854,	58,	62,	&c.	" " 6h p.m.
1855,	59,	63,	&c.	" " 12h midnight.

Example 1. Required the sun's declination in New York at 10 o'clock a.m. on the 13th of April, 1856?

From the table April 13 declin. $9^{\circ} 11' N.$

" " " " 14 " 9 32

Difference 21

Dec. $9^{\circ} 12'$

Correction 21 (10—6) : 24 — $3\frac{1}{2}$ add.

Required declination $9^{\circ} 15' 30''$ the answer.

NOTE. In leap years the declination and equation of time must be taken one day earlier in the tables for January, and February; for instance, declination on the 29th of February 1856 is $7^{\circ} 49'$.

EQUATION OF TIME.

Days:	JAN. Add	FEB. Add	MAR. Add	APR. Add	MAY Sub.	JUNE Sub.	JULY Add	AUG. Add	SEP. Sub.	OCT. Sub.	NOV. Sub.	DEC. Sub.	Days:
1	4 4	13 58	12 31	3 52	3 4	2 28	3 31	6 0	0 13	10 24	16 16	10 37	1
2	4 33	14 5	12 20	3 34	3 11	2 19	3 42	5 56	0 32	10 43	16 17	10 14	2
3	4 59	14 11	12 7	3 17	3 18	2 9	3 53	5 52	0 51	11 2	16 17	9 50	3
4	5 27	14 17	11 54	2 59	3 24	1 59	4 4	5 47	1 11	11 20	16 16	9 25	4
5	5 54	14 22	11 40	2 41	3 29	1 49	4 15	5 41	1 30	11 37	16 14	9 0	5
6	6 20	14 26	11 26	2 24	3 34	1 38	4 25	5 35	1 50	11 55	16 11	8 35	6
7	6 46	14 29	11 12	2 7	3 38	1 27	4 35	5 28	2 10	12 12	16 8	8 9	7
8	7 12	14 31	10 57	1 50	3 42	1 16	4 44	5 20	2 31	12 29	16 4	7 43	8
9	7 37	14 33	10 41	1 32	3 45	1 4	4 53	5 12	2 51	12 45	15 58	7 16	9
10	8 1	14 33	10 26	1 16	3 48	0 53	5 2	5 3	3 12	13 1	15 52	6 48	10
11	8 25	14 33	10 10	1 0	3 50	0 41	5 10	4 54	3 33	13 16	15 46	6 21	11
12	8 48	14 32	9 53	0 42	3 51	0 29	5 17	4 44	3 54	13 31	15 38	5 53	12
13	9 10	14 31	9 37	0 26	3 52	0 16	5 25	4 34	4 15	13 45	15 30	5 24	13
14	9 32	14 28	9 20	0 11	3 53	0 4	5 31	4 23	4 36	13 59	15 20	4 56	14
15	9 53	14 25	9 3	0 8	3 53	0 A 9	5 38	4 11	4 57	14 12	15 10	4 27	15
16	10 14	14 21	8 45	0 17	3 52	0 21	5 43	3 59	5 18	14 25	14 59	3 57	16
17	10 33	14 16	8 25	0 31	3 51	0 34	5 48	3 46	5 40	14 37	14 47	3 28	17
18	10 52	14 11	8 10	0 45	3 49	0 47	5 53	3 33	6 1	14 48	14 34	2 58	18
19	11 11	14 5	7 52	0 58	3 46	1 0	5 57	3 20	6 22	14 59	14 21	2 28	19
20	11 28	13 58	7 33	1 12	3 44	1 13	6 1	3 6	6 43	15 9	14 6	1 58	20
21	11 45	13 51	7 15	1 24	3 40	1 26	6 4	2 51	7 4	15 20	13 51	1 28	21
22	12 1	13 43	6 57	1 37	3 36	1 39	6 6	2 36	7 25	15 29	13 35	0 58	22
23	12 16	13 35	6 38	1 48	3 31	1 52	6 8	2 21	7 45	15 37	13 18	0 28	23
24	12 30	13 25	6 20	2 0	3 26	2 4	6 10	2 5	8 6	15 44	13 0	A 0 4	24
25	12 44	13 16	6 1	2 10	3 22	2 17	6 11	1 49	8 26	15 51	12 42	0 32	25
26	12 57	13 5	5 43	2 21	3 15	2 30	6 11	1 33	8 47	15 57	12 22	1 2	26
27	13 10	12 55	5 24	2 30	3 9	2 42	6 11	1 16	9 7	16 2	12 2	1 32	27
28	13 21	12 43	5 6	2 40	3 2	2 55	6 10	0 58	9 26	16 6	11 42	2 1	28
29	13 31		4 47	2 48	2 54	3 7	6 8	0 41	9 46	16 10	11 20	2 31	29
30	13 41		4 29	2 56	2 46	3 19	6 6	0 23	10 5	16 13	10 58	3 0	30
31	13 50		4 11		2 37		6 4	0 5		16 15		3 28	31

Example 2. Required the sun's declination in San Francisco at 4h 46m p.m., on the 5th of Aug. 1855.

From the table Aug. 5 declin. $16^{\circ} 55' N.$

" " " " 6 " $16^{\circ} 38'$

Difference 17

Long. of San Francisco 122°

" New York 74

$48 : 15 = 3h 12m$ difference in time.

Time in S. F 4 46

Midnight 12h — $8h = 4h \times 17 : 24 = 3'$ nearly.

$16^{\circ} 45'$

Correction add 3

Required declination $16^{\circ} 48'$ the answer.

Example 3. On the 5th day of Nov. 1857, at 2h 21m 56s apparent time p.m. Required the mean time?

Apparent time 2h 21m 56s

Equation of time, sub. 16 14

Mean time

2h 5m 42s the answer.

NOTE. Add the equation of time to or subtract from the apparent time, is the mean time.

EPACT OF THE YEAR.

d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.
1850	1851	1852	1853	1854	1855	1856	1857	1858	1859	1860	1861
17 17	28 8	9 10	20 1	1 3	11 18	23 10	4 12	15 3	25 17	7 21	18 12
d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.
1862	1863	1864	1865	1866	1867	1868	1869	1870	1871	1872	1873
29 3	10 6	21 21	2 23	13 15	24 6	6 8	16 23	27 18	8 20	19 15	0 17








EPACT OF THE MONTH.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
0 0	1 17	0 4	1 16	2 3	3 14	4 2	5 13	7	7 11	8 23	9 10

To the time of high water, or the time when the moon passes the meridian, add two minutes for every exceeding hour of the moon's age, for corrections.

MOON'S POSITION.

HIGH WATER IN DIFFERENT PLACES.

Moon.			High water in N. York.		Add to, or subtract from the time of high water in N. Y. is the time of high water in the desired place.		Rise Ft.
Q't'r'r	Face	Age d.	South. h. m.	h. m.			
New		0	12 0p.m.	8 37a.m.	New York		5
1st quarter		1	12 49 "	9 21 "	Quebec	add	17
		2	1 38 "	10 2 "	Halifax	sub.	8
		3	2 26 "	10 40 "	Boston	add	12
		4	3 26 "	11 16 "	New Haven	add	17
		5	4 4 "	11 54 "	Portsmouth	add	10
Half		6	4 55 "	12 36p.m.	Providence	sub.	0 41
		7	6 42 "	1 23 "	Albany	add	6 34
		8	6 30 "	2 16 "	Amboy	sub.	0 39
		9	7 19 "	3 16 "	Sandy Hook	sub.	1 8
		10	8 8 "	4 14 "	Philadelphia	add	5 15
2d quarter		11	8 57 "	5 9 "	Cape Henlopen	add	0 35
		12	9 46 "	6 0 "	Baltimore	sub.	4 14
		13	10 34 "	6 47 "	Cape Henry	add	0 57
		14	11 23 "	7 29 "	Washington	sub.	4 8
		15	12 12a.m.	8 7 "	Norfolk	sub.	0 52
Full		16	1 1 "	8 43 "	Charleston	sub.	0 22
		17	1 50 "	9 21 "	Key West	add	1 16
		18	2 38 "	10 4 "	Havana	add	1 35
		19	3 27 "	10 51 "	Rio Janeiro	sub.	6 35
		20	4 16 "	11 42 "	Buenos Ayres		0 0
3d quarter		21	5 5 "	12 37a.m.	Cape Horn	sub.	3 57
		22	5 54 "	1 37 "	Valparaiso	add	0 5
		23	6 42 "	2 39 "	San Francisco	add	2 23
		24	7 31 "	3 42 "	Liverpool	add	2 39
		25	8 20 "	4 43 "	London	sub.	6 30
Half		26	9 9 "	5 41 "	Hull	sub.	2 37
		27	9 58 "	6 36 "	Bremen	add	2 37
		28	10 46 "	7 27 "	Lisbon	sub.	4 37
		29	11 35 "	8 14 "	Cape Good Hope	sub.	5 37
		29 1/2	12 "	8 37 "			

Moon's Age. High Water. Moon South.

Example 1. Required the moon's age on the 25th day of September, 1855?

Date in September	25d	} add
Epact of September	7d	
Epact of 1855	11d 18h	

Reject 30 13d 18h is the moon's age at noon in New York.

Example 2. Required the time of high water in New York, and at what time the moon is south, on the same date as in preceding example?

In the annexed table we have given the moon's

Age.	South.	High water.
13 days	10h 34m	6h 47m
Correction add $18h \times 2 =$	36m	36m

Moon south at 11h 10m, and 7h 23m is the time of high water in New York.

Example 3. Find the time of high water in San Francisco on the 18th of July, 1856.

Date in July	18d	} add
Epact of July	3d 26h	
Epact of 1856	23d 10h	
Reject 30	<u>15d 12h</u>	the moon's age.

In the table	Age.	High water.
	15	8h 7m
Correction add $12h \times 2$		24m
For San Francisco add		<u>2h 23m</u>

Time of high water in San Francisco 10h 54m, July 18, 1856.

The strength and direction of the wind sometimes accelerates and sometimes retards the tide, in consequence the most careful and scientific calculations may differ an hour from the time of high water.

SOUNDINGS,

To Reduce Soundings to Low Water.

Letters denote.

T = time in hours between high and low water.

t = time in hours from low water to the time when the soundings are taken.

H = vertical rise of tide in feet from high to low water.

h = reduction of the sounding taken at the time, t , in feet.

$$v = \frac{180 t}{T}, \quad \text{and } h = \frac{1}{2} H (1 \mp \cos. v),$$

— $\cos. v$ when $v < 90$

+ $\cos. v$ when $v > 90$

Example. High water at 10h 15m p.m.

Low water at 3h 45m „

Time $T = 6h 30m$ „

The sounding taken at 5h 30m „ was 16 feet 6 inches.

Time $t = 1h 45m$

Vertical rise $H = 9.75$ feet.

Required the reduction $h = ?$ and true sounding at low water.

$$v = \frac{180 \times 1.75}{6.5} = 48^\circ 30', \quad \cos. v = 0.66262.$$

Reduction $h = \frac{1}{2} \times 9.75 (1 - 0.66262) = 1.6447$ feet.

Sounding taken at 5h 30m was 16.5 feet.

Reduction subtract $h = 1.6447$

True sounding at low water 14.8553 feet.

1

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z
a b c d e f g h i j k l m n o p q r s t
u v w x y z 1 2 3 4 5 6 7 8 9 0

2

A B C D E F G H I J K L M N O P Q R S
T U V W X Y Z.

3

A B C D E F G H I J K L M O P Q R S T U V W X Y Z

4

A B C D E F G H I J K L M N O P Q
R S T U V W X Y Z.

5

A B C D E F G H I J K L M N O P Q R S T U V
W X Y Z.

DEAF AND DUMB ALPHABET.

Keep your hand horizontal for the letters *a, g, h, r, s* and *t*.

For *j* and *z* describe the letter with the finger in the air.

For *x* make a motion up and down with the index finger.



9

A B C D E F G H I J K L M N O
P Q R S T U V W X Y Z
a b c d e f g h i j k l m n o p q r s t u v w x y z.

10

A B C D E F G H I J K L M N O P Q R S T
U V W X Y Z a b c d e f g h i j k l m n o p q r s t u v w x y z.

11

A B C D E F G H I J K L M N O P
Q R S T U V W X Y Z
a b c d e f g h i j k l m n o p q r s t
u v w x y z.

12

A B C D E F G H I J K L M N O P Q R S T
U V W X Y Z a b c d e f g h i j k l m n o p q r s t u v w x y z.

13

A B C D E F G H I J K L M N O P Q R S T U V W X
Y Z a b c d e f g h i j k l m n o p q r s t u v w x y z

14

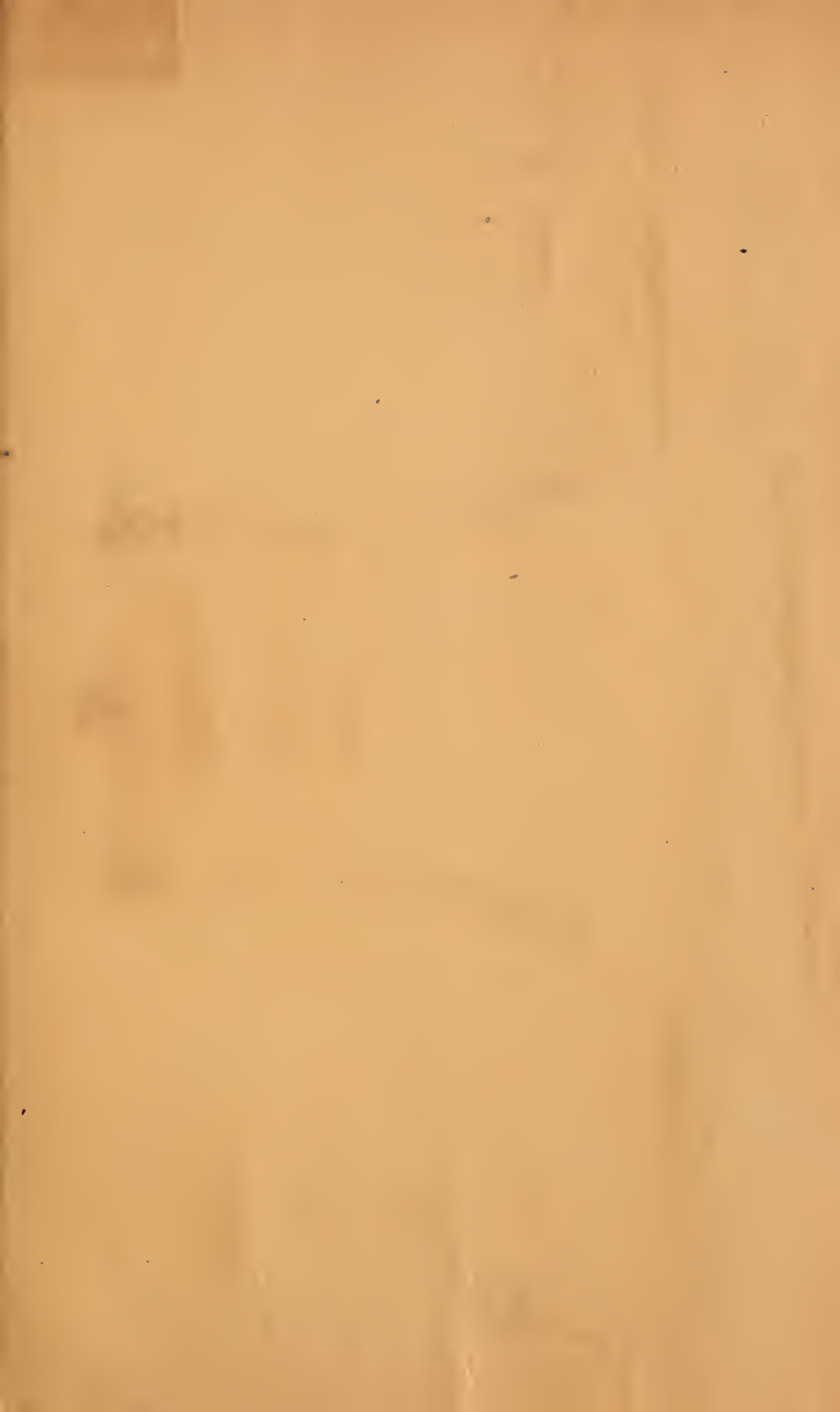
A B C D E F G H I J K L M N O P Q R S
T U V W X Y Z. 1 2 3 4 5 6 7 8 9 0
a b c d e f g h i j k l m n o p q r s t u v w x y z.

15

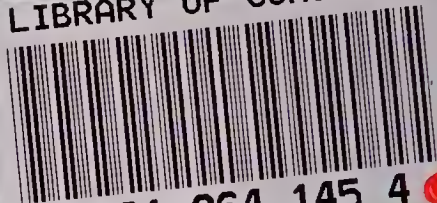
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

16

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z.



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